On the identification of a distribution for the arrivals to a queueing system: an application from bank data

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Abstract. Queues are an integral part of life. The literature available concentrates on Poisson arrival and exponential service time. The present paper focuses on statistical inference on the parameters of queueing models. The work is data driven. The dataset considered is the queueing system in banks which is publicly available. The number of servers has been estimated for a steady state system. The data has been tested for normality. The goodness of fit test has been done for the arrival pattern, and it is found that the discrete analog of smallest extreme value distribution is a good fit to the data which is an alternative to the widely used Poisson distribution.

Keywords: arrival time, model identification, queue, service time, waiting time

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1. Introduction

Queues are formed when there is heavy demand for service. The characteristics of a queue are the arrival pattern, the service rate, the system capacity and the queue discipline. The widely used distribution for arrival pattern is the Poisson distribution, and for the service rate, it is the exponential distribution. The system capacity may be finite or infinite, and the queue discipline may be first come first served or based on priority. These characteristics play a vital role in the waiting time of the customers, and consequently the queue length. The customers who need service may be either animate or inanimate. Extensive work on queueing theory can be found in Bhat [3], Gross et al. [8] and the references therein.

The recent pandemic COVID 19 was a period of crisis for the entire world. The application of queueing theory was conspicuously seen in the health sector. This is not a surprise as its history races back to Wolff [16] where the author discusses the problems of statistical inference for birth and death models. Besides the health sector, the effect was seen in banking and economy too despite digitalization. Dehmi et al. [6], Sreelatha et al. [13] and Ramesh and Manoharan [11] are some of the recent works in this direction. Adaptive Neuro-Fuzzy Inference System (ANFIS) was used to compute and optimize the cost of a queueing system [6]. A high performing probabilistic model was proposed for systems with dynamic service facility [13]. Comparison of blocking mechanisms with capacity restrictions was carried out in a two-station tandem network queueing model [11].

Some of the works related to the banking sector are that by Afolalu et.al. [1], Al-Jumaily and Al-Jobori [2], Cowdrey et al. [5], Eze and Odunukwe [7], Joel and Augustine [9], Sy et al.

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[15] and Yifter et al. [17]. An overview of applications of queueing models is given in [1], while development of automatic queueing system based on different queueing disciplines was carried out by [2]. Also, Markovian queueing model was applied to analyze customer management, optimize the waiting times and, improve service quality ([5], [7], [9], [15]). Recently, assessment of quality of services was examined using modeling and simulation [17].

Sampling design is an important framework for any probability or stochastic modeling. This includes estimation and inference of the parameters of the model. The terminology used for the process is known as identification. Queueing model is a stochastic model and the literature available on it widely focuses on Poisson distribution for the arrival of customers. Since the present-day studies are evident based or data driven, we decided to focus on fitting a probability distribution to an empirical data for validating whether Poisson distribution is always a good fit to the arrival pattern. The data used are available in Bishop et al. [4] on the arrivals of customers to three banks in Nigeria.

The main contribution of the present work is

- Estimation of the number of servers required at steady state
- Fitting a model to the arrival pattern of customers to a bank and
- Testing for goodness of fit of the model

The organization of the paper is as follows. Section 2 discusses the means of obtaining the various performance measures of a queueing system. Section 3 gives a brief description of the data. The identification of the distribution to the arrival pattern is done using goodness of fit test. This is shown in section 4. Section 5 gives the results of the analysis. The conclusion is given in section 6.

2. Performance measures of a queueing system

The following measures need to be considered in any general queueing system.

Let $N_q(t)$ and $N_s(t)$ denote respectively, the number of customers in the queue and the number at service, at time t, $t \geq 0$. Then, the total number of customers in the system is given by $N(t) = N_q(t) + N_s(t)$, $t \geq 0$.

Let $p_n(t) = P(N(t) = n)$ be the probability that there are n customers in the system at time t, $t \geq 0$, and its steady state probability be $p_n = P(N = n)$. Then, the expected number of customers in the system at steady state is $L = E(N) = \sum_{n=0}^{\infty} p_n$ and that in the queue is $L_q = E(N_q) = \sum_{n=s+1}^{\infty}(n-s)p_n$. Here, s represents the number of servers. Thus, the expected number of customers in service in the steady state is $E(N_S) = L - L_q$.

Let T_q be the time a customer spends waiting in the queue prior to entering service and T be the total time a customer spends in the system. Then $T = T_q + S$, where S is the service time. Since T, T_q and S are random variables, the mean waiting time in the queue is $W_q = E(T_q)$ and that in the system is $W = E(T) = E(T_q) + E(S)$.

The relationship between waiting time and queue length was given by J. D. C. Little. The equations are $L = \alpha W$ and $L_q = \alpha W_q$. Here, α is the average arrival rate of customers.

If β is the average service rate, then $\rho = \alpha/s\beta$ is a measure of traffic congestion. This is also known as traffic intensity. The system is said to be in a steady state when $\rho < 1$, i.e., the arrival rate is less than the service rate. Another important terminology in queueing theory is the busy period. This is the time from when a customer enters an empty system until it next empties out again. The empirical expression for arrival rate may also be written as $\alpha = \frac{N_c}{T}$, where N_c is the number of customers arriving over a period $(0, T)$.

Theoretically, the expression for p_n may be derived from the differential-difference equations of the general birth-death model. The equations are as follows.

$$\frac{dp_n(t)}{dt} = -(\alpha_n + \beta_n)p_n(t) + \alpha_{n-1}p_{n-1}(t) + \beta_{n+1}p_{n+1}(t), \;\; n \geq 1 \tag{1}$$

and

$$\frac{dp_0(t)}{dt} = -\alpha_0 p_0(t) + \beta_1 p_1(t) \tag{2}$$

where, α_n and β_n are the transition rates of birth and death respectively and n is the population size. Here, $p_n(t)$ is the unconditional probability that the system is in state n at time t, $t \geq 0$. Equations (1) and (2) are forms of the Chapman-Kolmogorov forward equations. Further information on these system of differential-difference equations are provided in [3].

The steady state solutions for the equations (1) and (2) are

$$p_n = p_0 \prod_{i=1}^{n} \frac{\alpha_{i-1}}{\beta_i}, \;\; n \geq 1 \tag{3}$$

and

$$p_0 = \left[1 + \sum_{n=1}^{\infty} \prod_{i=1}^{n} \frac{\alpha_{i-1}}{\beta_i}\right]^{-1} \tag{4}$$

respectively. Here, p_n is the probability of having n individuals in the system and p_0 is the probability that the system is empty initially. More precisely, the limiting distribution of the state of the birth-and-death queueing model are $\{p_n, \, n = 0, 1, \cdots\}$ and that $\{p_n, \, n = 0, 1, \cdots\}$ are nonzero if and only if $1 + \sum_{n=1}^{\infty} \prod_{i=1}^{n} \frac{\alpha_{i-1}}{\beta_i} < \infty$. Also, the normalizing condition $\sum_{n=1}^{\infty} p_n = 1$ holds.

If $\alpha_n = \alpha$ and $\beta_n = \beta$, the general birth-and-death model reduces to the simplest queueing model $M/M/s$, where M stands for Markovian. The arrival process Poisson and service process exponential are both Markovian. Thus, for a single server system, the probability of having no customer in the system is $p_0 = (1 - \frac{\alpha}{\beta})$ and the probability of having n customers in the system is $p_n = (1 - \frac{\alpha}{\beta}) \left(\frac{\alpha}{\beta}\right)^n$, $n \geq 1$. In this case, the traffic intensity will be $\rho = \frac{\alpha}{\beta}$, and system works in the steady state if $0 < \rho < 1$.

The details may be found in standard textbooks such as Bhat [3] and Gross et al. [8].

3. Data description

The dataset from Bishop et al. [4] consists of the number of customers arriving to three different banks of three different urban areas in Ogun State, Nigeria. The data give the number of customers arriving on twenty different days in four weeks. Let this be denoted as N. At each bank, the time range of a customer's arrival, the time his/her cheque/withdrawal booklet was collected, the time used to process it and the total time spent by the customers in the bank were recorded. Let these random variables be denoted respectively as W_q, S and W. Thus, using the terminologies of the queueing theory, W_q represents the waiting time of customers in queue, S the service time, and W the total time spent by the customer in the bank. The observations are made for seven hours each in all three banks.

The number of arrivals on twenty days to the three banks is given in Table 1. The corresponding bar chart is shown in Figure 1.

Day	1	2	3	4	5	6	7	8	9	10
Bank-I	880	720	1020	802	522	989	684	548	1021	789
Bank-II	1034	789	1002	910	931	748	924	872	764	890
Bank-III	767	930	921	878	790	876	923	910	1002	949
Day	11	12	13	14	15	16	17	18	19	20
Bank-I	1000	990	1001	1051	982	857	981	1057	899	996
Bank-II	971	685	724	873	605	1017	1009	891	948	901
Bank-III	934	1011	874	762	631	989	784	648	891	752

Table 1: *Number of arrivals to the three banks*

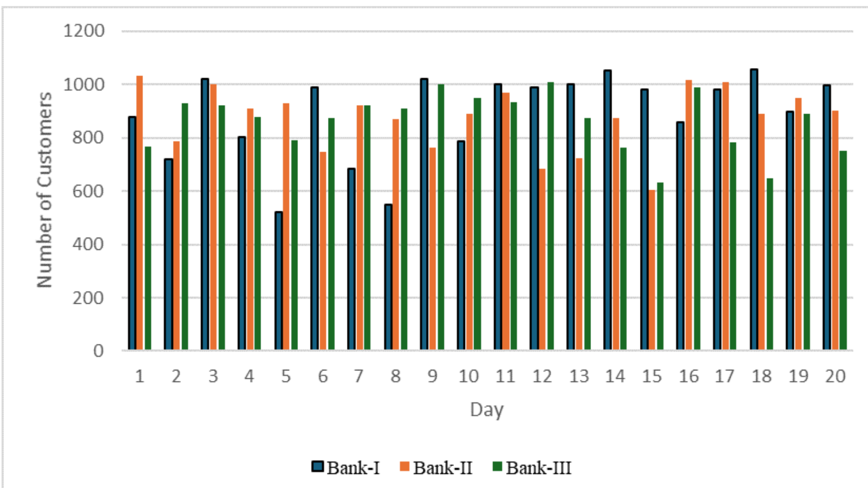

Figure 1: *Bar chart representation of the number of customers arriving to the three banks*

Table 2 gives the descriptive statistics of the data.

Bank	Statistic	Waiting Time in Queue (W_q) (in minutes)	Service Time (S) (in minutes)	Waiting Time in System (W) (in minutes)	Number of Arrivals per Day (N)
Bank-I	Average	12.65	15.1	27.75	889.45
	SD	6.64	5.46	9.07	162.71
Bank-II	Average	16.05	14.7	30.75	874.4
	SD	6.35	6.47	7.47	119.13
Bank-III	Average	11.85	11.15	23	861.1
	SD	3.94	3.51	5.77	109.49

Table 2: *Descriptive statistics*

Table 3 shows the arrival rate (α), service rate(β) and the number of servers required at each bank under the steady state.

Bank	Arrival Rate (α) (per hour=$N/7$)	Service Rate (β) (per hour=$60 diagup S$)	Number of Servers Required (s)
Bank-I	127.06	3.97	33
Bank-II	124.91	4.08	31
Bank-III	123.01	5.38	23

Table 3: *Arrival rate, service rate and number of servers at steady state*

In Bank-I, the average arrival is 127.06 per hour and the average service time is 3.97 per hour. Since, we are considering a steady state system, the condition $0 < \rho < 1$ should hold. Now, $\rho = \frac{\alpha}{\beta} = \frac{127.06}{3.97} = 32.005 > 1$.

Hence, if we set $s = 32$, and compute $\rho = \frac{\alpha}{s\beta} = \frac{127.06}{(32 \times 3.97)} = 1.00015 > 1$.

Since it is known that a queue will build up infinitely when $\rho \geq 1$ ([3], [8]), to bring the system under steady state, we increase the value of s and compute corresponding ρ until $\rho < 1$. When $s = 33$, $\rho = \frac{\alpha}{s\beta} = \frac{127.06}{(33 \times 3.97)} = 0.9698 < 1$, and when $s = 34$, $\rho = \frac{127.06}{(34 \times 3.97)} = 0.9413 < 1$.

It can be observed from the above calculations that, when the value of $s \geq 33$, the system is under steady state. The additional servers induce cost. Hence, if 33 servers are employed to attend to the large number of arrivals, the system will be maintained at steady state in Bank-I. The same explanation holds for Bank-II and Bank-III.

4. Goodness of fit test for arrival pattern

Since the main objective of this work is to find an appropriate distribution (which could also be an alternative distribution to Poisson) for the arrival pattern, the goodness of fit test was performed on the data. This was done using the trial version of SPC for Excel Software. Although arrival pattern is a discrete process, the software considers only the following continuous distributions. The probability density function (p. d. f.) and the corresponding log-likelihood function of the distributions tested for the arrivals are provided in Table 4.

The log-likelihood of the specified distribution is given for a sample size n with values $x_1, x_2, \cdots,$ x_n. In Table 4, μ. σ, α and θ refer to the location, scale, shape and threshold parameters respectively. More information related to these continuous distributions can be found in Johnson et al. [10].

The tests used were Shapiro-Wilk test (SW), Cramer-von Mises test (CVM) and Anderson-Darling test (AD). While SW test is used to test for the normality of the data, the CVM and AD tests are based on empirical distribution function (Stephens [14]). The hypothesis used in these goodness of fit tests is, H_0: The sample data follow the hypothesized distribution. If $x = x_1, x_2, \cdots, x_n$ denotes a random sample following a specified distribution, then the SW test statistic is given by

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

where $x_{(i)}$ is the i^{th} order statistic.
CVM test statistic is given by

$$W^2 = \sum_{i=1}^{n}\left[F(X_i) - \frac{(2i-1)}{2n}\right]^2 + \frac{1}{12n}$$

and
AD test statistic is given by

$$A^2 = -n - \frac{\left[\sum_{i=1}^{n}(2i-1)\left\{ln(F(X_i)) + ln(1 - F(X_{n+1-i}))\right\}\right]}{n}$$

where $F(.)$ is the cumulative distribution function of the specified distribution.

Distribution	p.d.f ($f(x)$)	Log-Likelihood
Smallest Extreme Value	$\frac{1}{\sigma}exp\left(\frac{x-\mu}{\sigma}\right)exp\left(-e^{\left(\frac{x-\mu}{\sigma}\right)}\right)$, $-\infty < x < \infty;\ \sigma > 0$	$\sum_{i=1}^{n}\left[\frac{x_i-\mu}{\sigma} - ln(\sigma) - exp(\frac{x_i-\mu}{\sigma})\right]$
Weibull	$\frac{\alpha}{\sigma}(\frac{x}{\sigma})^{(\alpha-1)}exp((-\frac{x}{\sigma})^{\alpha})$, $x \geq 0;\ \alpha,\ \sigma > 0$	$\sum_{i=1}^{n}[ln(\alpha) + (\alpha-1)ln(x_i) - \alpha ln(\sigma) - (\frac{x_i}{\sigma})^{\alpha}]$
Lognormal-Three Parameter	$\frac{1}{\sigma(x-\theta)\sqrt{2\pi}}exp\left(\frac{-(ln(x-\theta)-\mu)^2}{2\sigma^2}\right)$, $x > \theta;\ -\infty < \mu < \infty;\ \sigma > 0$	$-\sum_{i=1}^{n}\left[ln(\sigma\sqrt{2\pi}) + ln(x_i-\theta) + \frac{(ln(x_i-\theta)-\mu)^2}{2\sigma^2}\right]$
Normal	$\frac{1}{\sigma\sqrt{2\pi}}exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$, $-\infty < x < \infty;\ \sigma > 0$	$-\sum_{i=1}^{n}\left[ln(\sigma\sqrt{2\pi}) + \frac{(x_i-\mu)^2}{2\sigma^2}\right]$
Gamma	$\frac{x^{\alpha-1}}{\Gamma(\alpha)\sigma^{\alpha}}exp(\frac{-x}{\sigma}),\ \ x > 0;$ $\alpha,\ \sigma > 0$	$\sum_{i=1}^{n}\left[(\alpha-1)ln(x_i) - \frac{x_i}{\sigma} - ln(\Gamma(\alpha)) - \alpha ln(\sigma)\right]$
Loglogistic	$\frac{\alpha\sigma^{\alpha}x^{\alpha-1}}{(x^{\alpha}+\sigma^{\alpha})^2},\ x \geq 0;\ \alpha,\ \sigma > 0$	$\sum_{i=1}^{n}[ln(\alpha) + \alpha ln(\sigma) + (\alpha-1)ln(x_i) - 2ln(x_i^{\alpha} + \sigma^{\alpha})]$
Lognormal	$\frac{1}{\sigma x\sqrt{2\pi}}exp\left(\frac{-(ln(x)-\mu)^2}{2\sigma^2}\right)$, $x > 0;\ -\infty < \mu < \infty;\ \sigma > 0$	$-\sum_{i=1}^{n}\left[ln(\sigma\sqrt{2\pi}) + ln(x_i) + \frac{(ln(x_i)-\mu)^2}{2\sigma^2}\right]$
Largest Extreme Value	$\frac{1}{\sigma}exp\left(-\frac{(x-\mu)}{\sigma}\right)exp\left(-e^{-\left(\frac{x-\mu}{\sigma}\right)}\right)$, $-\infty < x < \infty;\ \sigma > 0$	$\sum_{i=1}^{n}\left[exp(\frac{x_i-\mu}{\sigma}) - ln(\sigma) - \frac{x_i-\mu}{\sigma}\right]$
Exponential-Two Parameter	$\frac{1}{\sigma}exp(\frac{-(x-\theta)}{\sigma}),\ \ x \geq \theta;\ \sigma > 0$	$-\sum_{i=1}^{n}\left[ln(\sigma) + \frac{x_i-\theta}{\sigma}\right]$
Exponential	$\frac{1}{\sigma}exp(\frac{-x}{\sigma}),\ \ x \geq 0;\ \sigma > 0$	$-\sum_{i=1}^{n}\left[ln(\sigma) + \frac{x_i}{\sigma}\right]$

Table 4: *Probability distributions and log-likelihood functions*

The functions *shapiro.test()* and *cvm.test()* in R [12] were used for SW and CVM tests.
The AD test was performed using SPC for Excel Software. This software reports the test statistic values, the p-values and the Akaike Information Criteria (AIC) values.

The AIC is given by $AIC = 2k - 2ln(\hat{L})$, where k is the number of estimated parameters and $ln(\hat{L})$ is the log likelihood function of the specified distribution.

The p-values are the size of the test or the probability of rejecting the null hypothesis when it is true. A small p-value leads to the rejection of the null hypothesis. The p-value is compared with a predetermined level of significance (LOS). If the computed p-value is less than the LOS, the null hypothesis is rejected at the specified LOS. The null hypothesis is the hypothesis to be tested. It is normally denoted as H_0 or H. In this paper, the LOS considered are 5% and 1%.

The AIC value is used to identify the model of best fit to the data. Among the models, the one which has the least value is considered as the best fit.

5. Results

Table 5 shows the test statistic and p-values for the three tests corresponding to the arrival pattern to the three banks. From the table, it is evident that the p-values of the test statistics SW and AD corresponding to Bank-I are less than 0.05 and 0.01. Thus, we reject the null hypothesis that the arrivals to the Bank-I follow normal distribution at 5% and 1% levels of significance.

Bank	SW Test		CVM Test		AD Test	
	W	p-value	W^2	p-value	A^2	p-value
Bank-I	0.84437	0.004296	0.4302	0.2066	1.242	0.002
Bank-II	0.93615	0.2026	0.10193	0.9727	0.509	0.175
Bank-III	0.92675	0.1337	0.18845	0.7553	0.585	0.110

Table 5: *Test statistic and p-value for testing normal distribution*

Tables 6-8 provide the summary of fitting the distribution to the number of customers who arrived at the three banks. The test used was the AD test. From the results, table 6 shows that the smallest extreme value distribution has a higher p-value compared to the rest. It is less than 0.05 but greater than 0.01. Hence it can be concluded that the data give enough evidence at 1% LOS to support that the smallest extreme value distribution is a good fit to the arrival pattern of customers to Bank-I. This is also evident from tables 7 and 8, corresponding to Bank-II and Bank-III and at a higher (i.e., 5%) LOS too. The p-values are greater than 0.25. Based on the same criteria, the next best distribution could be the Weibull distribution.

The software package also reports the AIC values and as depicted in tables 6-8, it is again clear that the smallest extreme value distribution has the least AIC value among all distributions. A slight difference of 0.5 on the higher side of the smallest extreme value distribution than the Weibull is seen in the AIC value, but this could be because of the number reporting to the banks. Despite this difference, it could still be concluded that the smallest extreme value distribution is a good fit for the arrival data followed by the Weibull.

Distribution	Log-Likelihood	AD	p - value	AIC
Smallest Extreme Value	-126.5	0.941	0.018	256.9
Weibull	-127.7	1.139	<0.01	259.4
Normal	-129.7	1.242	0.002	263.4
Lognormal- Three Parameter	-129.7	1.242	0.002	265.4
Gamma	-131.1	1.435	<0.005	266.3
Loglogistic	-131.4	1.280	<0.005	266.9
Lognormal	-132.0	1.541	<0.0001	268.0
Largest Extreme Value	-133.5	1.597	<0.01	271.0
Exponential- Two Parameter	-138.1	3.915	<0.001	280.3
Exponential	-155.8	6.361	<0.001	313.6

Table 6: *Distribution fitting summary for number of customers arrived at Bank-I*

Distribution	Log-Likelihood	AD	p - value	AIC
Smallest Extreme Value	-122.1	0.275	>0.25	248.2
Weibull	-122.3	0.335	>0.25	248.6
Normal	-123.5	0.509	0.175	250.9
Gamma	-124.1	0.621	0.109	252.3
Lognormal- Three Parameter	-123.5	0.509	0.175	252.9
Lognormal	-124.5	0.685	0.062	253.1
Loglogistic	-124.6	0.590	0.082	253.2
Largest Extreme Value	-126.4	0.893	0.023	256.7
Exponential- Two Parameter	-131.9	3.686	<0.001	267.8
Exponential	-155.5	6.955	<0.001	312.9

Table 7: *Distribution fitting summary for number of customers arrived at Bank-II*

Distribution	Log-Likelihood	AD	p - value	AIC
Smallest Extreme Value	-120.5	0.334	>0.25	245.0
Weibull	-120.6	0.399	>0.25	245.2
Normal	-121.8	0.585	0.110	247.6
Gamma	-122.4	0.691	0.075	248.7
Lognormal	-122.7	0.751	0.042	249.5
Lognormal- Three Parameter	-121.8	0.585	0.110	249.6
Loglogistic	-122.8	0.656	0.049	249.7
Largest Extreme Value	-124.6	0.978	0.015	253.2
Exponential- Two Parameter	-128.8	3.317	<0.001	261.5
Exponential	-155.2	7.104	<0.001	312.3

Table 8: *Distribution fitting summary for number of customers arrived at Bank-III*

6. Conclusion

On analyzing the data, it is evident that the minimum number of servers required for bank transactions is thirty-three at Bank-I, thirty-one at Bank-II and twenty-three at Bank-III. This seems to be a large number. Owing to the advancement of technology, the problem can be resolved as it is an optimization problem.

The focus of this paper was on the identification of a suitable distribution to the arrival pattern, and it is observed from the tests for normality as well as goodness of fit test that the smallest extreme value distribution is a good fit to the data.

The smallest extreme value distribution is also called the Gumbel distribution or type I extreme value distribution. It has two parameters, with location parameter as the mode. It belongs to the exponential family. It is a negatively skewed distribution and is platykurtic. Although the distribution is continuous, its discrete analog may be considered as the arrival process is a discrete setup. Further research in queueing theory can be done using the discrete analog of smallest extreme value distribution as an alternative to Poisson.

Data Availability

The dataset used is a secondary source and is publicly available at
https://doi.org/10.1016/j.dib.2018.05.101

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