Optimizing an imperfect production system with varying production cost under selling price and advertising-driven demand

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Abstract. This study develops a production inventory model that accounts for imperfect production processes, acknowledging that a certain percentage of items produced are defective, reason being labor inefficiencies and machinery malfunctions. These defective items are identified and are segregated during the quality check and inspection phase and sold at a discounted price. The model incorporates economies of scale, whereby production costs decreases as productivity increases, and represents production costs as a diminishing function of productivity. Furthermore, the demand rate is modeled as dependent on advertising frequency and selling price, impacting significantly the product demand. The primary objective of this research is to determine the optimal advertising frequency, selling price, and the production cycle that maximize overall system profit. A numerical illustration is provided to validate the model, and a sensitivity analysis is conducted to assess the stability of the system. The findings reveal that increased demand reduces production time and raises pricing potential that leads to boosting of profits. Higher production rates improve efficiency and lower unit costs, while a higher defect rate and rising holding costs adversely impact profitability, which illustrates the importance of quality control and efficient inventory management. These insights highlight the model's practicality and innovation, offering valuable guidance for optimizing operations, managing costs, and achieving sustainable growth.

Keywords: advertisement dependent demand, imperfect production, production model, selling price, variable production cost

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1. Introduction

Many inventory practitioners investigated numerous aspects of inventory modeling in the recent decades by assuming an unvarying demand rate. Yet, in reality, an item's demand, over the time remains in a dynamic condition. A product's selling price is an extremely crucial aspect in today's cut-throat market competition. In addition to price, advertising is another marketing factor that influences demand. Advertising enables businesses to reach a broader audience, including new or untapped markets. As a result, it can lead to increased demand from previously untargeted consumer segments. Thus, advertising is a vital tool for businesses to attract

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customers, increase sales, and stabilize success in competitive markets. Consequently, an item's demand may be thought of as being impacted by both its advertisement's frequency as well as the selling price. By incorporating price-reliant demand, researchers like [9], [11] and [20] presented their studies for deteriorating items. An inventory framework was discussed by [8] for products that do not deteriorate immediately, with a price-reliant demand. With sales returns, scrap as well as reworking, [24] created models in an imperfect production system. However, authors considered the demand rate as price-reliant in their study. [12] presented an inventory framework with deterioration as well as amelioration under trade-credit policy. Shortages were not permissible in this analysis, while the demand rate was thought to be price-reliant.

In all the above models the impact of advertising on demand was not taken into account in their studies. But authors like [19] and [17] projected their studies by incorporating demand as dependent upon the advertisement's frequency as well as the selling price. A retailer's joint pricing study was explored by [16] by utilizing a trade-credit strategy that incorporates price as well as advertisement's frequency dependent demand. Authors permitted shortages in this analysis while both the deterioration rate and the cost of holding of items were constant. Recently, a supply chain paradigm was presented by [6] incorporating demand as dependent upon advertisement's frequency as well as the selling price. Authors allowed shortages to occur in this presented study.

Among the most crucial components of a business tactic is its production system. An efficient production system ensures that resources are used optimally, minimizing waste of materials, time, and labor. This leads to cost savings and improved profitability for businesses. To minimize product overstocks and shortages, the firm should properly manage its production process in order to benefit the business. Researchers like [27] and [23] introduced production inventory models involving decaying products. An EPQ paradigm for items that were ameliorating deteriorating in nature by incorporating ramp type demand was formulated by [10]. For decaying goods which were non-instantaneous in nature, [25] gave an EPQ framework under no shortages. This study evaluated holding cost as contingent on time whereas the rate of production was treated as demand rate dependent. To identify the optimum rate of production, [1] proposed a production inventory framework for an agricultural commodity. Costs such as deterioration, transportation and holding cost were considered as unvarying in the proposed study. After that, [13] presented a production model by incorporating advertisement and price-reliant demand for defectives. As production rate was treated as higher in comparison of the demand rate, thus shortages were not permitted in this developed study.

In all the above models impact of varying production cost has not been considered in their study while it is essential in determining a firm's profitability, competitiveness, pricing strategy and overall financial health. It is commonly seen that when demand raises, production rates rise, which decreases the cost of production per unit. Hence, the unit production cost per unit depends upon production or demand. In line with this, only few authors considered variable production cost in their studies. [14] proposed a deterministic model by incorporating variable production cost as well as stock-reliant demand for damageable products. [22] explored an imperfect production system stating the unit production cost was treated as a function of the finite production rate. Two EPQ models with or without shortages were studied by [15] for deteriorating items. For developing models, authors treated that the production rate along with the unit production cost were inversely correlated whereas rate of production rate was proportional to the demand rate. [5] demonstrated a production replica by treating the production cost per unit that significantly increases with regard to production rate of the system, where as in the presented model we have considered that as the system's production rate increases, the cost of production per unit decreases.

A number of research studies that are available believe the products generated by a production system are perfect. While in reality it is not true. Due to the various challenges encountered in a lengthy production process, a certain ratio of produced items is defective. Therefore, the things that are defective may be addressed as a consequence of the production's imperfection. Authors, including [3] and [4] presented their inventory models incorporating imperfect production. After that, [21] presented imperfect production structure that incorporates an EPL framework under no shortages. In this study, the production's rate was used as a decision-making measure. [26] introduced their study for deteriorating and imperfect quality under no shortages. However, the demand rate, deterioration rate and the production rate was unvarying in nature. In support of imperfect quality products, [18] demonstrated an inventory paradigm by including partial trade credit. But demand for the products was treated to be reliant upon advertisement, only in the mentioned model. After that, [7] gave a framework than incorporates an imperfect production model under screening and shortage constraints, considering a non-linear time-reliant demand function in this study. In order to maximise total profit, [2] recommended an imperfect manufacturing system. This research was developed under shortages by incorporating demand as reliant upon selling price only.

2. Research gap

A key research gap identified in the existing literature on inventory and production models is the limited exploration of variable production costs in imperfect production systems, particularly where production costs are influenced by economies of scale. While some studies have incorporated price and advertisement-dependent demand and even addressed imperfect production and defect rates, many models assume fixed production costs, which do not account for the real-world scenario, where the production costs per unit decrease as production rates increase. Furthermore, although some studies have addressed imperfect production, few have considered the combined impact of both advertising and production costs on overall profitability. Existing models also often neglect fluctuating production costs as a function of both demand and production rate, which is crucial for accurate decision-making in dynamic and competitive markets.
Addressing this gap by developing a model that integrates variable production costs, selling price, advertisement-dependent demand, and imperfect production processes could lead to a more realistic approach to optimizing inventory and production systems. Thus,

- A production inventory model with imperfect production has been presented.
- Several business tactics, like advertisement frequency, selling price, and variable production cost are combined to create a system to support decisions.
- As the production process is not 100% reliable; during the inspection procedure, defective products are isolated and offered for sale at a reduced cost.
- The production rate is thought to be reliant on demand and reflects real-world scenarios.
- Production cost per unit reduces as the production rate of the system rises
- Optimal values of advertisement frequency, selling price, and production period are calculated, which optimize the system's overall average profit.

3. Assumptions and notations

The underlying presumptions that guided the creation of the suggested model are outlined in this part.
1. This is a single item inventory system with finite rate of replenishment.
2. The period between placing an order and its delivery is assumed to be negligible.
3. The rate of demand is reliant upon advertisement's frequency as well as selling price which is given by $(\alpha - \beta P^{\gamma})A^{\theta}$ where $\alpha > 0,\ 0 < \beta < 1,\ \lambda > 0,\ 0 < \theta < 1$.
4. Production rate is regarded as contingent upon demand rate which is given by $P = \eta(\alpha - \beta P^{\gamma})A^{\theta}$.

5. An inspection is done after the production of items. The inspection cost is given by $c_i = c_2 + c_3 \int_0^{t_1} P dt$, where c_2 represents the initial cost to set up the inspection process, regardless of the number of items produced; this cost remains constant and c_3 represents the cost per unit of inspection effort.
6. During assessment, a specific percentage of the total number of produced is discovered to be defective and these flawed products are available for sale at a discounted value.
7. This model assumes that the production cost per unit decreases as the system's production rate rises. It is given by the term $(\lambda - \mu P)$. Here $(\lambda - \mu P)$ must remain positive for meaningful production. If P exceeds a certain level, the cost per unit could become zero or negative, which is unrealistic. λ reflects the base cost of manufacturing one unit without considering production scale. μ is the cost sensitivity to production rate. As the production rate (P) increases, the per-unit cost decreases. This models situations where higher production reduces fixed costs per unit or achieves efficiencies (e.g., bulk purchasing, better utilization of machinery).
The key applications for this cost function are mentioned as follows.
Cost Optimization: Businesses can use this function to determine the optimal production rate (P) where per-unit costs are minimized, balancing economies and diseconomies of scale.
Pricing Strategies: Understanding how costs change with production allows firms to set prices competitively while ensuring profitability.

The notations are as follows:

$\alpha, \beta, \gamma, \theta$: demand parameters,
η : production parameter, $\eta > 1$,
A : number of advertisement frequencies conducted per month, $A \in \mathbb{N}$,
p : selling price in \$ / unit,
T : replenishment cycle,
t_1 : production period,
λ : initial production cost in \$ / unit,
μ : production cost parameter, $\mu > 0$,
ξ : set-up cost,
δ : percentage of imperfect products, $(0 < \delta < 1)$,
c_1 : cost for per advertisement,
P : production rate,
χ : percentage discount offered to imperfect products,
h : holding cost per unit per unit time,
c_2 : fixed cost of inspection,
c_3 : variable part of inspection cost.

4. Mathematical formulation

The production cycle (Figure 1) starts with zero-inventory and production begins at $t = 0$. During time $[0, t_1]$ inventory builds up, adjusting demand in the market. At $t = t_1$, production stops. After that, through the period $[t_1, T]$, this stock level reduces owing to market demand

and approaches the zero level at $t = T$.

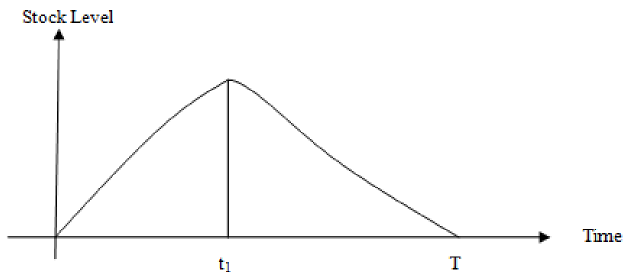

Figure 1: *Inventory behavior with respect to time.*

The differential equations showing the inventory system's behavior at any moment t are given by

$$\frac{dI(t)}{dt} = \eta(\alpha - \beta p^{\gamma})A^{\theta} - (\alpha - \beta p^{\gamma})A^{\theta}, \quad 0 \leq t \leq t_1, \tag{1}$$

$$\frac{dI(t)}{dt} = -(\alpha - \beta p^{\gamma})A^{\theta}, \quad t_1 \leq t \leq T, \tag{2}$$

with boundary condition $I(0) = 0, I(T) = 0$.
These equations' solutions are as follows

$$I(t) = (\eta - 1)(\alpha - \beta p^{\gamma})A^{\theta}t, \quad 0 \leq t \leq t_1, \tag{3}$$

$$I(t) = (\alpha - \beta p^{\gamma})A^{\theta}(T - t), \quad t_1 \leq t \leq T, \tag{4}$$

with the help of equation (3) and (4), the relation in and T as

$$T = \eta t_1. \tag{5}$$

Total production $= \int_0^t P dt$.

$$T.P. = \eta A^{\theta}(\alpha - \beta p^{\gamma})t_1 \tag{6}$$

The expense incurred by storing goods for a period of time is known as holding cost:

$$\begin{aligned} \text{Holding cost} &= h\left(\int_0^{t_1} I(t)\,dt + \int_{t_1}^{T} I(t)\,dt\right), \\ \text{H.C.} &= \frac{hD}{2}\left[(\eta - 1)t_1^2 + (T - t_1)^2\right]. \end{aligned} \tag{7}$$

If A is the advertisement's frequency then the cost associated with the advertisement will be

$$\text{Advertisement cost} = Ac_1. \tag{8}$$

If δ is the percentage of imperfect items, then the revenue ($R1$) generated from the perfect items will be

$$R_1 = p(1 - \delta)\eta t_1(\alpha - \beta p^{\gamma})A^{\theta}. \tag{9}$$

All the defective items are available for sale at discounted price χp where $0 < \chi < 1$ so the sales revenue (R_2) generated from the defective items will be

$$R_2 = \chi p \delta \eta t_1(\alpha - \beta p^{\gamma})A^{\theta}. \tag{10}$$

So, total sales revenue $= R_1 + R_2$,

$$\text{Sales revenue} = p(1-\delta)\eta t_1(\alpha - \beta p^{\gamma})A^{\theta} + \chi p\delta\eta t_1(\alpha - \beta p^{\gamma})A^{\theta}. \tag{11}$$

The expenses related to assembling all necessary equipment for manufacturing are provided by

$$\text{Set up cost} = \xi. \tag{12}$$

Since the production procedure is not 100% reliable, through production a few defective items are also produced. So to separate the imperfect items all the produced items has to undergo an inspection procedure. The cost associated for the inspection of items is given by

$$\begin{aligned} c_i &= c_2 + c_3\int_0^{t_1} P dt, \\ c_i &= c_2 + c_3\eta A^{\theta}(\alpha - \beta P^{\gamma})t_1. \end{aligned} \tag{13}$$

If λ is the initial production cost per unit in that case the total production cost will be

$$\begin{aligned} P.C. &= \int_0^{t_1} P(\lambda - \mu P)dt, \\ P.C. &= \eta(\alpha - \beta p^{\gamma})A^{\theta}(\lambda - \mu\eta(\alpha - \beta p^{\gamma})A^{\theta})t_1. \end{aligned} \tag{14}$$

The system's overall average profit may now be calculated as

$$\begin{aligned} \text{T.A.P.}(t_1, A, P) = \frac{1}{\eta t_1}\big[&\text{Sales revenue} - \text{Set up cost} - \text{Inspection cost} \\ &- \text{Holding cost} - \text{Advertisement cost} - \text{Production cost}\big], \end{aligned} \tag{15}$$

$$\begin{aligned} \text{T.A.P.}(t_1, A, P) = \frac{1}{\eta t_1}\Big(&P(1-\delta)\eta t_1(\alpha - \beta p^{\gamma})A^{\theta}) + \chi p\delta\eta t_1(\alpha - \beta p^{\gamma})A^{\theta} - \xi - c_2 \\ &-c_3\eta A^{\theta}(\alpha - \beta p^{\gamma})t_1 - \frac{h(\alpha - \beta p^{\gamma})A^{\theta}}{2}[(\eta - 1)t_1^2 + (\eta t_1 - t_1^2)] - Ac_1 \\ &-\eta(\alpha - \beta p^{\gamma})A^{\theta}(\lambda - \mu\eta(\alpha - \beta p^{\gamma})A^{\theta})t_1\Big). \end{aligned} \tag{16}$$

5. Solution procedure and algorithm

Now the problem can be formulated as:

Max T.A.P.(t_1, A, p)
with constraints $0 \le t_1 \le T$ and $p > \lambda$
Under the following conditions
$\alpha > 0,\ 0 < \beta < 1,\ \gamma > 0,\ 0 < \theta < 1,\ \lambda > \mu P,\ 1 \le A \le 10$
We use the following algorithm to determine the model's optimal solution.

5.1. Algorithm

For all the constant parameters employed in the model
Step 1:Initialize $A = 1$
Step 2: Determine optimal values of decision variables t_1 and p, subject to given constraints

Step 3: Calculate the optimal solution T.A.P. (t_1, A, p)
(a) For a fixed value of A, put

$$\frac{\partial \text{T.A.P}(t_1, A, p)}{\partial t_1} = 0, \qquad \frac{\partial \text{T.A.P}(t_1, A, p)}{\partial p} = 0. \tag{17}$$

(b) Solve these equations simultaneously for critical points.
The determinant of the Hessian exists, but it's symbolic form is prohibitively complex for presentation. Thus, we determine negative definiteness by examining the leading principal minors according to Sylvester's criterion and find the optimal value.
Step 4: Set $A_1 = A + 1$
Recalculate T.A.P(t_1, A_1, p_1) using new A_1 & compare T.A.P(t_1, A_1, p_1) with T.A.P(t_1, A, p)
Step 5: If T.A.P$(t_1, A_1, p_1) >$ T.A.P(t_1, A, p), then: Set $A = A_1$
Repeat Step 3 and Step 4
Else: Proceed to Step 6
Step 6: Set $(t_1, A_1, p_1) = (t_1^*, A^*, p^*)$ as the best solution.
Step 7: Calculate T.A.P$^*(t_1^*, A^*, p^*)$
Return the optimal solution (t_1^*, A^*, p^*)
End Algorithm.

5.2. Hessian matrix

For optimal value of p and t_1. We differentiate the total profit function with respect to p and t_1 and equating to zero:

$$\frac{\partial \text{T.A.P}(t_1, A, p)}{\partial t_1} = -\frac{1}{2} A^\theta h(\alpha - p^\gamma \beta)(-1 + \eta) + \frac{\xi + Ac_1 + c_2}{\eta t_1^2}, \tag{18}$$

$$\begin{aligned}\frac{\partial \text{T.A.P}(t_1, A, p)}{\partial p} =& \frac{1}{2p} A^\theta \Big(2p\alpha\{1 + (-1 + \chi)\delta\} - 2p^{1+\gamma}\beta(1 + \gamma)\{1 + (-1 + \chi)\delta\} \\ &+ 4A^\theta p^2 \gamma \beta^2 \gamma \eta \mu + 2p^\gamma \beta \gamma (\lambda - 2A^\theta \alpha \eta \mu) \\ &+ 2p^\gamma \beta \gamma c_3 + hp^\gamma \beta \gamma (-1 + \eta) t_1 \Big).\end{aligned} \tag{19}$$

The optimal solutions p^* and $t_1{}^*$ can be obtained by solving the resultant system after setting the equations (18) and (19) to zero:

$$H = \begin{pmatrix} \frac{\partial^2 T.A.P}{\partial t_1^2} & \frac{\partial^2 T.A.P}{\partial t_1 \partial p} \\ \frac{\partial^2 T.A.P}{\partial p \partial t_1} & \frac{\partial^2 T.A.P}{\partial p^2} \end{pmatrix}, \tag{20}$$

$$\frac{\partial \text{T.A.P}(t_1, A, p)}{\partial t_1^2} = -\frac{2(\xi + Ac_1 + c_2)}{\eta t_1^3} < 0, for \quad \xi > 0, A > 0, c_1 > 0, c_2 > 0, \eta > 0. \tag{21}$$

It shows $D_1 < 0$ (negative definite):

$$\frac{\partial^2 \text{T.A.P.}(t_1, A, P)}{\partial p^2} = \frac{1}{2} A^\theta p^{-2+\gamma} \beta \gamma B(p, t_1), \tag{22}$$

where

$$\begin{aligned}B(p, t_1) =& -2p(1 + \gamma)(1 + (-1 + \chi)\delta) + 4A^\theta p^\gamma \beta(-1 + 2\gamma)\eta\mu \\ &+ 2(-1 + \gamma)(\lambda - 2A^\theta \alpha \eta \mu) + 2(-1 + \gamma)c_3 + h(-1 + \gamma)(-1 + \eta)t_1\end{aligned} \tag{23}$$

$$\frac{\partial^2 \text{T.A.P.}(t_1, A, P)}{\partial p \partial t_1} = \frac{\partial^2 \text{T.A.P.}(t_1, A, P)}{\partial t_1 \partial p} = \frac{1}{2} A^{\theta} h p^{-2+\gamma} \beta\gamma J(p, t_1). \tag{24}$$

The determinant of the Hessian is given by

$$|H| = \frac{1}{16\eta^2 t_1^3} A^{\theta} p^{-2+\gamma} \beta\gamma J(p, t_1), \tag{25}$$

where

$$\begin{aligned} J(p,t_1) = \Big(&32p\xi\eta + 32p\gamma\xi\eta - 32p\delta\xi\eta + 32p\chi\delta\xi\eta - 32p\gamma\delta\xi\eta + 32p\chi\gamma\delta\xi\eta + 32\xi\eta\lambda \\ &- 32\gamma\xi\eta\lambda - 64A^{\theta}\alpha\xi\eta^2\mu + 64A^{\theta}p^{\gamma}\beta\xi\eta^2\mu + 64A^{\theta}\alpha\gamma\xi\eta^2\mu - 128A^{\theta}p^{\gamma}\beta\gamma\xi\eta^2\mu \\ &+ 32\xi\eta c_3 - 32\gamma\xi\eta c_3 + 8h\xi t_1 - 8h\gamma\xi t_1 - 24h\eta\xi t_1 + 24h\gamma\xi\eta t_1 + 16h\xi\eta^2 t_1 \\ &- 16h\gamma\xi\eta^2 t_1 - A^{\theta}h^2p^{\gamma}\beta\gamma t_1^3 + 6A^{\theta}h^2p^{\gamma}\beta\gamma\eta t_1^3 - 13A^{\theta}h^2p^{\gamma}\beta\gamma\eta^2 t_1^3 \\ &+ 12A^{\theta}h^2p^{\gamma}\beta\gamma\eta^3 t_1^3 - 4A^{\theta}h^2p^{\gamma}\beta\gamma\eta^4 t_1^3 + 8Ac_1(4\eta(p(1+\gamma)(1+(-1+\chi)\delta) \\ &- 2A^{\theta}p^{\gamma}\beta(-1+2\gamma)\eta\mu - (-1+\gamma)(\lambda - 2A^{\theta}\alpha\eta\mu)) - 4(-1+\gamma)\eta c_3 \\ &- h(-1+\gamma)(1-3\eta+2\eta^2)t_1) \qquad + 8c_2(4\eta(p(1+\gamma)(1+(-1+\chi)\delta) \\ &- 2A^{\theta}p^{\gamma}\beta(-1+2\gamma)\eta\mu \qquad - (-1+\gamma)(\lambda - 2A^{\theta}\alpha\eta\mu)) - 4(-1+\gamma)\eta c_3 \\ &- h(-1+\gamma)(1-3\eta+2\eta^2)t_1)\Big). \end{aligned} \tag{26}$$

The second derivative with respect to t_1 has been shown analytically to be negative under the stated parameter assumptions. The symbolic determinant of the Hessian is prohibitively large for closed-form presentation. Consequently, we verified the sign of the determinant numerically: we evaluated D_2=det(H), across the feasible ranges of p and t_1 and for the parameter values considered in this study. In all tested cases the determinant was strictly positive while $D_1 < 0$. Hence, by Sylvester's criterion the Hessian is negative definite on the feasible domain and the stationary point is a (global) maximizer in the admissible region.

6. Numerical example

These following are the inputs used to calculate the numerical results. Mathematical software Mathematica 11.3 has been used to solve the numerical example.
α=500 units, β=0.02, η=1.5, θ=0.001, γ=2.5, μ=0.1, χ=0.5, δ=0.2, ξ=1000\$ /setup, c_1=5\$ /advertisement, λ=5, c_2=500\$ /Inspection, c_3=1\$ /unit, h=0.2\$ /unit.

We treat A as an integer (ads per month) with a practical upper bound $A_{\max}$. Because $A_{\max}$ is small in realistic scenarios, we use an exhaustive linear search over $\{1, \ldots, A_{\max}\}$.This approach is exact for the discrete problem, computationally trivial, and avoids errors from continuous approximations.

A	t_1	p	T.A.P.
1	7.81631	36.568	9010.57
2	**7.93887**	**36.5699**	**9012.79**
3	8.06058	36.5718	9012.40
4	8.18089	36.5738	9010.98
5	8.29967	36.5758	9009.02
6	8.41690	36.5777	9006.73
7	8.53261	36.5797	9004.23
8	8.64685	36.5816	9001.6
9	8.75964	36.5835	8998.86
10	8.87104	36.5854	8996.06

Table 1: *Solution search procedure for optimal ‘A’.*

The results show that when the value of A increases, the cycle time t_1 also increases steadily. This means that higher values of A require more time for operations. Price p does not show any major change and remains nearly constant across all values of A. The Total Annual Profit (T.A.P.) first rises and reaches its highest level when $A = 2$. After this point, the profit starts decreasing slowly as A continues to increase. Therefore, the value $A = 2$ gives the best profit outcome, and further increase in A reduces the profitability even though the cycle time becomes longer. Thus, the maximum value obtained for T.A.P. is the optimal value of the system with optimal frequency of advertisement and corresponding to it the value of production period t_1 and selling price p are also the optimal values for the given system. Here the optimal values are: $A^* = \mathbf{2}, t_1^* = \mathbf{7.93887}$ months, $p^* = \mathbf{36.5699}$\$, T.A.P$^* = \mathbf{9012.79}$\$, $T = 11.90$ months.

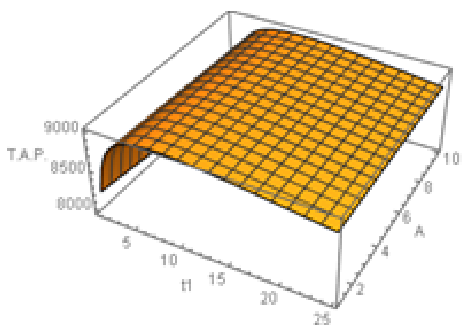

Figure 2: *Concavity of T.A.P. function for fix value of ‘p’.*

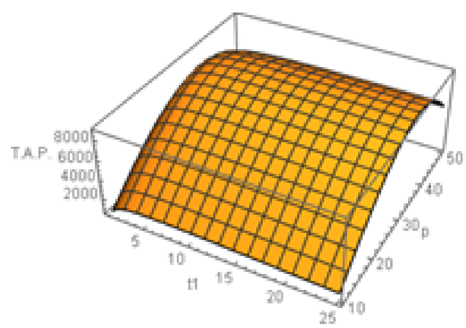

Figure 3: *Concavity of T.A.P. function for fix value of ‘A’.*

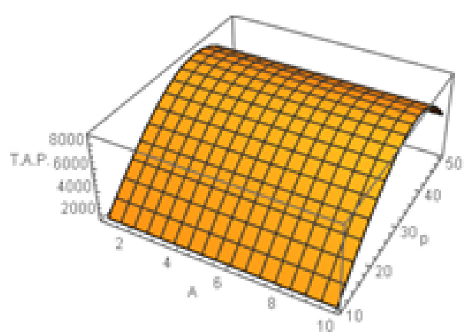

Figure 4: *Concavity of T.A.P. function for fix value of ‘t_1’.*

7. Sensitivity analysis

To look into how distinct parameters affect the system's total average profit, a sensitivity analysis is carried out. The results are given in Table 2. The following points are noticed:

1. With the increment in initial demand parameter 'α'', t_1 decreases whereas p and T.A.P. increases. The value of A remains constant up to a certain range of variation and then shows a slight increase. This happens because higher demand leads to higher sales, which results in increased profit. The rationale is that when demand of a product raises it boost the sale of the product which result an increase in profit.

2. We noticed that as initial production cost parameter 'λ'' increases, t_1 & p increases, whereas T.A.P. decreases. The rationale is that as initial production cost/unit increase, the profit per unit decreases due to reduced profit margins. As a result, the overall average profit, which is a measure of the average profit per unit produced or sold, reduces. As a result, the manufacturer needs to monitor the initial production cost parameter, since it has a quick impact on earnings.

3. As the production parameter (η) increases, t_1 and p decreases, and the Total Average Profit (T.A.P.) also decreases, which aligns with practical expectations. The reason is that when the production rate increases, the cost per unit decreases, leading to lower pricing. This ultimately reduces the overall profit earned.

4. Variation in parameter 'δ'' (fraction of imperfect items) is presented in Table 2. As parameter 'δ'' increases, t_1 and p increases, while T.A.P. decreases. The rationale is that a higher fraction of imperfect items reduces product quality, prompting customers to switch to competitors, offering better-quality products. This leads to reduced sales, ultimately decreasing profit. Therefore, manufacturers should pay close attention to this parameter, as it has a significant impact on earnings.

Parameter	**% Variation**	**Value**	A	t_1	p	**T.A.P.**
α	-20	400	2	8.91552	33.6938	6397.18
	-10	450	2	8.38547	35.1804	7669.40
	0	500	2	7.93887	36.5699	9012.79
	10	550	2	7.55594	37.8773	10423.2
	20	600	3	7.33358	39.1158	11897.4
λ	-20	4	2	7.89495	36.2262	9353.18
	-10	4.5	2	7.91666	36.3974	9182.51
	0	5	2	7.93887	36.5699	9012.79
	10	5.5	2	7.96158	36.7435	8844.03
	20	6	2	7.98480	36.9183	8676.24
η	-20	1.20	3	14.2561	36.6012	9057.96
	-10	1.35	3	10.1585	36.5905	9027.44
	0	1.50	2	7.93887	36.5699	9012.79
	10	1.65	2	6.63630	36.5467	9006.58
	20	1.80	2	5.72479	36.5207	9005.63
δ	-20	0.16	2	7.93363	36.5294	9260.39
	-10	0.18	2	7.93622	36.5494	9136.58
	0	0.20	2	7.93887	36.5699	9012.79
	10	0.22	2	7.94159	36.5908	8889.02
	20	0.24	2	7.94438	36.6122	8765.25

Table 2: *Sensitivity analysis for some crucial parameters.*

8. Conclusion

A realistic inventory problem with varying production costs is addressed by taking the selling price and advertisement dependent demand into consideration. Here, production cost is deemed to be a decreasing function of production rate, as it is realistic that if anyone produces more items, then the production cost per unit decreases, which results in an overall profit of the system. The results presented in sensitivity analysis confirmed this fact also. Throughout the production phase, a few flawed goods are manufactured, that are separated during the inspection procedure. These defective goods are sold at discounted price to generate profit. This research investigates the effects of key operational factors on efficiency and profitability in the production and inventory systems. The proposed model introduces a novel approach by incorporating real-world factors such as imperfect production systems, demand-dependent rates, and strategies for optimizing advertisement frequency and pricing. It addresses defective item management and demonstrates how higher production rates contribute to cost efficiency. These findings offer practical insights into optimizing production operations and achieving sustainable business growth.

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