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Estimating Short-term Default Probabilities Conditional to Economic Conditions: Applications of Regularisation Approach and Economic Adjustment Coefficients

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Abstract

Background: Corporate bonds are crucial for corporations as they provide a flexible and often less costly alternative to equity financing. However, rising corporate debt levels, along with rating downgrades and economic uncertainty, can cause corporations to face financial distress, exacerbating the probability of default. **Objectives:** The purpose of this paper is to estimate bond default probabilities conditional on fluctuations in economic growth over short-term frequencies using inputs from rating transitions. **Methods/Approach:** The estimation is based on a Markov chain framework and the incorporation of economic growth by utilizing specifications of the economic adjustment coefficient. Further, quasi-optimisation of the roots matrix is utilized to extend the model within a quarterly domain. **Results:** Economic growth (proxied by GDP) carries little informational content on the future default probabilities. Non-investment grade ratings depict higher default probability, while investment-grade ratings yield default propensity of less than 1.1% in the next quarters and exhibit higher distance between default probabilities by tenor points and neighbouring states as the time horizon lengthens. **Conclusions:** First, practitioners can measure forward-looking bond exposure across different tenure buckets using the estimation approach developed in this study. Second, by considering historical fluctuations in the economic cycle as an additional factor for estimating future default probability, this study informs financial market regulators by providing entities with an alternative reference point to their in-house generated models, helping them meet regulatory requirements.

Keywords: credit ratings; default probability; Markov framework; regularisation approach; economic adjustment coefficients

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Introduction

Following the global financial crisis 2008, large companies in major economies and emerging markets prioritized the bond market for financing as commercial bank lending was subdued (Lund et al., 2018). This shift led to a remarkable increase in bond issuance post-crisis, expanding the corporate bond market by almost 40 percent since 2007. Global corporate bond issuances totalled approximately USD 21 trillion in 2008 and grew to USD 33.6 trillion outstanding in 2023, with an increase of USD 13 trillion (OECD, 2024). This expansion benefited from dovish monetary policy, a favourable market infrastructure, a supportive tax environment, and access to longer-term financing due to a diverse investor profile. Unwavering, multipronged initiatives by regulators to develop the corporate bond market as part of efforts to stabilize the unfavourable feedback loop to banking institutions post-crisis have also boosted the global corporate bond market.

Although the rise in corporate bonds facilitates access to a broader financing base for expansion, it also harbours risks, as economic activities increasingly rely on leverage. High levels of corporate debt and predicted rating downgrades raise concerns, particularly for speculative bonds, which could face financial distress in unfavourable economic conditions (Fernandes, 2024). Volatile currencies, oil prices, and a global tightening of monetary policy post-COVID-19 are increasing pressure on economic growth. The pandemic-related decline in stock prices (Bouri et al., 2022) has also disproportionately affected the markets (Topcu & Gulal, 2020). Ongoing global financial uncertainty, exacerbated by the global crisis, increases the risk of rising corporate defaults worldwide, which could impact highly open economies. This is particularly evident in the report by S&P Global Ratings, where 153 defaults occurred in 2023, 80 percent more than in 2022.

Given the surge in global corporate bankruptcies, evaluating a corporation's default risk has gained heightened importance. Many methods have been developed over the past half-century to derive default probabilities from various observations, reflecting the ongoing evolution of the field and underscoring its continued importance. One such method is the rating transition approach, a subset of the reduced-form model introduced by Jarrow et al. (1997), which utilizes a Markov framework to construct credit risk spread term structures. This method, used in literature by studies such as Möstel et al. (2020), Boreiko et al. (2019), Baena-Mirabete and Puig (2018), Xing and Yu (2018), Pfeuffer et al. (2019), and Pasricha et al. (2017), employs credit rating transition matrices based on Markovian principles.

Long-term default curves are traditionally used in credit risk analysis as part of the rating system. One notable limitation is their inability to comprehensively derive the long-term dynamics of credit states of obligors, as they cannot incorporate the appearance of new companies with randomly assigned initial credit states. However, it is increasingly recognized that a short-term default curve is required to accurately capture the details of default risk. Extensive research emphasizes the importance of the short-term default curve for credit risk management, including financial decisions such as accurate pricing of debt instruments like speculative grade bonds and credit default swaps, and ensuring appropriate valuation of obligations sensitive to market conditions (Cathcart et al., 2020). Hence, this paper aims to estimate conditional default probabilities in relation to economic growth. Specifically, quarterly transition matrices are utilized to account for varying bond market maturities and similar credit risks. Since maturities often occur non-annually, smaller-frequency transition matrices are essential for accurate risk assessment.

Furthermore, we contend that existing studies generally treated the modelling of the multi-period default curve in isolation with complex procedures (see Kreinin and Sidelnikova, 2001; Bladt and Sorensen, 2005, 2009; Crommelin and Vanden-Eijnden, 2006; dos Reis and Smith, 2018). Studies on joint consideration of external factors are mostly derived within the context of long-term dynamics (see Blümke, 2022; Zhu, 2016). This prompted us to examine the current research to demonstrate the applicability of constructing short-term default probabilities conditioned to economic changes based on discrete observations.

This study focuses on the estimation of default probabilities as a function of economic conditions at a short-term frequency for the following reasons: (1) the literature on modelling default probability taking into account external factors is extensive, but the joint consideration of both factors over a shorter time horizon is limited despite its importance; (2) the construction of such a model is crucial as it allows the distinction between defaults due to systematic and idiosyncratic shocks and provides an explicit channel and model for default correlation. For this purpose, we first generate the multi-period (intra-year) forecasts of the discrete credit loss distribution using a regularization procedure developed by Kreinin and Sidelnikova (2001), known as quasi-optimization of the root matrix (QOM). This method proved to be more powerful than the conversion of discrete data into continuous time, such as the quasi-optimization of generators, and is therefore a preferred approach for the calculation of short-term periods. We further hybridized the short-term default curve with economic adjustment coefficients based on Vaněk and Hampel (2017)'s framework to establish a conditional model. Through this, the decomposition of economic adjustment coefficients is divided proportionately between the directions (upgrades and downgrades) of rating states. Key features include discrete-time and state space modelling, forward-looking estimation, and incorporation of the correlated economic cycle in forming the conditional term structure.

The remainder of the paper proceeds as follows: Following the introduction, Section 2 presents a review of previous works in the area of credit risks. Section 3 presents the data and methodology utilized in this paper to estimate the forward-looking default probability. Section 4 discusses the results, and the conclusion and future works are given in Section 5.

Literature Review

Default probabilities

Default probabilities have been an active research area in credit risk environments. Regulatory bodies have specified that the estimation of credit losses must be based on default measurements, with one of the components being the probability of default (PD) to act as the weights. Two major branches related to credit loss estimation are structural and reduced-form models, pioneered by Black and Scholes (1973) and Jarrow and Turnbull (1995), respectively. The difference between these two models is mainly in the inputs they use. Merton (1974) formalized the earlier models by incorporating information from a company's equity as a call option on its assets to assess the credit risk of a company. Some studies that have employed Merton's model include those of Derbali and Jamel (2019), Pfeuffer et al. (2019), Afik et al. (2016), Munandar (2014), and Anuwar and Jaffar (2017).

Under the reduced-form model, the assessment of credit risk does not depend on company parameters; instead, it deals directly with observable market data. The rating transition approach, a sub-branch of the reduced-form model studied by Jarrow et al. (1997), uses a Markov framework to build the term structure of credit risk spreads. Since then, the practice of credit rating transition matrices based on Markovian models has been popular in the literature, including studies by Möstel et al. (2020), Boreiko et al. (2019), Baena-Mirabete and Puig (2018), Xing and Yu (2018), Pfeuffer et al. (2019), and Pasricha et al. (2017).

Parameterizing the model from rating information is more relevant in practice since such data is observable, permitting the estimation of default rates without the need to analyse a company's financial statements. Rating data provided by rating agencies are often used as information to gauge credit risk (Kariya et al., 2019). Hence, ratings data are more informative, especially for speculative grade issuances (Gredil et al., 2022). Global rating agencies, such as Moody's and Standard and Poor's, now publish annual updates of historical transition matrices. This information, which shows the historical rating migration and default experience, is prevalently leveraged as estimates of default probabilities among analysts and risk managers (Cappon et al., 2018).

Default probabilities conditional to external factors based on rating observations

Various studies treat transition and default as functions of other external factors—either systematically or idiosyncratically (e.g., Zhu, 2016; Petrov and Pomazanov, 2009; Xing and Yu, 2018; Blümke, 2022). Authors integrate these multifaceted variables and construct models to explore the systematic and idiosyncratic elements that characterize specific industries or companies. For example, Petrov and Pomazanov (2009) and Kaniovski and Pflug (2007) develop a coupled model that integrates transition probabilities with correlated and uncorrelated assets into the distribution of defaults within sectors. A Markov chain model is used as the marginal law, but correlation coefficients within and between sectors and between rating classes are introduced for the joint law of migration of all portfolio components.

Xing and Yu (2018) have developed a continuous-time modulated Markov model to project the behaviour of rating transitions in the presence of unobserved structural market breaks (such as macroeconomic conditions). Modelling such an effect involves the integration of market fluctuations derived from data on rating history, corporate accounting, market instability, and the risk of structural breaks as indicated by the estimated time-varying coefficients.

Studies that incorporate ratings transitions to past economic cycles include Figlewski et al. (2012), Weißbach and Strohecker (2016), and Couderc and Renault (2004). These studies have documented that corporate defaults are susceptible to economic conditions. However, the level of default intensities across different markets and economies varies. Figlewski et al. (2012), for instance, utilized the Cox model to examine the effects of general economic conditions on rating transitions and defaults in the case of corporate issuers in the US market. They found a significant increase in the explanatory power of defaults after accounting for economic variables in the estimation. Studies along similar lines, but using different inputs to account for default measurements, such as Duffe et al. (2007) and Virolainen (2004), also supported this claim. Among the variables used, economic growth is observed to be of prime importance, with most works proxied by gross domestic product (GDP).

Table 1 summarizes various studies that deal with aspects of modelling and estimating credit risk using rating histories as inputs.

Table 1
Studies related to estimating default probabilities conditional on external factors using rating histories as inputs

Author	Issue	Model	Context of application
Blümke (2022)	Requirement of the accounting standard to take into account the macroeconomy aspect in estimating future average of default probabilities	Extend the existing discrete-time survival model and to incorporate additional time- and covariate-dependent shape parameter into the hazard function	Accounting standard with respect to the estimating future average of default probabilities
Xing & Yu (2018)	The channel through which market structural breaks (i.e., crises) affects firms' credit risk is too complex	Continuous-time modulated Markov model	Model's capability to capture the full effect of market structural breaks might be limited by the unknown magnitude of such events
Zhu (2016)	Credit risk must be captured at granular level that accounts for both regional and sectoral influence	Credit index model that is conditional to macroeconomic scenarios	Applicable for stress testing of banks with large loan portfolios
Rubtsov & Petrov (2016)	Construction of through-the-cycle ratings and point-in-time probability of default needed for regulation requirement	Two-step process to obtain default probability estimates, namely rating classification and calibration	Basel regulations and IFRS 9
Petrov & Pomazanov (2009)	Maturity effect and probability of default time structure in the implementation of Basel II	Method of maturity adjustment calculation	Validate the maturity adjustment formula for Basel II capital requirement
Figlewski et al. (2012)	Structural approach does not capture important credit default events such as rating downgrades into the model	Reduced form Cox intensity model	Fit the reduced-form Cox intensity models with a broad macroeconomic and firm-specific ratings to analyse the influence of these factors on the risk of credit events

Weißbach & Strohecker (2016)	Introduce a low-dimensional model that accounts for defaults triggered by cascading rating downgrades and immediate defaults from good ratings using local bank portfolio of medium sized	Time-continuous discrete-state Markov process	Low rated corporate debtors tend to improve their credit quality, while highly rated debtors are more likely to experience downgrades, resulting in a convergence towards medium rating grades.
Couderc & Renault (2006)	Default probability models often neglect past information and lack of consideration for lagged effects.	Fully non-parametric estimator of default intensities based on the Gamma kernel with a parametric component using Cox proportional hazard methodology, and maximum likelihood estimations	Two-factor models with time-varying trends is appropriate for capturing the dynamics of default probabilities over time

Source: Authors' work.

Existing studies address issues such as the inclusion of external factors to meet accounting standards, capturing the complexity of market structure breaks, and modelling credit risk at a granular level, considering regional and sectoral influences. The methods range from the extension of survival models to the inclusion of time-dependent parameters using continuous-time Markov models and the use of the reduced form of the Cox hazard model. These approaches aim to fulfil various applications, including stress tests for banks, compliance with regulatory requirements such as Basel regulations and IFRS 9, and the calculation of capital requirements. The models shed light on phenomena such as maturity effects on the probability of default and the dynamics of credit events influenced by economic factors. An investigation that estimates short-term default probabilities conditioned to economic changes, despite its importance, is conspicuously absent in the literature. This paper attempts to fill this void in the field.

Methodology

This paper aims to construct quarterly transition probabilities first unconditionally and then link them to the economic conditions represented by GDP growth. Consequently, this conditional model builds a quarterly term structure of default probability. The resulting estimate of the conditional loss distribution is then analysed. To achieve this, a common method for modelling credit risk is used, namely a finite Markov chain, and quarterly default probabilities are derived by directly fitting the roots of the annual transition matrices using the QOM approach. We also use the Markov chain to specify the migration between individual states in each quarter. Finally, we integrate the economic effects into the transition matrices by decomposing the specifications of the economic adjustment coefficients.

Markov chain

The Markov process is characterized by the assumption that the prediction of future distributions depends on the present state and that the present state only depends on the most recent previous state. Following the credit rating agencies' classification, we categorize state space into eight credit classes $S \in \{1, 2, \dots, D\}$, with state 1 being the highest (AAA) and state D being the lowest. Let X_n be the discrete-time Markov chain defined in a finite state space, S with transition matrix, P, where each n and every i_0, $\dots i_n$ and $j \in S$. The following form describes the movement of states from $X_0 = i$ to $X_1 = j$ within the Markov framework (Kijima, 2002):

$$P\{X_{n+1} = j | X_0 = i_0, X_1 = i_1, X_2 = i_2, \dots, X_n = i_n\} = P\{X_{n+1} = j | X_n = i_n\} = P_{i,j}, \qquad (1)$$

Since the current state is enough to predict the future of all distributions, this property ensures the feasibility of estimating the multi-period default probabilities once the current state is known. For this purpose, we use the cumulative average one-year transition rate sourced from S&P's default study to serve as a base or initial transition matrix. If an obligor currently has a high credit risk, the probability of default in the future is higher than for an obligor with a low credit risk. Assuming that the transition is Markovian, the matrix of the one-year transition matrix is of the form:

$$\mathrm{P} = \begin{bmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,n} \\ \vdots & & \ddots & \vdots \\ P_{n-1,1} & P_{n-1,2} & \cdots & P_{n-1,n} \\ 0 & 0 & & 1 \end{bmatrix}$$

Let P be the one-year transition matrix, where $P_{i,j}$ is the probability that the credit risk of an issuer changes from rating grade I to rating grade j within one year. The following properties govern the transition matrix under Markovian assumptions (Vaněk & Hampel, 2017; Engelmann & Ermakov, 2011):

- The entries in the transition matrix are all probabilities, $0 \leq P_{i,j} \leq 1$ *for i,j = 1,..., n*
- The sum of all probabilities of each row equals one, $\sum_{j=1}^{n} P_{i,j} = 1$ *for j= 1, 2,..., n*
- The probabilities in the rightmost column represent the default probability (*n*-th state) for a given *i*.
- The rating grade *n-th* (last row) represents the absorbing state, $P_{j,n} = 0$ for $j < n$ and $P_{n,n} = 1$

The first property reflects the dependency of probability on the length of observation period, which is one year. The second property ensures that for every rating grade *j for j = 1, 2..., n*, the distribution of probabilities in every row, *j* state must equal to one. If one obligor is rated AA at the beginning of the year, it either remains in the same rating grade or moves to another rating grade (could be upgrade or downgrade or default) at the end of the period. Hence, sum of the transition rate for each row must be equal to 1. The columns to the right of rating in Table 2 show the default rates for each rating grade. The last row represents the absorbing state, where it is impossible to migrate backward once this state is achieved. Taking the transition matrix in Table 2 as the base matrix, P with eight rating grades and assuming that the process is currently in the fourth one, this can be written as S_t = (0 0 0 1 0 0 0 0). The calculation of transition probability of the process from the current state to another state within the possible grades at t_2, using the 8x8 transition matrix P, can be expressed as $S_{t+n} = S_t . P_{t+1}^{n}$. The migrating state probabilities from current state *i* to another state *j* within one year at a_n can be computed recursively by matrix multiplication.

Regularisation approach

Often, rating agencies provide a one-year transition matrix to indicate the credit migration of a pool of borrowers with equivalent rating characteristics from one state to another. This one-year interval is considered most relevant as the frequency of credit ratings generally occurs once (or probably twice) per year. Observing transition probabilities in a period shorter than one year (i.e. quarterly) would generally be too short to estimate a reliable transition matrix (Israel et al., 2001). Moreover, access to time series of raw default data by credit rating at specific granularity is usually not possible (Hughes & Werner, 2016). Therefore, practitioners must rely on annual transition matrices. However, a bond often has a residual maturity that is not precisely specified in years, so mapping such a bond to the probabilities of an annual transition matrix would be less accurate. Consider a bond with a BBB rating having a 3-quarter lifespan before the expiry date; it must allocate the annual PD rate at a specific reporting date, an amount that is higher than it is supposed to carry. A higher PD rate means the bondholders need to put aside a greater loss allowance because of the higher risk weight perceived by the matrix.

To derive probabilities for a transition matrix that can satisfy different maturity profiles of individual bonds, we need to compute transition matrices over quarterly time horizons. Mathematically, this can be done by finding the fractional roots of the matrix. However, there are two general problems with simply raising the matrix to the power *n*. First, the resulting matrix typically would contain negative elements, making the matrix invalid. Second, even when it does not contain negative elements, it might create an identification (non-unique) problem, whereby it can produce several possible transition matrices. This leads to the problem of determining which matrices should be used for valuation purposes. Prior studies (Israel et al., 2001; Kreinin & Sidelnikova, 2001) have proposed an alternative regularization approach to obtain a valid transition matrix over arbitrary time horizons from the roots of the annual transition matrix (i.e., QOM) that is feasible for discrete-time models. Specifically, approximating the roots of the matrix is calculated as the problem of minimizing the distance between any two points x and y in a simplex, R^n,

$$dist(x,y) = \sqrt{\sum_{t=1}^{n}(y_i - x_i)^2}\,, x, y \in R^n \tag{2}$$

The above problem can be solved on a row-by-row basis by projecting arbitrary points to the simplex. The following procedure for deriving the quarterly probability of default term structure is a simplification of the regularization procedure conducted in an iterative manner suggested by the authors, with an additional iterative condition of setting the difference ε of each row to specific decimals, $\mathrm{x}10^{-10}$.

Step 1: Compute the quarterly roots of the transition matrix using the equation of spectral decomposition

$$P^{1/4} = Q\Lambda Q^{-1} \tag{3}$$

where P denotes square nxn matrix whose n values can be 1, 2,..., n, Q is the square nxn matrix whose *i*th column is the eigenvector, Λ is the diagonal matrix whose diagonal components are the eigenvalues, i.e. $(\mathrm{p_{ii}})^{1/4}$, and Q^{-1} is the matrix inverse of the square nxn matrix whose *i*th column is the eigenvectors, $\mathrm{Q}^{-1} = \frac{1}{|\mathrm{Q}|}\mathrm{adj}(\mathrm{Q})$. To get the eigenvalues and eigenvectors of a matrix, it follows the following adaptation:

$$(\mathrm{P} - \lambda_{\mathrm{i}}\mathrm{I})\mathrm{v} = 0 \tag{4}$$

where λ_{i} are the eigenvalues of the matrix P, I is identity matrix which is the matrix equivalent to 1, and v indicates a nonzero vector corresponding to scalar λ. Raising the matrix X to $P^{1/12}$ may result in the matrix to contain negative entries and hence invalid. To remove the negative elements, step 2 is used.

Step 2: Set all negative elements to zero and once any elements are fixed to zero, it remains fixed until all elements in the matrix are all non-negative.

$$p_{ij} = \begin{cases} 0 \text{ if } (i \neq j) \text{ and } p_{ij} < 0 \\ p_{ij} \text{ otherwise} \end{cases} \text{for } i,j = 1, 2, \ldots, n \quad (5)$$

After the removal of negative elements, the new row sums will be greater than 1,

$$\sum_{j=1}^{n} p_{ij} > 1 \qquad for\ i = 1,2,\ldots,n \quad (6)$$

Step 3: Thus, we have to adjust the non-negative elements in p_{ij} so that each row sum, $\sum_{j=1}^{n} p_{ij} = 1$. To solve this, construct p_i^{k+1} by projecting $p_i^{k+1} = p_i^k - \lambda$, where

$$\lambda = \frac{1}{n}\left(\sum_{i=1}^{n} p_i - 1\right) \quad (7)$$

Means, for any k=1 to n, do

$$p_i^{k+1} = p_i^k - \frac{1}{n}\left(\sum_{i=1}^{n} p_i - 1\right) \quad (8)$$

Calculate Step 3 for all the rows over each time interval (quarters) until the convergence of row sums closely equals 1. For this purpose, we set the differential of row sums,

$$\sum_{i=0}^{n} p_{ij} - 1 < \varepsilon \quad (9)$$

where $\varepsilon = x10^{-10}$, then stop the procedure. The p_i^{k+1} is considered the optimal solution to the distance minimisation problem. This limit was imposed given the error difference of row sums is close enough to 1. The iterative process of finding the optimal solution continues until it satisfies the above equation. This approach is adopted by iterative row projection, on a row-by-row basis. By permuting the row elements, one can observe that p_i^{k+1} moves in a descending order.

Step 4: Simulate the regularised $P^{1/4}$ using matrix multiplication up to four quarters based on the Markov chain. The effect of this adjustment on the quarterly roots of transition matrix will be measured using maximum absolute deviation (MAX) (Kreinin & Sidelnikova, 2001) and mean absolute deviation (MAD) (Ibrahim & Wong, 2006), expressed as follow:

$$\text{MAX}(P', P) = \|P' - P\|_\infty = \max_{i,j}|p'_{i,j} - p_{i,j}| \quad (10)$$

$$\text{MAD}(P', P) = \|P' - P\|_1 = \frac{1}{n^2}\sum_{i,j}|p'_{i,j} - p_{i,j}| \quad (11)$$

Integrating the impact of economic conditions

To capture the impact of economic growth on transition probabilities, the transition matrix at each time interval needs to be adjusted. The decomposition of economic effect into transition probabilities takes the following form:

$$S_{t+n} = S_t . \sum_{y=1}^{n} P_{t+y} \quad (12)$$

where P_{t+y} is a transition matrix at anytime *t*, specifically adjusted by the decomposition of the EAC. Identification of *y* factors to be used to represent economic conditions in this paper is GDP growth. To get the coefficient value, the relationship between the GDP growth and the default patterns must be investigated first. This test is crucial because it ensures whether this economic factor can be a candidate to explain the variations that influence the forward-looking default probabilities in the matrix $P_{i,j}$. For this purpose, we adopt a simple linear regression, as expressed in the following form:

$$Y = \beta_0 + \beta_1 X_1 + \varepsilon, \tag{13}$$

where Y denotes default rate, β_0 is the intercept, β_1 = regression coefficients of the GDP growth, and X_1 represents GDP growth. The difference between the forecasted GDP growth and the baseline values will then be used together with the coefficient value β_n of GDP growth (notated henceforth as $\Delta_{t+y}.\mathrm{EAC}$ effect) to adjust the annual transition probabilities. The distribution of Δ. EAC explicitly depends on the state of forecast values, that is, whether it is higher or lower than the baseline values. If in the next quarters, the economic conditions are better than the current conditions, the distribution on the lower grade from the diagonal elements will be reduced, and the higher grade will be increased. On the contrary, if the economic conditions are expected to perform poorer than the current conditions, the distribution on the lower grade from the diagonal elements will be increased, and the higher grades will be decreased

Following the framework generalisation of Alternative III by Vaněk and Hampel (2017), this alternative decomposes the $\Delta_{t+y}.\mathrm{EAC}$ effect between the directions of transition probabilities to both better and worse grades from the diagonal entries as a starting point. This decomposition is done on a row-by-row basis. The sum of 'plus' changes will equal the sum of 'minus' changes in each corresponding row; hence the row sum is zero,

$$P_{i,j} - 2.\frac{\gamma}{i}.\frac{2(i-j)+1}{i} \; for\ i,j < r-1\ and\ j \leq i \tag{14}$$

where

$$\gamma = \frac{\Delta_{t+k}.EAC}{2(r-1)}.\frac{2i-1}{r-1} \tag{15}$$

The above equation adjusts the entries of the better grades and on the diagonal with an assumption that the $\Delta_{t+y}.\mathrm{EAC}$ effect will have a greater influence on the higher rating grades from the initial state in an increasing trend.

$$P_{i,j} + \frac{\sigma}{r-i}.\frac{2(r-j-1)+1}{r-i} for\ i,j < r-1\ and\ j > i \tag{16}$$

On the contrary, the above equation adjusts those entries in the worse grades with an assumption that the $\Delta_{t+y}.\mathrm{EAC}$ effect will have a greater influence on the lower rating grades from the initial state, in a decreasing trend. Where

$$\sigma_i = \sum_{j=i+1}^{r} \delta_{i,j} \tag{17}$$

$\delta_{i,j}$ represents 'plus' changes associated with probabilities $P_{i,j}$, thus σ_i denotes the sum of these changes in each row. If the sum of $\Delta_{t+y}.EAC$ effect is negative, the increasing transition probabilities starting from the diagonal entries will depict negative values, while the decreasing transition probabilities under the diagonal elements will depict positive values. Using the above assumptions, we can observe that the variance of decreasing and increasing trend from both directions at each r-stopping point move such that the value difference is similar and remains constant.

Data

Initial transition matrix. We use the cumulative average of one-year transition rates sourced from S&P's default study as the base to calculate the short-term multi-period transition matrix. Usually, credit rating agencies provide rating transition matrices to indicate the credit rating behaviours of a pool of obligors with similar rating characteristics and publish an annual update of historical issuer defaults. A static pool approach is employed whereby a pool of issuers, called a cohort, holding similar credit ratings is grouped at a specific formation date (beginning of the year), and the constitution of every cohort remains fixed over the sample period. The average transition percentage aggregates historical credit rating migration of a group of obligors (latest pool consists of 23,288 issuers) that were rated from 1981 to 2023. The data was also corrected to withdrawals, redemptions, or suspensions. This means that if the issuance is withdrawn, matured, or suspended at the time formation starts, it will no longer serve as a constituent of the pool.

The inclusion of new constituents into the pool starts typically at the beginning of the year and observation of rating stability will be tracked until the end of a specific time horizon, which could be one year, two years or longer. In each time interval, some obligors of the pool that have survived to specific horizon, *t*, might be upgraded, downgraded or at default. The symbols for rating grades are AAA, AA, A, BBB, BB, B, C and D. In practice, transition matrices have a greater number of rating states. As an example, Table 2 shows the historical cumulative weighted average of one-year transition matrix for ratings from AAA up until D, based on the time frame from 1981 to 2023. Each row represents an initial rating at the beginning of the year, and each column corresponds to ratings at the end of the year. These ratings will change continuously based on the time frame.

Table 2
Adjusted unconditional transition matrix obtained from S&P ratings for the period spanning1981-2023.

	AAA	AA	A	BBB	BB	B	C	D
AAA	0.900	0.092	0.0053	0.0003	0.0010	0.0003	0.0005	0.0000
AA	0.005	0.904	0.0781	0.0046	0.0005	0.0006	0.0002	0.0002
A	0.000	0.015	0.9205	0.0487	0.0025	0.0010	0.0001	0.0005
BBB	0.000	0.001	0.0318	0.8990	0.0338	0.0042	0.0009	0.0014
BB	0.0000	0.0002	0.0010	0.0460	0.8109	0.0660	0.0053	0.0059
B	0.0000	0.0002	0.0006	0.0016	0.0460	0.7741	0.0500	0.0308
C	0.0000	0.0000	0.0008	0.0014	0.0044	0.1376	0.4638	0.2681

Source: S&P Ratings Global (2024).

Default rate. S&P defines default as any breach of the binding obligations under the original terms of an agreement between the issuer and the bondholders, based on the premise that the bondholders are likely to be exposed to monetary losses. We take the historical default rates, a ratio between several defaults against several firms, as the proxy for the likelihood of corporate defaults. Data is sourced from S&P's default study report, and the time frame spans from 1981 to 2023.

Economic growth. The default probability model in this paper integrates the variation of GDP growth into the estimation of default propensity. We use an annual change of GDP to serve as a proxy to reflect economic conditions, considering its role as a broad aggregate variable in ascertaining the behaviour of an economy at large. GDP measures the total value of national's output, whereby it is a common variable to reflect economic strength. When GDP growth deteriorates during a contraction period, the probability of defaults among corporations will be relatively high because slower GDP growth is often accompanied by lower corporate earnings expectations hence lower ability to generate cash flows to service their debt obligations. In contrast, fewer defaults are expected during expansionary times.

Results

To illustrate this application, we will show the conditional model by conditioning the "unconditional" model on explicit macroeconomic factors. For this purpose, we use gross domestic product (GDP) to allow the model for a joint consideration. We incorporate the variation in GDP growth into the estimate of the propensity to default. We use quarterly forecast GDP as an indicator of future economic conditions because it serves as a comprehensive aggregate variable to determine the behaviour of an economy at large. GDP measures the total value of national output, making it a common variable to represent economic strength. When GDP growth deteriorates during a contraction, the likelihood of debtors defaulting is likely to be relatively high, as slower GDP growth is often associated with lower profit expectations for companies, making them less able to generate cash flows to service their debt obligations. In contrast, fewer defaults are expected during expansionary times.

The shared exposure of each rating class towards future GDP growth is determined by the correlation coefficients value derived from Hu *et al*. (2021). Since the values reported in their study cover only three rating classes, representing respectively the investment, medium and speculative quality grades, we assume that the decomposition of the GDP effects on the default distribution for the remaining rating classes follows this classification. Table 3 shows the classification of such a decomposition on other rating classes with the respective estimated GDP coefficient values.

Table 3
Relationship between historical default rate and GDP growth across rating grade

Rating	Decomposition to other rating grades	Estimated GDP coefficient (%)
A	AAA, AA	-12.094
Baa	BBB	-16.988
Ba	BBB, BB and C	-12.985

Note: A-rated bonds are investment grade, Baa-rated bonds are medium grades, and Ba-rated bonds are speculative grades. *Source:* Authors' work.

Table 4
Quarterly economic forecast of US GDP for year 2023

	Baseline and forecast GDP	Difference (Δ)	Coeff. value	$\Delta_{t+y}eac$ effect
Q1 2023	1.10	5.5749		
Q2 2023	2.90*	0.0180	-0.1209**1	-0.0022
			-0.1699**2	-0.0031
			-0.1299**3	-0.0023

Note: * Quarterly GDP Estimate sourced from Federal Reserve Bank of Atlanta. **denotes 1 percent significance level. 1, 2, 3 represent coefficient values for A-rated, Baa-rated, and Ba-rated, respectively. *Source:* Authors' work.

To determine the impact of GDP changes on the transition, we use the quarterly forecast values of the matrix of credit transition and default probabilities constructed from S&P rating data as shown in Table 4.

Decomposition of economic adjustment coefficient into forward quarter-end transition

The impact of GDP growth movement (Δ EAC effect) is decomposed on the quarterly transition matrix using the EAC specifications of the alternative III proposed by Vaněk and Hampel (2017). This adjustment allocates the effect proportionately between the directions of *i* to another state *j* (can be towards better grades or to the worse grades) from the P_{ii} – diagonal components. As shown in Tables 5 and 6, the proportion of Δ. EAC effects are distributed between the directions of transition from the initial rating grades that is from the diagonal elements to both upgrade and downgrade directions. This breakdown of the GDP's impact on different credit ratings shows that when economy contracts, the propensity of each grade moving to better ratings will reduce. In contrast, the propensity towards lower ratings will increase. Applying the whole process described above, we obtain the quarterly probability of default term structure estimates that are conditional on GDP growth for each rating grade (AAA, AA, BBB, BB, B, and C) in the next quarter, as shown in Table 7 below.

Table 5
Decomposition of economic adjustment coefficient onto quarterly transition and default distribution to better grades

	AAA	AA	A	BBB	BB	B	C	D
AAA	0.0000							
AA	0.0002	0.0001						
A	0.0003	0.0002	0.0001					
BBB	0.0003	0.0002	0.0001	0.0000				
BB	0.0002	0.0002	0.0002	0.0001	0.00003			
B	0.0003	0.0003	0.0002	0.0001	0.0001	0.0001		
C	0.0002	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	

Source: Authors' work.

Table 6
Decomposition of economic adjustment coefficient onto quarterly transition and default distribution to lower grades

	AAA	AA	A	BBB	BB	B	C	D
AAA		-0.00001	-0.00001	-0.00001	-0.00001	-0.00000	-0.00000	-0.00000
AA			-0.00009	-0.00007	-0.00006	-0.00004	-0.00002	-0.00000
A				-0.00017	-0.00013	-0.00009	-0.00006	-0.00002
BBB					-0.00017	-0.00014	-0.0001	-0.00006
BB						-0.00044	-0.00026	-0.00009
B							-0.00084	-0.00028
C								-0.00063

Note: Transition to lower grades depict reduction in default rate because the changes have been allocated to initial rating, shown in diagonals. Source: Authors' work.

Table 7
Adjusted transition probabilities conditional to GDP growth across rating grades for the next first quarter

	AAA	AA	A	BBB	BB	B	C	D
AAA	0.8111	0.1665	0.0168	0.0013	0.0018	0.0007	0.0007	0.0002
AA	0.0091	0.8192	0.1425	0.0120	0.0011	0.0011	0.0003	0.0005
A	0.0011	0.0287	0.8499	0.0884	0.0057	0.0019	0.0001	0.0011
BBB	0.0000	0.0023	0.0582	0.8114	0.0575	0.0089	0.0014	0.0032
BB	0.0008	0.0009	0.0037	0.0791	0.6632	0.1048	0.0095	0.0138
B	0.0000	0.0009	0.0017	0.0051	0.0735	0.6109	0.0609	0.0674
C	0.0000	0.0001	0.0015	0.0026	0.0122	0.1709	0.2223	0.3956

Note: This adjusted transition probabilities matrix is called 'conditional' matrix proportionate to changes in economic factors, GDP. *Source:* Authors' work.

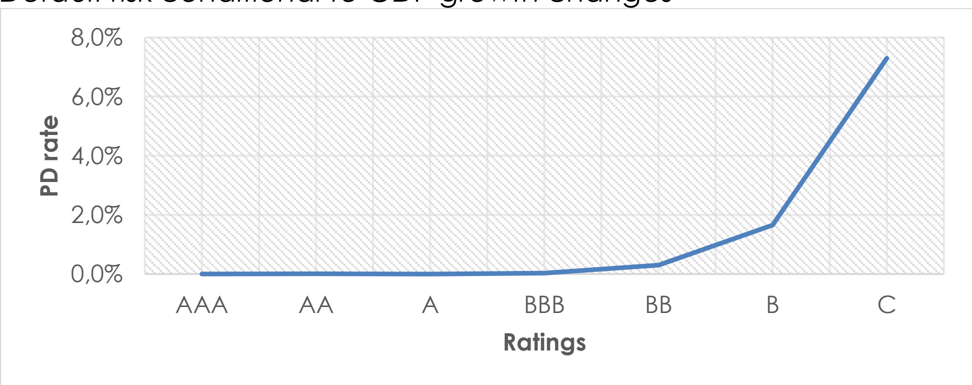
For the next quarter, the conditional model describes the following; first, corporate obligors with high investment grade yield lowest default risk among the quality scales. Obligors rated AAA, AA, and A show estimates of default probability conditional on GDP growth less than 0.11 percent with AAA-rated obligors has 0.02 percent, AA-rated obligors less than 0.049 percent, and A-rated obligors less than 0.107 percent. Second, obligors with medium-investment grade yielded default risk less than 3.19 percent, whereas speculative grade bonds showed highest default risk with of BB-rated obligors less than 1.4 percent, of B-rated obligors less than 6.7 percent and C-rated obligors above 30 percent.

Note that the conditional default probability curve for all rating classes, as shown by Figure 1 shows a linear upward trend. The analysis provided also reinforces that speculative-grade or non-investment issuances have higher propensity to default in the future. This propensity, depending on the credit quality of the obligors, is driven by the changes in the economic growth which points that lower-grade bonds are more subject to higher exposure of systematic risk compared to investment grade bonds. In particular, the increment in default probability, as measured within the quarterly interval, shown a variation if compared to non-speculative-grade bonds.

This result is in line with Samsuddin *et al.* (2011), Figlewski *et al*. (2012) and Weißbach and Strohecker (2016) who reported lower rating grades often demonstrate higher propensity to default, except that Figlewski *et al*. (2012) found that economic variables (with additional variables incorporated) have more explanatory power in downgrade transitions. In addition, this study also analysed the distance to default measured based on the difference in values of default probability between quarters and rating states. In terms of duration, that is within quarters, the result showed that non-speculative grade or investment-grade bonds tend to pose higher distance to default over time, whereas the speculative one yielded an opposite pattern. The value of default probability transited between rating states for each rating grade also pointed an ascending pattern when period lengthens.

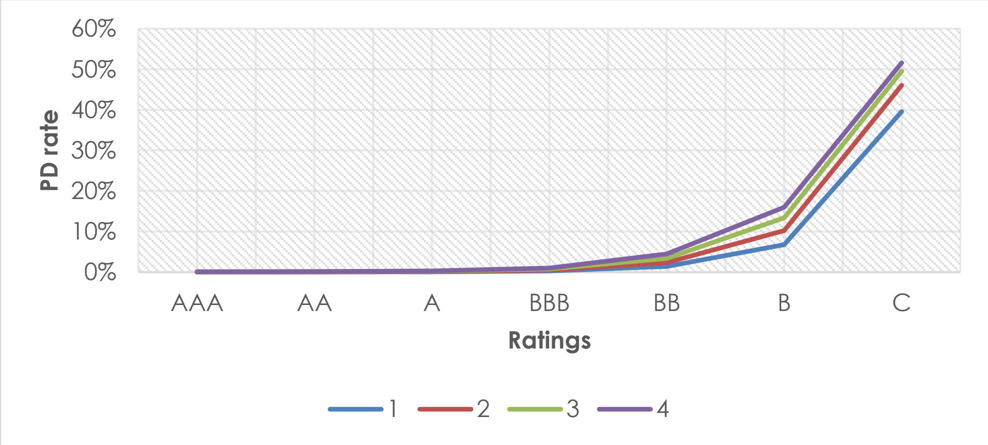
The estimated PD curve in the figure 2 plots the PD rate for rating grades with different time to maturity in quarters. The trend depicted by the curve estimates the monthly average credit risk conditions of corporate obligors for the next five years. The values along the curve reflect the PD rate the obligors carry at each quarter. As shown in the graph, PD rate slope increases in a positive linear trend when the maturity lengthens. This makes sense as investors may expect a higher risk level for obligors with longer maturity duration to compensate for the volatility associated with the borrowing time.

Figure 1
Default risk conditional to GDP growth changes



Source: Authors' work.

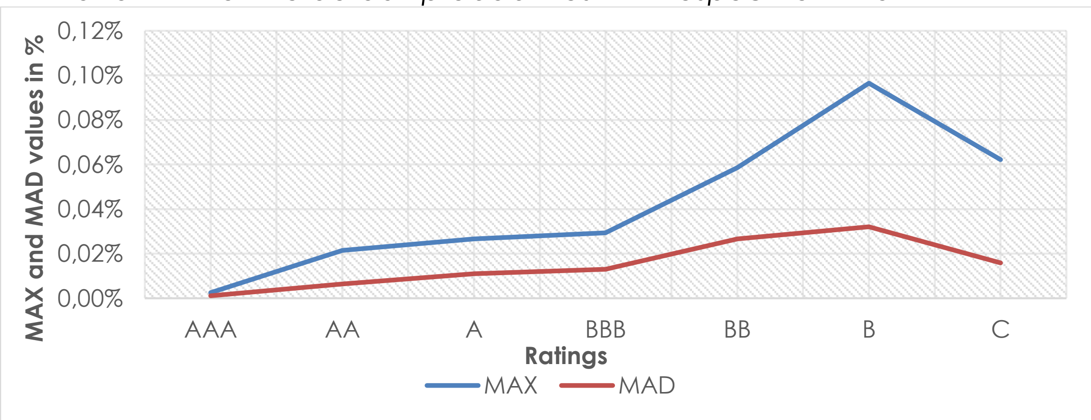
Figure 2
Quarterly term structure of default probability



Source: Authors' work.

Figure 3 shows the results of MAX (blue line) and MAD (orange line). The error difference, as measured by the maximum absolute deviation between the annual transition matrix with the matrix adjusted by regularisation, increases as the period lengthens.

Figure 3
MAX and MAD of the default probabilities with respect to P matrix



Source: Authors' work.

In particular, the error difference above was strongly affected by the deviation arising from the post-regularisation whereby we took P^x rather than taking the actual annual transition matrix for an integral number of years, P^2x. Therefore, when the period spans for several quarters, the error difference is carried forward on each time interval making the trend move upward for both MAX and MAD.

Discussion

Overall, the results showed that speculative-grade or non-investment issuances have higher propensity to default in the future. This propensity however, slightly driven by the changes in the economic growth specifically, suggesting that low-grade bonds are more subject to idiosyncratic risks compared to systemic risk. In particular, the increment in default probability, as measured within the quarterly interval, shown a huge gap if compared to non-speculative-grade bonds. This result is in line with Samsuddin et al (2011), Figlewski et al. (2012) and Weißbach & Strohecker (2016) who reported lower rating grades often demonstrate higher propensity to default, except that Figlewski et al. (2012) found that economic variables (with additional variables incorporated) have more explanatory power in downgrade transitions. In addition, this study also analysed the distance to default measured based on the difference in values of default probability between quarters and rating states. In terms of duration, that is within quarters, the result showed that both investment-grade and speculative-grade bonds tend to pose higher distance to default over time. The value of default probability transited between rating states for each rating grade also pointed an ascending pattern when period lengthens.

It is important to note that; first, the estimation is built based on the aggregate rating transitions frequencies by rating class data and thus, do not reflect the state of individual firm-level differences. Nonetheless, the data inputs used in this paper covered full population of rated firms over a long period of time, hence, this does not impact the reliability of the results. Second, the integration of external factor onto the estimation only considers economic fluctuations and do not include the effects of other variables. We cannot exclude the possibility that changes in other factors such as inflation or unemployment rate might be plausible to be included, hence future work along these lines could seek different modelling techniques, firm-level variables and macroeconomic factors for default probability estimation based on different credit risks.

The results shown highlight opportunities for banks and financial regulators as follows. Banks especially those in the credit risk side can apply the model we presented here using their internal ratings data to quantitatively measure a forward-looking bond exposure across different tenure buckets, which might yield results that are more aligned to their respective silos. With the historical fluctuations in economic cycle taken as additional factor to estimate future default probability, it can device the regulators by providing entities with alternative reference to the in-house generated models to meet regulatory requirements. In addition, short-term estimates allow investors to assess credit risk at granular basis and can effectively utilise the proposed method tailored to discrete estimations.

Conclusion

In this paper, we show how the estimation of future state of credit loss distribution conditional to fluctuations in economic growth at short-term frequencies can be constructed using rating histories as inputs. In particular, the estimation is conducted using Markov chain and we use quasi-optimisation of the roots matrix to extend the model within a quarterly domain. We further integrated the effect of economic growth using specifications of economic adjustment coefficient.

There results reveal several interesting outcomes. First, variations in GDP fluctuations minimally affect historical default events, suggesting firm-specific factors drive default propensity rates. Investments grade corporate bond obligors, particularly those rated AAA, AA, A, and BBB-rated obligors exhibit low default propensities. Non-investment grade obligors as well indicate results with descending order, with C ratings indicating highest default propensity. Additionally, the distance between default probabilities over time show that investment-grade obligors maintain higher distance as the time horizon lengthens compared to non-investment obligors. The difference in default rates between its neighbouring states, on the other hand, posted an upward trend over the longer horizons.

The approach used in this paper to estimate the probability of default term structure has several features of relevance to credit risk management. First, the integration of economic growth onto the credit rating transition probabilities using the EAC method is very straightforward and at the same time, allows for the allocation of the effects on different rating grades. This enables for the analysis of the scale of economic shocks on credit risks. Second, developing quarterly PD term structure can satisfy the mapping of individual bonds with varying maturity profiles that poses similar credit risks, making credit loss estimation more accurate and precise in values.

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