A new selection operator for genetic algorithms that balances between premature convergence and population diversity

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Abstract. The research objective is to find a balance between premature convergence and population diversity with respect to genetic algorithms (GAs). We propose a new selection scheme, namely, split–based selection (SBS) for GAs that ensures a fine balance between two extremes, i.e. exploration and exploitation. The proposed selection operator is further compared with five commonly used existing selection operators. A rigorous simulation–based investigation is conducted to explore the statistical characteristics of the proposed procedure. Furthermore, performance evaluation of the proposed scheme with respect to competing methodologies is carried out by considering 14 diverse benchmarks from the library of the traveling salesman problem (TSPLIB). Based on t-test statistic and performance index (PI), this study demonstrates a superior performance of the proposed scheme while maintaining the desirable statistical characteristics.

1. Introduction

Development of GAs stems from the seminal work of Holland [20], which exploited the philosophical basis of Darwin's understanding of the evolution process [1, 46]. Since then, GAs have attracted a great attention and, as a result, GAs maintain its key role in optimization literature. For example, [39] highlight the applicability of GAs in the field of behavioral ecology to explore the vigilance behavior in animals. Further, [44] used GAs in modelling of membership behavior in stock market and to increase the scalability in credit risk assessment. More recently, [43] employed GAs to achieve optimal satellite selection for global positioning system (GPS). Other than these, a stream of applications of GAs can be witnessed in multidisciplinary fields, interlocking medicine [14], artificial intelligence [37], etc. The popularity of GAs is most commonly coined with its ability of solving complex multidimensional and multi–models optimization problems with minimum information required about objective function, see for example [6]. Furthermore, [2] recommended GAs to process multi objective optimization. A comperhensive and detailed overview of GAs features are presented by [35, 47].

Unfortunately, GAs suffer from the premature convergence in pursuit of finding optimal solutions [13], regardless of the their utility in all aspects. The problem of premature convergence

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is in fact rooted in the philosophical orientation of GAs as summarized by [24], who noted that "GAs are characterized by selection algorithms that favor the more fit chromosomes in their populations and by crossover and mutation operators that combine and modify existing chromosomes to generate novel offspring". The fondness of more fit chromosome is more likely to result in compromising the population diversity. Optimization literature acknowledges the diversity of population as a vital component in search of an optimal solution. This is evident by the discussion on the relevance of premature convergence and population diversity by [9, 25] and a more recent by [1, 11, 35]. Based on these studies, it is clear that the performance of GA is most affected by the choice of selection operators. Therefore developing better selection scheme remains the most important issue in the body of contributions associated with GAs.

Acknowledging the importance of selection phase of GAs, this research contributes to the literature by introducing a novel selection operator, namely the split–based selection (SBS). Our proposition mainly focuses on facilitating the convergence process by maintaining desirable level of population diversity. The objective is achieved by attaining a balance between the key concepts of exploration and exploitation. We used fitness rank of participants in concordance with normality of generations to aid the selection process. The use of underling mathematics of the operator, enables the opportunity to all fitness classifications members be selected, from worst to best. The encouraging results of this delicate selection scheme are documented throughout the article.

This article is divided into six main sections. Section 2 briefly introduces existing selection operators. In Section 3 we propose new selection operator and give its theoretical and mathematical foundations. Further, Section 4 investigates stochastic properties of the newly proposed operator. Inspired by the stochastic features, Section 5 delineates the applicability of proposed methodology in solving practical problems. New proposed operator is employed to the 14 famous benchmarks instances from the library of traveling salesman problem. Lastly, Section 6 concludes the study along with a brief discussion of future research.

2. Tradational selection procedures: theory and methods

In this section, we provide a brief review of the most commonly employed selection operators in the GAs literature.

The first one, noted as the most popular by [1], is known as fitness proportional selection (FPS). The methodological orientation of this scheme is based on the understanding that fitter individuals should have a higher chance of selection as a member of parent population. For this purpose, first we calculate the fitness of all individuals using the following rule:

$$f_i = \beta(1 - \beta)^i, \qquad i \in \{1, 2, ..., K\}, \tag{1}$$

where f_i is the $i - th$ individual of ascending order population and $\beta \in (0, 1)$ and generally suitable within the range of 0.01 to 0.3 [51]. The selection probability of $i - th$ individual p_i is directly proportional to it's fit:

$$p_i = \frac{f_i}{\displaystyle\sum_{i=1}^{K} f_i}, \qquad i \in \{1, 2, ..., K\}, \tag{2}$$

where K represents the size of the population. In statistical literature, one may recognize that the operational mandate of FPS is equivalent to that of the probability proportional to size (pps) sampling scheme. The straightforward nature of FPS procedure make it a feasible candidate in various GA applications, see for example [34, 45].

The next procedure is the linear rank selection (LRS), which was first introduced by [5] to cater for the issue of premature convergence attributed with FPS. In his seminal work Baker

[5] emphasized the use of rank–based selection criteria to provide better opportunity to select weaker individuals and thus offered a smoother selection function. Based on LRS procedure, the selection probability of the $i - th$ individual is assigned according to the following rule:

$$p_i = \frac{1}{K}\left(\eta^- + (\eta^+ - \eta^-)\frac{i-1}{K-1}\right), \qquad i \in \{1, 2, ..., K\}, \tag{3}$$

where i is the rank of the individual based on fitness status and K is the population size, while η^- and η^+ are the parameters depicting the selection probabilities of worst and best individuals based on their ranks, respectively. For the estimation purpose of the function (3), Baker imposed a constraints $\eta^+ = 2 - \eta^-$ and $\eta^- \geq 0$. A well documented weakness of LRS is slower convergence of the algorithm [1, 35]. This drawback is rooted in its methodology based on ranks instead of fitness values directly for selection. As a result, even if individuals differ notably in fitness status, the ranks remain the same unable to reflect the difference with desirable intensity and therefore naturally compromise relevant information. This issue becomes more obvious in the case of a larger population where ranks can be considered as a realization from the uniform distribution.

To resolve the issue of LRS slower convergence, [31] proposed another rank–based selection scheme named exponential rank selection (ERS). To distinguish from LRS, [31] suggested that the selection probabilities increase exponentially from worst individual to best:

$$p_i = \frac{r^{K-i}(1-r)}{1-r^K}, \qquad i \in \{1, 2, ..., K\} \tag{4}$$

where r is a constant ratio defining the inclusion weights of individuals based on their fitted ranks. The constant ratio, r, is capable of taking values over the range $0 < r < 1$, but for maximum gain values of r closer to 1 is recommended by [28, 31, 40]. The acceptability of ERS as a popular selection method is evidenced by various applications, for example see [28, 40].

Another selection procedure BTS (binary tournament selection) was introduced by [3], who argued that instead of relying on probabilities, selection of individuals should be based on direct competition. Recognizing the importance of population diversity, [3] urged lower tournament size. Thereby, pair–wise comparison becomes the most common theme in tournament selection schemes [4, 27]. The selection probability of $i - th$ ordered individual is given as:

$$p_i = \frac{1}{K^t}\left((i)^t - (i-1)^t\right), \qquad i \in \{1, 2, ..., K\} \tag{5}$$

where t represents the array of tournament size. Many have appreciated the logical orientation of binary tournament selection (BTS) in persuasion of the population comprises of fitter individuals [27].

The selection oriented literature was further enriched by [24] employing a probability based threshold level to select the winner of the tournament called probabilistic 2–tournament selection (PTS). Julstrom showed that the competition winner will survive with a probability $0.5 < q < 1$, where the loser will get another chance of competing, with probability $1 - q$. The mathematical expression providing the selection probability of the $i - th$ ordered individual is calculated as:

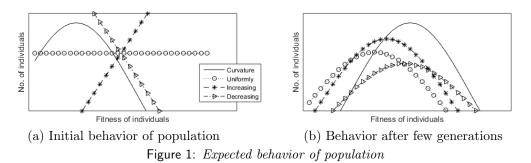$$p_i = \frac{2(i-1)}{K(K-1)}q + \frac{2(K-i)}{K(K-1)}(1-q) \qquad i \in \{1, 2, ..., K\}. \tag{6}$$

This selection procedure has been used in various applications [28, 40].

3. Proposed selection procedure

3.1. Motivation

From the aforementioned studies, one may recognize the urge of adaption of selection pressure while maintaining population diversity in the selection process to facilitate the achievement of optimal conditions. Figure 1(a) presents four hypothetical and most likely scenarios depicting the initial fitness status of individuals in a population. The fitness of individuals is represented through (i) moving along curvature, (ii) uniform distribution, (iii) increasing trend and (iv) decreasing trend. Figure 1(b) highlights the resultant fitness behavior after few generations.

It is noticeable that regardless of initial population trends, fitness of generations ultimately approaches to a normal distribution. The essence of our proposed approach lies in achieving the expected individuals fitness behavior in more cohesive manner. To formally proceed, let us consider two highly employed extremes of selection in the GA literature; LRS and FPS. The LRS emphasizes on maintaining higher levels of population diversity (in more technical language also known as exploration) at the cost of selection pressure and results in slowest convergence of GAs. On the other hand, FPS emphasizes high selection pressure (also known as exploitation) while scarifying the diversity and as a result remains the prime candidate suffering from premature convergence. Given the delicate nature of the matter, in the next subsection, we propose a new operator capable of achieving more balance between exploration and exploitation. This new scheme not only eliminates the fitness scaling problem but also provides an adequate selection pressure throughout the selection process.

(a) Initial behavior of population   (b) Behavior after few generations

Figure 1: *Expected behavior of population*

3.2. Proposed scheme: split-based selection (SBS) procedure

Let us consider that a population of size K (usually it is even) is ranked on the basis of fitness status of individuals from worst to best. In our proposition, to ensure the population diversity each individual gets a unique rank based on fitness status. Keeping Figure 1(b) in view, one may agree that rank–based populations usually remain classified into three categories; lower fit, average fit and best fit. In order to maintain selection pressure, our proposed scheme pursues the assigning of probabilities of selection within the fitness categories, in systematic way. In our scheme, the bottom 40% ranked individuals are labeled as lower fit. The resultant proportion of this category in a population of size K is:

$$\frac{K}{5}\left(\frac{2K}{5}+1\right). \tag{7}$$

Further, the average fit category contains individuals covering the middle 20% ranks such as:

$$\frac{K}{10}(K+1). \tag{8}$$

The remaining 40% individuals comprise the best fit category. The proportion of the highest ordered category is calculated as:

$$\frac{K}{25}(8K + 5). \tag{9}$$

The general expression of selection probability associated with every individual to be a participant of parent population is:

$$p_i = \begin{cases} \alpha\left(\dfrac{25i}{K(2K+5)}\right), & i \leq \dfrac{2K}{5} \\ \beta\left(\dfrac{5}{K}\right), & \dfrac{2K}{5} < i \leq \dfrac{3K}{5} \\ \gamma\left(\dfrac{25i}{K(8K+5)}\right), & i > \dfrac{3K}{5}, \end{cases} \tag{10}$$

where α, β and γ are selection parameters and remain subject to the constraints such as, $0 \leq \alpha, \beta, \gamma \leq 1$ and $\alpha + \beta + \gamma = 1$.

One may appreciate the generality and flexibility of proposed scheme as the different values of α, β and γ parameters, permit the investigator to adjust the selection pressure at a desired level. For demonstration purposes, in this article, we considered $\alpha = \beta = 0.2$ and $\gamma = 0.6$. This setting indicates that the bottom 40% individuals are assigned the weights equals to 0.2, whereas the middle 20% individuals are assigned a weight of 0.2 and the weight of most fit 40% individuals is set as 0.6. This gives:

$$p_i = \begin{cases} \dfrac{5i}{K(2K+5)}, & i \leq \dfrac{2K}{5} \\ \dfrac{1}{K}, & \dfrac{2K}{5} < i \leq \dfrac{3K}{5} \\ \dfrac{15i}{K(8K+5)}, & i > \dfrac{3K}{5}. \end{cases} \tag{11}$$

In the next section, we explore the statistical characteristics of this new operator with other competing selection schemes.

4. Empirical analysis

The selection process in genetic algorithms can be divided into two steps. The first step is to assign the selection probability to each individual with respect to its fitness level. The expected number of offspring rate e_i of individual i for the next generation is calculated as $e_i = K \times p_i$ where K is the population size and p_i denotes the selection probability of individual i. The second step entails the selection of K individuals from the current population using various sampling algorithms (roulette wheel or universal sampling, etc.). The sampling algorithms then proceed by providing the observed number of offspring of individual i, let us say o_i, such that $E[o_i] = e_i$.

Next, we explore the statistical characteristics of our proposed operator (11) while comparing with aforementioned four rank–based operators (LRS, ERS, BTS and PTS). The first operator, that is FPS is not considered in this section because it is a purely population dependent operator and not rank–based, see also [28, 40]. Moreover, for a fair comparison, we consider optimal parametric values for the above mentioned operators to ensure their maximal performance. For example, in the case of LRS, Backer [5] recommended the value of $\eta^+ = 1.1$ in equation (3) to achieve optimal performance of the operator. For ERS, the value of r closer to one is advised in the literature to gain better performance [28, 31]; we use $r = 0.99$ in this study. Finally, for

PTS, the recommended range of the parameter q in (6), ensuring higher performance of the operator, is $(0.5 - 1)$ [28]; we use $q = 0.8$.

To resolve the issue of approximation and to achieve higher accuracy, we follow the recommendations of [28, 40] suggesting the population size $K \geq 250$ with at least 10 categories. Thus, we generate a population of 300 individuals and ordered them according to their fitness status. In the next phase, through each operator, we assigned the selection probability p_i to each individual and distributed them into 10 categories to attain the expected number e_i. To estimate the observed frequencies o_i of each category, we repeat this process 300 times, where sampling is conducted through the roulette wheel (RW) sampling mechanism, see also [20]. The discrepancies between expected and observed numbers of offspring are then quantified using a χ^2 goodness–of–fit measure. Let us consider, there are c disjoint classes, such as $\{C_1, C_2, ..., C_c\}$, where $C_j \subset \{1, 2, ..., K\}$ and $\cup_{j=1}^{c} C_j = \{1, 2, ..., K\}$. Also let $\xi_j = \sum_{i \in C_j} e_i$ denote the overall expectation, where $O_j = \sum_{i \in C_j} o_i$ represents the observed copies of individuals in the mating pool after the sampling procedure. Ideally, ξ_j should be of the order K/c for $1 \leq j \leq c$, so that each class contains the same number of individuals on average. [40] defined the χ^2 test as a measure to determine the accuracy of the sampling process as:

$$X = \sum_{j=i}^{c} \frac{(\xi_j - O_j)^2}{\xi_j}. \tag{12}$$

Table 1 presents the expected counts associated with each category with respect to all selection procedures. As noted by [28, 40], under the assumptions of $C_j \geq 10$, $\xi_j \geq 10$ and $K \geq 100$, the sampling distribution of X in (12) will follow a χ^2 distribution with $c-1$ degrees of freedom, such that $E[X] = c-1$ and $Var[X] = 2(c-1)$. Table 2 confirms this anticipated distributional behavior of the sampling operators.

	LRS		ERS		BTS		PTS		SBS	
j	C_j	ξ_j	C_j	ξ_j	C_j	ξ_j	C_j	ξ_j	C_j	ξ_j
1	1–33	30.05	1–107	29.88	1–95	30.08	1–75	29.85	1–85	30.21
2	34–65	29.84	108–158	30.36	96–134	29.77	76–117	29.86	86–120	29.79
3	66–96	29.56	159–191	29.78	135–164	29.80	118–150	30.09	121–150	30.00
4	97–127	30.20	192–216	30.13	165–190	30.68	151–178	30.09	151–180	30.00
5	128–157	29.84	217–236	30.19	191–213	30.90	179–203	30.42	181–205	30.09
6	158–187	30.44	237–253	30.89	214–233	29.73	204–225	29.53	206–227	29.71
7	188–216	30.00	254–267	29.72	234–252	30.72	226–246	30.61	228–247	29.63
8	217–245	30.56	268–279	29.02	253–269	29.52	247–265	29.73	248–266	30.46
9	246–273	30.04	280–290	29.86	270–285	29.55	266–283	29.94	267–283	29.16
10	274–300	29.47	291–300	30.16	286–300	29.25	284–300	29.87	284–300	30.96

Table 1: *The overall expected counts ξ_j with respect to their classes C_j $(j = 1, 2, ..., 10)$*

	LRS	ERS	BTS	PTS	SBS
$\hat{\mu}$	8.70	9.32	9.40	9.08	9.31
$\hat{\sigma}^2$	17.53	19.96	19.96	19.05	17.90

Table 2: *Simulated means and variances of the χ^2 test statistics*

5. Performance evaluation

5.1. Traveling salesman problem and state–of–the–art settings

In this section, we evaluate the performance of the proposed scheme in comparison with the aforementioned competing schemes. The test problem is one of the most famous and popular benchmarks available in the optimization literature. We considered the traveling salesman problem (TSP), first documented by Euler in 1759 to resolve the knight's tour problem. The applicability of this classical problem is extensive in multidisciplinary research. For example in the fields of bioinformatics [12], transportation [17, 19], and genetics [14]. The unified nature of the TSP enables its prominence at the core of GA literature. Many researchers, such as, [8, 22, 23, 33, 38] have used the TSP to test the performance of new search algorithms.

To ensure the generality of performance comparison, we considered 14 diverse problems from the library of the traveling salesman problem (TSPLIB). Our choice of problems include Euclidean and two–dimensional problems incorporating symmetric as well as asymmetric in nature, where the number of cities varies from 52 to 442. In addition, we considered two most widely used crossover schemes, namely the partially–mapped crossover (PMX) and order crossover (OX) along with two common mutation operators, which are the exchange mutation (EM) and inversion mutation (INV). Table 3 details these state–of–the–art settings, for further information one may consult to Larranaga et. al [26]. In our simulation experiments, all GA programs were implemented in MATLAB. Moreover, we used two stopping criteria for our simulation experiments, i.e. attaining the maximum number of generations and if the tour, shorter than the current optimal tour is not being found during last 300 consecutive generations.

Parameter	Setting
Representation	Path
Population size	300
Crossover criteria	PMX and OX
Crossover rate	80%
Mutation method	EM and INV
Mutation rate	5%
Maximum generation	10000
Number of trails	30
Replacement in GA	Steady-state GA

Table 3: *Parameters configuration for GA*

5.2. Results and discussion

Since GAs belong to the class of stochastic search algorithms [49], therefore, for comparative purposes, we recored average values, standard deviations (S.D.) and relative errors (R.E) based on 30 runs and for all 14 problems while considering six contemporary selection schemes. Moreover, to evaluate the relative performance of existing approaches with respect to proposed scheme, we used two criteria: **(i)** t-test statistics [49, 23] and **(ii)** performance index (PI) [10, 46].

Criterion (i): The t-test statistic under the null hypothesis of "SBS is at least as good (at least as small) as the solution obtained with the competing operator" is employed for pairwise comparisons of proposed scheme with existing techniques. The expression given in equation (13) provides the test–statistic:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S_p \sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}, \tag{13}$$

where S_p is pooled standard deviation from both samples, $\bar{x}_1$ represents the average of proposed operator and $\bar{x}_2$ belongs to the contemporary approach. If the null hypothesis is true, the test–statistic will follow a t–distribution with 58 degrees of freedom.

For 14 problems across four combinations of crossover and mutation operators for studying six procedures, we obtain 336 average optimal distance values. We observe that, out of 336 instances, our proposed procedure (SBS) outperforms competing schemes 318 times. Moreover, t–test statistics indicate that in 162 cases, our selection strategy provides statistically significantly better performance when compared with contemporary operators. We observe no case where significantly lower performance can be attributed to our proposition. These results are not presented here to preserve the space, but they are available upon request.

Criterion (ii): The performance index (PI) is aslo used – a comprehensive and widely used criterion comparing the performance of population–based heuristic algorithms [10, 46]. The overall relative performances of competing schemes using PI is calculated by considering average values, S.D and R.E. The expression of PI with respect to aforementioned statistics is given as:

$$PI = \frac{1}{Np} \sum_{i=1}^{Np} (k_1 \alpha_1^i + k_2 \alpha_2^i + k_3 \alpha_3^i), \tag{14}$$

where

$$\begin{aligned}
\alpha_1^i &= \frac{A^i}{MA^i} \\
\alpha_2^i &= \frac{S^i}{MS^i} \\
\alpha_3^i &= \frac{R^i}{MR^i}, \qquad i = 1, 2, ..., N_p
\end{aligned} \tag{15}$$

and

N_p: total number of problems analyzed

A^i: least average values of objective function of ith problem for all competing selection operators and for nominated state-of-art settings

MA^i: minimum of average values of the array of A^i

S^i: least standard deviations of objective function of ith problem for all competing selection operators and for nominated state-of-art settings

MS^i: minimum of Standard deviations of the array of S^i

R^i: least relative errors of objective function of ith problem for all competing selection operators and for nominated state-of-art settings

MR^i: minimum of relative errors of the array of R^i.

The weights of aforementioned criteria are k_1, k_2 and k_3, such that $k_1 + k_2 + k_3 = 1$ for all $0 \le k_1,\ k_2,\ k_3 \le 1$. From the above definition, it is clear that PI is a function of k_1, k_2 and k_3. Since $k_1 + k_2 + k_3 = 1$, one of the k_i, $i = 1, 2, 3$ can be wipe out to lessen the quantity

depended variables from the expression of PI. We adopt the strategy of [32] of assigning equal weights to two terms at a time to aid the visualization of PI. Along these lines, the resultant cases are as follows:

$$case(a): \quad k_1 = w, \ k_2 = k_3 = (1-w)/2, \ 0 \le w \le 1$$
$$case(b): \quad k_2 = w, \ k_1 = k_3 = (1-w)/2, \ 0 \le w \le 1$$
$$case(c): \quad k_3 = w, \ k_2 = k_3 = (1-w)/2, \ 0 \le w \le 1.$$

The graphs corresponding to each of the cases (a), (b) and (c) are shown in **Figures 2–4** respectively, where the horizontal axis represents the weights and the vertical axis present the PI with respect to considered attribute. The case (a) evaluates the PI with respect to average values while considering S.D. and R.E. with equal weights. In case (b), we consider average values and R.E. of the same weights where the PI is calculated for S.D. Lastly, case (c) denotes the situation where the PI is quantified for R.E. while taking average values and S.D. of equal weights. For all cases, a superior overall performance of the proposed scheme is evident. From **Figures 2–4**, it is clear that over the permissible range of PI, i.e. (0 1), the proposed operator shows higher performance while comparing with existing schemes. Moreover, the consistent and least fluctuated behavior of the PI associated with proposed operator also highlights its robustness with respect to the assigned weights to average values, S.D. and R.E.
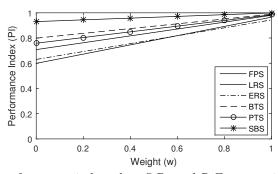

Figure 2: *The performance index when S.D. and R.E. are assigned equal weights*
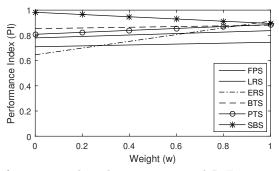

Figure 3: *The performance index when averages and R.E. are assigned equal weights*
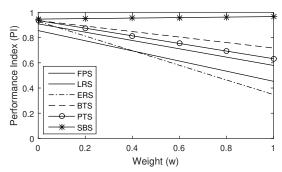
Figure 4: *The performance index when averages and S.D. are assigned equal weights*

6. Conclusion

Throughout the optimization literature, a major concern of experts lies in the trade–off between population diversity and selection pressure, see for example [1, 9, 23, 35]. This article presents a novel selection operator–split–based selection (SBS). Our proposed technique facilitates the optimization problem by ensuring a balance between diversity and selection pressure and thus avoids premature convergence. In its essence, our SBS technique is rank–based, where individuals are prioritized according to their fitness status. The ranked individuals are then assigned selection weights by classifying them into three fitness categories; lowest–fit, average–fit and best–fit. The selection weights are further employed within each class to ensure population diversity. At the same time, higher weights are offered to the most fit individuals and thus selection pressure is maintained. Throughout this article, we demonstrated a superior performance of the newly proposed operator in comparison to the existing operators. Based on a rigorous performance evaluation study, considering 14 highly regarded benchmarks in optimization literature, the proposed strategy is shown to not only cope with the fitness selection but also maintain the selection pressure and thus facilitates the optimization processes. In future, it will be interesting to explore the performance of selection operator when there exists a group-structure at the population level. Moreover, the use of auxiliary information to aid the selection operator is also a plausible avenue for future research.

References

[1] Aibinu, A. M., Bello Salau, H., Rahman, N. A., Nwohu, M. N. and Akachukwu, C. M. (2016). A novel clustering based genetic algorithm for route optimization. *Engineering Science and Technology, an International Journal*, 19(4), 2022–2034. doi: 10.1016/j.jestch.2016.08.003

[2] Amsa, M. G. B. A., Aibinu, A. M. and Salami, M. J. E. (2013). A novel hybrid artificial intelligence technique for colpitts oscillator design. *Journal of Control, Automation and Electrical Systems*, 25(1), 10–21. doi: 10.1007/s40313-013-0084-4

[3] Back, T. (1996). *Evolutionary algorithms in theory and practice: Evolution strategies, evolutionary programming, genetic lgorithms.* Oxford University Press.

[4] Baeck, T. (2000). *Evolutionary computation 1 – Basic algorithms and operators.* Boca Raton: CRC Press. doi: 10.1201/9781482268713

[5] Baker, J. E. (1985). *Adaptive Selection Methods for Genetic Algorithms.* In Grefenstette J. J. (Ed.), Proceedings of the First International Conference on Genetic Algorithms and their Applications (pp. 101–111). New York: Psychology Press. doi: 10.4324/9781315799674

[6] Beyer, H.–G. (1997). An alternative explanation for the manner in which genetic algorithms operate. *Biosystems*, 41(1), 1–15. doi: 10.1016/s0303-2647(96)01657-7

[7] Bhanu, B., Lee, S. and Ming, J. (1995). Adaptive image segmentation using a genetic algorithm. *IEEE Transactions on Systems, Man, and Cybernetics*, 25(12), 1543–1567. doi: 10.1109/21.478444

[8] Bhattacharyya, M. and Bandyopadhyay, A. K. (2008). Comparative study of some solution methods for traveling salesman problem using genetic algorithms. *Cybernetics and Systems*, 40(1), 1–24. doi: 10.1080/01969720802492967

[9] Burke, E. K., Gustafson, S. and Kendall, G. (2004). Diversity in genetic programming: An analysis of measures and correlation with fitness. *IEEE Transactions on Evolutionary Computation*, 8(1), 47–62. doi: 10.1109/tevc.2003.819263

[10] Deep, K. and Thakur, M. (2007). A new crossover operator for real coded genetic algorithms. *Applied Mathematics and Computation*, 188(1), 895–911. doi: 10.1016/j.amc.2006.10.047

[11] Deng, Y., Liu, Y. and Zhou, D. (2015). An improved genetic algorithm with initial population strategy for symmetric TSP. *Mathematical Problems in Engineering*, 2015, ID212794, 1–6. doi: 10.1155/2015/212794

[12] Ezziane, Z. (2006). Applications of artificial intelligence in bioinformatics: A review. *Expert Systems with Applications*, 30(1), 2–10. doi: 10.1016/j.eswa.2005.09.042

[13] Fogel, D. B. (1994). An introduction to simulated evolutionary optimization. *IEEE Transactions on Neural Networks*, 5(1), 3–14. doi: 10.1109/72.265956

[14] Forrest, S., Javornik, B., Smith, R. E. and Perelson, A. S. (1993). Using genetic algorithms to explore pattern recognition in the immune System. *Evolutionary Computation*, 1(3), 191–211. doi: 10.1162/evco.1993.1.3.191

[15] Goldberg, D. E. and Deb, K. (1991). *A comparative analysis of selection schemes used in genetic algorithms*. In Foundations of Genetic Algorithms (pp. 69–93). Elsevier Inc. doi: 10.1016/b978-0-08-050684-5.50008-2

[16] Grefenstette, J. J. (1999). Evolvability in dynamic fitness landscapes: A genetic algorithm approach. *Proceedings of the Congress on Evolutionary Computation – CEC 99*, 3, 2031–2038. doi: 10.1109/cec.1999.785524

[17] Hà, M. H., Bostel, N., Langevin, A. and Rousseau, L.–M. (2014). An exact algorithm and a metaheuristic for the generalized vehicle routing problem with flexible fleet size. *Computers and Operations Research*, 43, 9–19. doi: 10.1016/j.cor.2013.08.017

[18] Helsgaun, K. (2000). An effective implementation of the Lin–Kernighan traveling salesman heuristic. *European Journal of Operational Research*, 126(1), 106–130. doi: 10.1016/s0377-2217(99)00284-2

[19] Ho, W. and Ji, P. (2009). An integrated scheduling problem of PCB components on sequential pick–and–place machines: Mathematical models and heuristic solutions. *Expert Systems with Applications*, 36(3), 7002–7010. doi: 10.1016/j.eswa.2008.08.025

[20] Holland, J. H. (1992). *Adaptation in natural and artificial systems*. The MIT Press. doi: 10.7551/mitpress/1090.001.0001

[21] Huang, H.-X., Li, J.–C. and Xiao, C.–L. (2015). A proposed iteration optimization approach integrating backpropagation neural network with genetic algorithm. *Expert Systems with Applications*, 42(1), 146–155. doi: 10.1016/j.eswa.2014.07.039

[22] Hussain, A., Muhammad, Y. S., Nauman Sajid, M., Hussain, I., Mohamd Shoukry, A. and Gani, S. (2017). Genetic algorithm for traveling salesman problem with modified cycle crossover operator. *Computational Intelligence and Neuroscience*, 2017, 1–7. doi: 10.1155/2017/7430125

[23] Hussain, A. and Muhammad, Y. S. (2019). Trade–off between exploration and exploitation with genetic algorithm using a novel selection operator. *Complex and Intelligent Systems*, 6(1), 1–14. doi: 10.1007/s40747-019-0102-7

[24] Julstrom, B. A. (1999). It's all the same to me: Revisiting rank–based probabilities and tournaments. *Proceedings of the Congress on Evolutionary Computation – CEC 99*, 2, 1501–1505. doi: 10.1109/cec.1999.782661

[25] Koumousis, V. K. and Katsaras, C. P. (2006). A saw–tooth genetic algorithm combining the effects of variable population size and reinitialization to enhance performance. *IEEE Transactions on Evolutionary Computation*, 10(1), 19–28. doi: 10.1109/tevc.2005.860765

[26] Larrañaga, P., Kuijpers, C. M. H., Murga, R. H., Inza, I. and Dizdarevic, S. (1999). Genetic algorithms for the travelling salesman problem: A review of representations and operators. *Artificial Intelligence Review*, 13(2), 129–170. doi: 10.1023/A:1006529012972

[27] Lavinas, Y., Aranha, C., Sakurai, T. and Ladeira, M. (2018). Experimental analysis of the tournament size on genetic algorithms. *IEEE International Conference on Systems, Man, and Cybernetics*, 3647–3653. doi: 10.1109/smc.2018.00617

[28] Lee, S., Soak, S., Kim, K., Park, H. and Jeon, M. (2007). Statistical properties analysis of real world tournament selection in genetic algorithms. *Applied Intelligence*, 28(2), 195–205. doi: 10.1007/s10489-007-0062-2

[29] Mahajan, A., Pilch, A. and Chu, T. (2000). Intelligent image correlation using genetic algorithms for measuring surface deformations in the autonomous inspection of structures. *IEEE Proceedings of the American Control Conference*, 1, 460–461. doi: 10.1109/acc.2000.878942

[30] Mashohor, S., Evans, J. R. and Arslan, T. (n.d.). Elitist selection schemes for genetic algorithm based printed circuit board inspection system. *IEEE Congress on Evolutionary Computation*, 2, 974–978. doi: 10.1109/cec.2005.1554796

[31] Michalewicz, Z. (1992). *Genetic algorithms + data structures = evolution programs*. Artificial Intelligence book series. Berlin: Springer. doi: 10.1007/978-3-662-02830-8

[32] Mohan, C. and Nguyen, H. T. (1999). A controlled random search technique incorporating the simulated annealing concept for solving integer and mixed integer global optimization problems. *Computational Optimization and Applications*, 14(1), 103–132. doi: 10.1023/A:1008761113491

[33] Nagata, Y. and Soler, D. (2012). A new genetic algorithm for the asymmetric traveling salesman problem. *Expert Systems with Applications*, 39(10), 8947–8953. doi: 10.1016/j.eswa.2012.02.029

[34] Pal, S. K. and Wang, P. P. (2017). *Genetic Algorithms for Pattern Recognition*. Boca Raton: CRC Press. doi: 10.1201/9780203713402

[35] Pandey, H. M., Chaudhary, A. and Mehrotra, D. (2014). A comparative review of approaches to prevent premature convergence in GA. *Applied Soft Computing*, 24, 1047–1077. doi: 10.1016/j.asoc.2014.08.025

[36] Pandey, H. M., Shukla, A., Chaudhary, A. and Mehrotra, D. (2016). *Evaluation of genetic algorithm's selection methods*. In Satapathy S., Mandal J., Udgata S. and Bhateja V. (Eds), Information Systems Design and Intelligent Applications, Advances in Intelligent Systems and Computing (pp. 731–738). New Delhi: Springer. doi: 10.1007/978-81-322-2752-6_72

[37] Rubio, F., Llopis–Albert, C., Valero, F. and Suñer, J. L. (2016). Industrial robot efficient trajectory generation without collision through the evolution of the optimal trajectory. *Robotics and Autonomous Systems*, 86, 106–112. doi: 10.1016/j.robot.2016.09.008

[38] Ruiz, E., Albareda-Sambola, M., Fernández, E. and Resende, M. G. C. (2015). A biased random-key genetic algorithm for the capacitated minimum spanning tree problem. *Computers and Operations Research*, 57, 95–108. doi: 10.1016/j.cor.2014.11.011

[39] Ruxton, G. D. and Beauchamp, G. (2008). The application of genetic algorithms in behavioural ecology, illustrated with a model of anti–predator vigilance. *Journal of Theoretical Biology*, 250(3), 435–448. doi: 10.1016/j.jtbi.2007.10.022

[40] Schell, T. and Wegenkittl, S. (2001). Looking beyond selection probabilities: Adaptation of the χ^2 measure for the performance analysis of selection methods in GAs. *Evolutionary Computation*, 9(2), 243–256. doi: 10.1162/106365601750190424

[41] Shapiro, J. L., Prügel–Bennett, A. and Rattray, M. (1997). *A statistical mechanics analysis of genetic algorithms for search and learning*. In Ellacott S. W., Mason J. C. and Anderson I. J. (Eds.), Mathematics of Neural Networks, Operations Research/Computer Science Interfaces Series (pp. 318–322). Boston: Springer. doi: 10.1007/978-1-4615-6099-9_55

[42] Shilane, D., Martikainen, J., Dudoit, S. and Ovaska, S. J. (2008). A general framework for statistical performance comparison of evolutionary computation algorithms. *Information Sciences*, 178(14), 2870–2879. doi: 10.1016/j.ins.2008.03.007

[43] Song, J., Xue, G. and Kang, Y. (2016). A novel method for optimum global positioning system satellite selection based on a modified genetic algorithm. *PLOS ONE*, 11(3), e0150005. doi: 10.1371/journal.pone.0150005

[44] Soui, M., Gasmi, I., Smiti, S. and Ghédira, K. (2019). Rule–based credit risk assessment model using multi–objective evolutionary algorithms. *Expert Systems with Applications*, 126, 144–157. doi: 10.1016/j.eswa.2019.01.078

[45] Tang, K. S., Man, K. F., Kwong, S. and He, Q. (1996). Genetic algorithms and their applications. *IEEE Signal Processing Magazine*, 13(6), 22–37. doi: 10.1109/79.543973

[46] Thakur, M. (2014). A new genetic algorithm for global optimization of multimodal continuous functions. Journal of Computational Science, 5(2), 298–311. doi: 10.1016/j.jocs.2013.05.005

[47] Yee Leung, Yong Gao and Zong–Ben Xu. (1997). Degree of population diversity – a perspective on premature convergence in genetic algorithms and its Markov chain analysis. *IEEE Transactions on Neural Networks*, 8(5), 1165–1176. doi: 10.1109/72.623217

[48] Yu, M., Eua–anant, N., Saudagar, A. and Udpa, L. (n.d.). Genetic algorithm approach to image segmentation using morphological operations. *Proceedings of the International Conference on Image Processing – ICIP 98*, 3, 775–779. doi: 10.1109/icip.1998.999063

[49] Yuan, S., Skinner, B., Huang, S. and Liu, D. (2013). A new crossover approach for solving the multiple travelling salesmen problem using genetic algorithms. *European Journal of Operational Research*, 228(1), 72–82. doi: 10.1016/j.ejor.2013.01.043

[50] Zhang, X., Zhang, Y., Hu, Y., Deng, Y. and Mahadevan, S. (2013). An adaptive amoeba algorithm for constrained shortest paths. *Expert Systems with Applications*, 40(18), 7607–7616. doi: 10.1016/j.eswa.2013.07.054

[51] Wang, J., Ersoy, O. K., He, M. and Wang, F. (2016). Multi–offspring genetic algorithm and its application to the traveling salesman problem. *Applied Soft Computing*, 43, 415–423. doi: 10.1016/j.asoc.2016.02.021