Croatian Operational Research Review
CRORR **14**(2023), 15–28

A novel modified Khatter's approach for solving Neutrosophic Data Envelopment Analysis

Kshitish Kumar Mohanta[1,*], Deena Sunil Sharanappa[1] and Abha Aggarwal[2]

[1] *Department of Mathematics, Indira Gandhi National Tribal University, Lalpur, Amarkantak, 484887, Madhya Pradesh, India*
Email: ⟨kshitishkumar.math@gmail.com, deena.sunil@igntu.ac.in⟩
[2] *School of Basic and Applied Sciences, Guru Gobind Singh Indraprastha University Delhi, 110078, India*
E-mail: ⟨abhaaggarwal27@gmail.com⟩

Abstract. The evaluation of the performance of decision-making units (DMUs) that use comparable inputs to produce related outputs can be accomplished through a non-parametric linear programming (LP) technique called Data Envelopment Analysis (DEA). However, the observed data are occasionally imprecise, ambiguous, inadequate, and inconsistent which may result to incorrect decision-making when these criteria are ignored. Neutrosophic Set (NS) is an extension of fuzzy sets which is used to represent unclear, erroneous, missing, and wrong information. This paper proposes a neutrosophic version of the DEA model, and a novel solution technique for Neutrosophic DEA (Neu-DEA) model. The possibility mean for triangular neutrosophic number (TNN) is redefined and modified the Khatter's approach to convert directly the Neu-DEA model into its crisp DEA model. As a result, the Neu-DEA model is simplified to a crisp LP problem with a risk parameter ($\delta \in [0,1]$) that represents the attitude of the decision-maker towards taking risk. The efficiency score of the DMUs is computed by using various risk factors and divided into efficient and inefficient groups. The ranking of DMUs is determined by calculating the mean efficiency score of DMUs, which is based on various risk parameters. A numerical example is illustrated here to describe the suggested approach's flexibility and authenticity and compared with some of the existing approaches.

Keywords: Efficiency Analysis, Triangular Neutrosophic Number, Neutrosophic Data Envelopment Analysis, Possibility mean, Khatter's Approach.

Received: June 27, 2022; accepted: December 30, 2022; available online: July 10, 2023

DOI: 10.17535/crorr.2023.0002

1. Introduction

The DEA is one of the most popular performance evaluation techniques used to evaluate the efficiency of a group of comparable DMUs that generate similar outputs using similar inputs. Charnes et al. [4] introduced a linear mathematical programming model based on Farrell's model to estimate the relative efficiency of DMUs, assuming constant returns to scale (CRS). Since then, many researchers have employed DEA to estimate the efficiency of DMUs using various models, including BCC, SBM, Additive, Super Efficiency, and Undesirable DEA. Recently, Taeb et al. [24] established a DEA model to measure the efficiency of time-dependent input-output of DMUs. The DEA method has been widely applied in various sectors such as banking, insurance, education, SCM, crisis management, sustainability, agriculture, energy, and healthcare services [5, 12, 16], making it a powerful and efficient MCDM approach. The

*Corresponding author.

traditional DEA models assume that the input and output variables are precise and deterministic, and do not account for uncertainties and vagueness in the data. However, in real-world situations, the inputs and outputs variables may be imprecise or uncertain due to incomplete or inaccurate data or subjective assessments. The most concerning problem for DM is to create a mathematical model that handles uncertainty data in the evaluation of DMU performance. First time Sengupta [22] in 1992 created the technique to solve the DEA model in the presence of uncertain/imprecise data. After that, many authors developed different techniques to solve fuzzy DEA (FDEA) models. Fuzzy stochastic approaches [20] and interval FDEA models are commonly used to address inaccurate input-output data. Zhou & Xu [30] provided a summary of fuzzy DEA models and their practical applications. Fuzzy DEA has been classified into seven methods, which include "tolerance, α-cut, fuzzy ranking, possibility, fuzzy arithmetic, fuzzy random/type-2 fuzzy set approaches, and multi-objective approach" [10]. These methods have been applied in diverse areas, including portfolio optimization and industries [21, 26].

Smarandache [23] developed the Neutrosophic Set, which is an extension of Fuzzy Set and Intuitionistic Fuzzy Set. It includes an indeterminacy-membership grade based on IFS and is suitable for modeling problems with high levels of uncertainty. NS theory is capable of managing ambiguity, indeterminacy, and inconsistency in decision-making and can control the DM process and manage nonlinear complex systems, making it useful for solving various MCDM problems. The essential features of NS theory, which independently define "truth-membership, false-membership, and indeterminacy-membership grades," and their sum lies between 0 and 3, make it an effective tool in decision-making in today's competitive environment where DM problems are usually complicated by multiple opposing criteria. Possibility theory, introduced by Zadeh [29], is crucial in handling incomplete information and uncertainty, similar to probability theory in crisp set. It has been used to solve optimization problems under uncertain conditions in MCDM. The possibility means, variance, and standard deviation of TIFNs [27] and SVTrNNs [11] have been defined to tackle MCDM problems. Recently, Khatter [14] defined the possibility mean for TNNs and employed it to solve neutrosophic LP problems in which first converts the TNN into corresponding crisp value. Then, it is directly replaced the crisp value in the Neu-LPP to convert the corresponding crisp LP problem and find the optimum solution of the Neu-LPP. However, this method does not directly convert Neutrosophic constraints or objective functions into crisp constraints or objective functions, nor does it satisfy the possibility mean of each real crisp number must be the same crisp number. These are the major drawback with the Khatter method.

To address these problems, we redefined the possibility mean for TNN such that it can be applied to both TNN and crisp real numbers. The Khatter approach for solving neutrosophic LP problem is modified such that it may be used directly in the neutrosophic optimization problem to convert the equivalent crisp optimization problem. This paper presents a unique, efficient solution strategy for solving a neutrosophic DEA model having TNN input and output data. The modified Khatter's method is implemented to obtain the solution for Neu-DEA model. The proposed solution technique for the Neu-DEA model has the significant advantage of allowing the decision-maker to a greater flexibility in determining the efficiency of DMUs with risk parameter. The risk parameter indicates whether decision-makers should be pessimistic, optimistic, or neutral in an uncertain environment.

The following is an outline of the remaining manuscript: In section 2, a brief overview of the literature review of Neutrosophic DEA is presented. Section 3 delves into the fundamental concepts of NSs, including terminology, notations, and mathematical operations. The possibility mean for TNN is redefined, and the neutrosophic DEA model is developed from the Conventional DEA model. Section 4 outlines the step-by-step procedure for solving the proposed neutrosophic DEA model and utilizes the modified Khatter's approach to convert it into the crisp DEA model. Section 5 illustrates the effectiveness and validation of the suggested model and draws a comparison with the pre-existing model through a numerical example. Finally, in section 6, the advantages and future possibilities of the proposed approach are discussed.

2. Literature Review

In this section, a survey of relevant literature is presented regarding the development and utilization of the DEA within a neutrosophic fuzzy environment. The DEA is widely utilized within the field of operational research to calculate DMU efficiency scores. However, standard DEA models are not suitable when the precise values of all input-output data are not known in advance. Several theories, such as stochastic techniques, interval DEA model, and FS approach, have been suggested to address this issue. Nonetheless, these techniques do not take into account the certainty level of the data. The neutrosophic theory can handle not only unclear, imprecise, and incomplete data, but also indeterminate data. Neutrosophic sets (NSs) and their expansions have subsequently been used in a number of domains, including computer science, mathematics, engineering, medical [17]. Edalatpanah [7] has designed the first theoretical Neutrosophic DEA model in 2018 using TNN inputs and outputs. Following that, several authors attempted to develop and apply the Neu-DEA model in many fields to solve uncertainty problems. Kahraman et al. [13] suggested an integrated Neutrosophic version of the AHP and DEA models to analyse the efficiency of 15 private universities in Turkey. Abdelfattah [1] suggested a ranking and parametric approach to solve the Neu-DEA model with inputs and outputs are the neutrosophic numbers. Edalatpanah & Smarandache [8] applied natural logarithms to transform an input-oriented Neu-DEA model into a corresponding crisp DEA model based on simplified neutrosophic numbers. Again, Edalatpanah [9] investigated the Neu-DEA model based on TNNs and devised a ranking technique to solve it. Mao et al. [15] used single-valued neutrosophic sets (SVNSs) in undesirable DEA model based on logarithm approach. Consequently, Tapia [25] created a new MCDA method that utilizes a neutrosophic DEA model to assess NETs and their associated risks, accounting for the uncertainties within these technologies. Yang et al. [28] used a single-valued triangular neutrosophic number to measure hospital efficiency based on Neu-DEA model, which is the first study to demonstrate real-world Neu-DEA application. Again, Abdelfattah [2] in 2021 applied his proposed model [1] as an application for measuring the performance of the 32 regional hospitals of in Tunisia. Recently, other extension of FS like Fermatean fuzzy set [3] and Spherical fuzzy set [18, 19] are used to construct the DEA models and its solution techniques are developed.

3. Development of Neutrosophic Data Envelopment Analysis (Neu-DEA)

This section discusses the earlier findings, results and operation of the Neutrosophic set (NS). Also, the possibility mean for TNN is redefined and the neutrosophic DEA (Neu-DEA) model is developed to assess the efficiency of DMUs having neutrosophic inputs-outputs.

3.1. Preliminary

Definition 1 ([23]). *Let Ω be the universal set. The neutrosophic set (NS) $\hat{N}$ over Ω is defined as*

$$\hat{N} = \{\langle x; \phi_N, \varphi_N, \psi_N\rangle : x \in \Omega\} \tag{1}$$

where $\phi_N, \varphi_N, \psi_N : \Omega \to [0,1]$ are the truth, indeterminate and falsity membership grades and satisfy $0 \leq \phi_N + \varphi_N + \psi_N \leq 3$.

Definition 2 ([6]). *The Triangular neutrosophic number (TNN) is denoted by $\widehat{X} = \langle x^L, x^M, x^U;$*

$\phi_x, \varphi_x, \psi_x\rangle$, in which there are three membership grades of x are given below:

$$\mathfrak{T}(x) = \begin{cases} \dfrac{x - x^L}{x^M - x^L}\phi_x, & x^L \leq x \leq x^M \\ \phi_x, \quad x = x^M \\ \dfrac{x^U - x}{x^U - x^M}\phi_x, & x^M \leq x \leq x^U \\ 0, \quad otherwise \end{cases} \qquad \mathfrak{I}(x) = \begin{cases} \dfrac{x^M - x + \varphi_x(x - x^L)}{x^M - x^L}, & x^L \leq x \leq x^M \\ \varphi_x, \quad x = a^M \\ \dfrac{x - x^M + \varphi_x(x^U - x)}{x^U - x^M} & x^M \leq x \leq x^U \\ 1, \quad otherwise \end{cases}$$

$$\mathfrak{F}(x) = \begin{cases} \dfrac{x^M - x + \psi_x(x - x^L)}{x^M - x^L}, & x^L \leq x \leq x^M \\ \psi_x, \quad x = x^M \\ \dfrac{x - x^M + \psi_x(x^U - x)}{x^U - x^M}, & x^M \leq x \leq x^U \\ 1, \quad otherwise \end{cases}$$

where $0 \leq \mathfrak{T}(x) + \mathfrak{I}(x) + \mathfrak{F}(x) \leq 3,\ x \in \Omega$.

Definition 3 ([6]). *Suppose $\widehat{X}_1 = \langle x_1^L, x_1^M, x_1^U; \phi_{x_1}, \varphi_{x_1}, \psi_{x_1}\rangle$ and $\widehat{X}_2 = \langle x_2^L, x_2^M, x_2^U; \phi_{x_2}, \varphi_{x_2}, \psi_{x_2}\rangle$ are two TNNs. The arithmetic operations are given as*

1. $\widehat{X}_1 \oplus \widehat{X}_2 = \langle x_1^L + x_2^L, x_1^M + x_2^M, x_1^U + x_2^U; \phi_{x_1} \wedge \phi_{x_2}, \varphi_{x_1} \vee \varphi_{x_2}, \psi_{x_1} \vee \psi_{x_2}\rangle$.
2. $\widehat{X}_1 - \widehat{X}_1 = \langle x_1^L - x_2^U, x_1^M - x_2^M, x_1^U - x_2^L; \phi_{x_1} \wedge \phi_{x_2}, \varphi_{x_1} \vee \varphi_{x_2}, \psi_{x_1} \vee \psi_{x_2}\rangle$.
3. $\widehat{X}_1 \otimes \widehat{X}_1 = \langle x_1^L x_2^L, x_1^M x_2^M, x_1^U x_2^U; \phi_{x_1} \wedge \phi_{x_2}, \varphi_{x_1} \vee \varphi_{x_2}, \psi_{x_1} \vee \psi_{x_2}\rangle$.
4. $\lambda\widehat{X}_1 = \begin{cases} \langle \lambda x_1^L, \lambda x_1^M, \lambda x_1^U; \phi_{x_1}, \varphi_{x_1}, \psi_{x_1}\rangle, & \lambda > 0 \\ \langle \lambda x_1^U, \lambda x_1^M, \lambda x_1^L; \phi_{x_1}, \varphi_{x_1}, \psi_{x_1}\rangle, & \lambda < 0 \end{cases}$
   where $x \wedge y = \min(x, y)$ and $x \vee y = \max(x, y)$.

Definition 4 ([14]). *The α, β & $\gamma-$cut for a TNN $\widehat{X} = \langle x^L, x^M, x^U; \phi_x, \varphi_x, \psi_x\rangle$, is defined as*

$$\widehat{X}^{(\alpha,\beta,\gamma)} = \{x : \phi_x \geq \alpha, \varphi_x \leq \beta, \psi_x \leq \gamma\}, \tag{2}$$

where $0 \leq \alpha \leq \phi_x, \varphi_x \leq \beta \leq 1$ and $\psi_x \leq \gamma \leq 1$.

From definition 3 and equation (2) , the lower limits $L_{\widehat{X}}(\alpha), L_{\widehat{X}}(\beta)$ and $L_{\widehat{X}}(\gamma)$, and upper limits $U_{\widehat{X}}(\alpha), U_{\widehat{X}}(\beta)$ and $U_{\widehat{X}}(\gamma)$ of α, β and γ-level cut for TNN $\widehat{X}$ are defined as

$$\begin{aligned} \widehat{X}_\alpha &= [L_{\widehat{X}}(\alpha), U_{\widehat{X}}(\alpha)] = \Big[x^L + \alpha(\frac{x^M - x^L}{\phi_x}), x^U - \alpha(\frac{x^U - x^M}{\phi_x})\Big], \\ \widehat{X}_\beta &= [L_{\widehat{X}}(\beta), U_{\widehat{X}}(\beta)] = \Big[\frac{(\beta - \varphi_x)x^L + (1-\beta)x^M}{1 - \varphi_x}, \frac{(\beta - \varphi_x)x^U + (1-\beta)x^M}{1 - \varphi_x}\Big], \\ \widehat{X}_\gamma &= [L_{\widehat{X}}(\gamma), U_{\widehat{X}}(\gamma)] = \Big[\frac{(\gamma - \psi_x)x^L + (1-\gamma)x^M}{1 - \psi_x}, \frac{(\gamma - \psi_x)x^U + (1-\gamma)x^M}{1 - \psi_x}\Big]. \end{aligned}$$

Definition 5 ([14]). *Let $\widehat{X} = \langle x^L, x^M, x^U; \phi_x, \varphi_x, \psi_x\rangle$, be a TNN. The possibility mean for truth, indeterminate, and falsity membership grades of a TNN $\widehat{X}$ is defined in equation (3), equation (4) and equation (5) and denoted by $\mu(\widehat{X}_\alpha)$, $\mu(\widehat{X}_\beta)$ and $\mu(\widehat{X}_\gamma)$ respectively.*

1.

$$\mu(\widehat{X}_\alpha) = \frac{\mu^L(\widehat{X}_\alpha) + \mu^U(\widehat{X}_\alpha)}{2} \tag{3}$$

where

$$\mu^L(\widehat{X}_\alpha) = 2\int_0^{\phi_x} Pos[\widehat{X} \leq L_{\widehat{X}}(\alpha)]L_{\widehat{X}}(\alpha)d\alpha = \frac{x^L + 2x^M}{3}\phi_x^2$$

and

$$\mu^U(\widehat{X}_\alpha) = 2\int_0^{\phi_x} Pos[\widehat{X} \geq U_{\widehat{X}}(\alpha)]U_{\widehat{X}}(\alpha)d\alpha = \frac{x^U + 2x^M}{3}\phi_x^2$$

2.

$$\mu(\widehat{X}_\beta) = \frac{\mu^L(\widehat{X}_\beta) + \mu^U(\widehat{X}_\beta)}{2} \tag{4}$$

where

$$\begin{aligned}\mu^L(\widehat{X}_\beta) &= 2\int_{\varphi_x}^{1} Pos[\widehat{X} \leq L_{\widehat{X}}(\beta)]L_{\widehat{X}}(\beta)d\beta \\ &= (x^M - \varphi_x x^L)(1+\varphi_x) - \frac{2(x^M - x^L)}{3}(1+\varphi_x+\varphi_x^2)\end{aligned}$$

and

$$\begin{aligned}\mu^U(\widehat{X}_\beta) &= 2\int_{\varphi_x}^{1} Pos[\widehat{X} \geq U_{\widehat{X}}(\beta)]U_{\widehat{X}}(\beta)d\beta \\ &= (x^M - \varphi_x x^U)(1+\varphi_x) + \frac{2(x^U - x^M)}{3}(1+\varphi_x+\varphi_x^2)\end{aligned}$$

3.

$$\mu(\widehat{X}_\gamma) = \frac{\mu^L(\widehat{X}_\gamma) + \mu^U(\widehat{X}_\gamma)}{2} \tag{5}$$

where

$$\begin{aligned}\mu^L(\widehat{X}_\gamma) &= 2\int_{\psi_x}^{1} Pos[\widehat{X} \leq L_{\widehat{X}}(\gamma)]L_{\widehat{X}}(\gamma)d\gamma \\ &= (x^M - \psi_x x^L)(1+\psi_x) - \frac{2(x^M - x^L)}{3}(1+\psi_x+\psi_x^2)\end{aligned}$$

and

$$\begin{aligned}\mu^U(\widehat{X}_\gamma) &= 2\int_{\psi_x}^{1} Pos[\widehat{X} \geq U_{\widehat{X}}(\gamma)]U_{\widehat{X}}(\gamma)d\gamma \\ &= (x^M - \psi_x x^U)(1+\psi_x) + \frac{2(x^U - x^M)}{3}(1+\psi_x+\psi_x^2)\end{aligned}$$

Definition 6 ([14])**.** *Let $\widehat{X} = \langle x^L, x^M, x^U; \phi_x, \varphi_x, \psi_x\rangle$, be a TNNs, the possibility means over the risk parameter $\delta \in [0,1]$ of truth, indeterminate and falsity grades is defined as*

$$\Im(\widehat{X}) = \delta\ \mu(\widehat{X}_\alpha) + (1-\delta)\Big(\mu(\widehat{X}_\beta) + \mu(\widehat{X}_\gamma)\Big) \tag{6}$$

and the value of δ indicates the decision maker's risk attitude, as follows:

(a) If δ is in the range of $[0, 0.5)$, it suggests that the decision maker is willing to take risks and considers uncertainty.

(b) If $\delta = 0.5$, it indicates that the decision-maker is neutral in their parameter selection.

(c) If δ is in the range of $(0.5, 1]$, it suggests that the decision maker is cautious about taking risks when making decisions.

Example 1. *If $\widehat{X} = x \in \mathbb{R}$ is a crisp number, it is possible to express it in the form of TNN i.e., $\widehat{X} = \langle x, x, x; 1, 0, 0\rangle$ then $\Im(\widehat{X}) = \delta x + (1-\delta)2x = 2x - x\delta \neq x$.*

Definition 7. *The possibility means over the risk parameter $\delta \in [0, 1]$ of truth, indeterminate, and falsity membership grades of $\widehat{X} = \langle x^L, x^M, x^U; \phi_x, \varphi_x, \psi_x\rangle$ are redefined as:*

$$\widetilde{\Im}(\widehat{X}) = \delta\ \mu(\widehat{X}_\alpha) + (1-\delta)\Big(\frac{\mu(\widehat{X}_\beta) + \mu(\widehat{X}_\gamma)}{2}\Big) \tag{7}$$

That implies

$$\begin{aligned}\widetilde{\Im}(\widehat{X}) = \delta\Big(\frac{x^L + 4x^M + x^U}{6}\Big)\phi_x^2 + \frac{1-\delta}{2}\Bigg[\frac{1}{6}\left(\begin{matrix}2[x^L + x^M + x^U] - [x^L - 2x^M + x^U]\varphi_x \\ \qquad - [x^L + 4x^M + x^U]\varphi_x^2\end{matrix}\right) \\ + \frac{1}{6}\left(\begin{matrix}2[x^L + x^M + x^U] - [x^L - 2x^M + x^U]\psi_x \\ \qquad - [x^L + 4x^M + x^U]\psi_x^2\end{matrix}\right)\Bigg]\end{aligned} \tag{8}$$

Definition 8. *For two TNNs $\widehat{X_1}$ and $\widehat{X_2}$, we say that*

1. *$\widehat{X_1} \preccurlyeq \widehat{X_2}$ if and only if $\widetilde{\Im}(\widehat{X_1}) \leq \widetilde{\Im}(\widehat{X_2})$,*
2. *$\widehat{X_1} \approx \widehat{X_2}$ if and only if $\widetilde{\Im}(\widehat{X_1}) = \widetilde{\Im}(\widehat{X_2})$,*

where $\widetilde{\Im}(.)$ is the possibility mean function for TNN.

Example 2. *If we consider $\widehat{X} = x \in \mathbb{R}$ is a crisp number, it is possible to express it in the form of TNN i.e., $\widehat{X} = \langle x, x, x; 1, 0, 0\rangle$ then $\widetilde{\Im}(\widehat{X}) = \delta x + (1-\delta)x = x$.*

Lemma 1. *Let us consider there are n TNNs represented by $\widehat{X_i} = \langle x_i^L, x_i^M, x_i^U; \phi_{x_i}, \varphi_{x_i}, \psi_{x_i}\rangle$ where each $\widehat{X_i}$ has an associated scalar value $\alpha_i \in \mathbb{R}$. Then*

$$\begin{aligned}\widetilde{\Im}\left(\sum_{i=1}^n \alpha_i\widehat{X_i}\right) = \sum_{i=1}^n \Bigg[\delta\Big(\frac{x_i^L + 4x_i^M + x_i^U}{6}\Big)\Big(\bigwedge_{i=1}^n \phi_{x_i}\Big)^2 \\ + \frac{1-\delta}{2}\Bigg(\frac{2\Big[x_i^L + x_i^M + x_i^U\Big] - \Big[x_i^L - 2x_i^M + x_i^U\Big]\Big(\bigvee_{i=1}^n \varphi_{x_i}\Big) - \Big[x_i^L + 4x_i^M + x_i^U\Big]\Big(\bigvee_{i=1}^n \varphi_{x_i}\Big)^2}{6} \\ + \frac{2\Big[x_i^L + x_i^M + x_i^U\Big] - \Big[x_i^L - 2x_i^M + x_i^U\Big]\Big(\bigvee_{i=1}^n \psi_{x_i}\Big) - \Big[x_i^L + 4x_i^M + x_i^U\Big]\Big(\bigvee_{i=1}^n \psi_{x_i}\Big)^2}{6}\Bigg)\Bigg]\alpha_i\end{aligned}$$

Proof.

$$\sum_{i=1}^n \alpha_i\widehat{X_i} = \left\langle \sum_{i=1}^n \alpha_i x_i^L, \sum_{i=1}^n \alpha_i x_i^M, \sum_{i=1}^n \alpha_i x_i^U; \bigwedge_{i=1}^n \phi_{x_i}, \bigvee_{i=1}^n \varphi_{x_i}, \bigvee_{i=1}^n \psi_{x_i}\right\rangle$$

Using above Definition (7) and equation (8) , we have

$$\widetilde{\Im}\left(\sum_{i=1}^{n}\alpha_i\widehat{X_i}\right)=\delta\Big(\frac{\sum_{i=1}^{n}\alpha_i x_i^L+4\sum_{i=1}^{n}\alpha_i x_i^M+\sum_{i=1}^{n}\alpha_i x_i^U}{6}\Big)\Big(\bigwedge_{i=1}^{n}\phi_{x_i}\Big)^2$$
$$+\frac{1-\delta}{2}\left[\frac{1}{6}\left(\begin{array}{l}2\Big[\sum_{i=1}^{n}\alpha_i x_i^L+\sum_{i=1}^{n}\alpha_i x_i^M+\sum_{i=1}^{n}\alpha_i x_i^U\Big]\\ \quad -\Big[\sum_{i=1}^{n}\alpha_i x_i^L-2\sum_{i=1}^{n}\alpha_i x_i^M+\sum_{i=1}^{n}\alpha_i x_i^U\Big]\Big(\bigvee_{i=1}^{n}\varphi_{x_i}\Big)\\ \qquad -\Big[\sum_{i=1}^{n}\alpha_i x_i^L+4\sum_{i=1}^{n}\alpha_i x_i^M+\sum_{i=1}^{n}\alpha_i x_i^U\Big]\Big(\bigvee_{i=1}^{n}\varphi_{x_i}\Big)^2\end{array}\right)\right.$$
$$\left.+\frac{1}{6}\left(\begin{array}{l}2\Big[\sum_{i=1}^{n}\alpha_i x_i^L+\sum_{i=1}^{n}\alpha_i x_i^M+\sum_{i=1}^{n}\alpha_i x_i^U\Big]\\ \quad -\Big[\sum_{i=1}^{n}\alpha_i x_i^L-2\sum_{i=1}^{n}\alpha_i x_i^{M_1}+\sum_{i=1}^{n}\alpha_i x_i^U\Big]\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big)\\ \qquad -\Big[\sum_{i=1}^{n}\alpha_i x_i^L+4\sum_{i=1}^{n}\alpha_i x_i^M+\sum_{i=1}^{n}\alpha_i x_i^U\Big]\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big)^2\end{array}\right)\right]$$

that implies

$$\widetilde{\Im}\left(\sum_{i=1}^{n}\alpha_i\widehat{X_i}\right)=\sum_{i=1}^{n}\left[\delta\Big(\frac{x_i^L+4x_i^M+x_i^U}{6}\Big)\Big(\bigwedge_{i=1}^{n}\phi_{x_i}\Big)^2\right.$$
$$+\frac{1-\delta}{2}\left(\frac{2\Big[x_i^L+x_i^M+x_i^U\Big]-\Big[x_i^L-2x_i^M+x_i^U\Big]\Big(\bigvee_{i=1}^{n}\varphi_{x_i}\Big)-\Big[x_i^L+4x_i^M+x_i^U\Big]\Big(\bigvee_{i=1}^{n}\varphi_{x_i}\Big)^2}{6}\right.$$
$$\left.\left.+\frac{2\Big[x_i^L+x_i^M+x_i^U\Big]-\Big[x_i^L-2x_i^M+x_i^U\Big]\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big)-\Big[x_i^L+4x_i^M+x_i^U\Big]\Big(\bigvee_{i=1}^{n}\psi_{x_i}\Big)^2}{6}\right)\right]\alpha_i.$$

□

3.2. Neutrosophic Data Envelopment Analysis (Neu-DEA)

Let us consider the input and output for the DMU_o, $(o=1,2,\cdots,n)$ are defined as x_{io}, $(i=1,2,\cdots,m)$ and y_{ko}, $(k=1,2,\cdots,r)$ respectively. The Input and Output matrix for the system is defined by $X=[x_1,\cdots,x_m]\in R^{m\times n}$, and $Y=[y_l,\cdots,y_r]\in R^{r\times n}$ respectively.
The LPP model given below is used to evaluate the efficiency score of DMU_o:

$$\begin{aligned}\max_{u,v}\ \theta&=\sum_{k=1}^{r}u_k y_{ko},\\ \text{subject to }\ &\sum_{i=1}^{m}v_i x_{io}=1,\\ &\sum_{k=1}^{r}u_k y_{kj}\le\sum_{i=1}^{m}v_i x_{ij}\quad j=1,2,\cdots n,\\ \text{and }\ &u_k\ge 0,\ \forall k\ ;v_i\ge 0,\ \forall i\end{aligned}\tag{9}$$

which is known as traditional DEA or CCR Model [4]. The traditional DEA model may lead to incorrect evaluations of DMU performance due to the presence of faulty, imprecise, or ambiguous observational data. Moreover, if a DMU is found to be the most efficient, it may not be a reliable reference point for other less efficient DMUs. Hence, a robust method is necessary to address such situations, and Neutrosophic Set theory can be employed to achieve this goal. Neutrosophic Set theory deals with the concept of indeterminacy, which allows for handling uncertain and imprecise information by incorporating the degrees of truth, indeterminate, and falsity. Therefore, the utilization of Neutrosophic Set theory in DEA can lead to more accurate and reliable evaluations of DMU performance.

Assuming that the input and output weights (i.e., v_i & u_k) are crisp real numbers and the inputs and outputs data are TNNs. Thus, the Neutrosophic DEA (Neu-DEA) model is defined as

$$\begin{aligned}
&\max_{u,v} \theta = \sum_{k=1}^{r} u_k \widehat{y_{ko}}, \\
\text{subject to } &\sum_{i=1}^{m} v_i \widehat{x_{io}} = \widehat{1}, \\
&\sum_{k=1}^{r} u_k \widehat{y_{kj}} \leq \sum_{i=1}^{m} v_i, \widehat{x_{ij}}, \quad j = 1, 2, \cdots n, \\
\text{and } &u_k \geq 0, \ \forall k \ ; v_i \geq 0, \ \forall i,
\end{aligned} \tag{10}$$

where $\widehat{x_{ij}} = \langle x_{ij}^L, x_{ij}^M, x_{ij}^U, \phi_{x_{ij}}, \varphi_{x_{ij}}, \psi_{x_{ij}} \rangle$ and $\widehat{y_{kj}} = \langle y_{kj}^L, y_{kj}^M, y_{kj}^U, \phi_{y_{kj}}, \varphi_{y_{kj}}, \psi_{y_{kj}} \rangle$, $\forall\ i, j, k$ are the TNNs. The efficiency of the DMUs is optimal solution of the above Neu-DEA model i.e., $\theta^* \in [0, 1]$.

Definition 9. *A DMU is said to be efficient if its efficiency score is one (i.e., $\theta^* = 1$); Otherwise the DMU is called inefficient.*

Theorem 1. *The Neu-DEA model given in equation (10) is equivalent to the crisp LP model given in equation (12).*

Proof. If the aggregation operator and possibility mean approach, which are described in Step 3 and Step 4 of Section 4, are used and it clear that the optimal feasible solution of each Neu-DEA model is also an optimal feasible solution for the corresponding crisp DEA model, and conversely, the optimal feasible solution of the crisp DEA model is also an optimal feasible solution for the Neu-DEA model. Hence, the two models have identical optimal feasible solutions when using this approach. □

4. Method for Solving Neutrosophic DEA (Neu-DEA) model

Consider the inputs $\widehat{x_{ij}}$ and outputs $\widehat{y_{kj}}$ are the TNNs that is $\widehat{x_{ij}} = \langle x_{ij}^L, x_{ij}^M, x_{ij}^U, \phi_{x_{ij}}, \varphi_{x_{ij}}, \psi_{x_{ij}} \rangle$ and $\widehat{y_{kj}} = \langle y_{kj}^L, y_{kj}^M, y_{kj}^U, \phi_{y_{kj}}, \varphi_{y_{kj}}, \psi_{y_{kj}} \rangle$, $\forall\ i, j, k$. The Neu-DEA model defined in equation (10) can't be solved directly using the usual LPP approach. The following steps are proceed to solve the Neu-DEA model and obtain the efficiency score of the DMUs.

Step 1: Fuzzifier or Fuzzification is the process of changing crisp input-output data into neutrosophic input-output data using information from a knowledge base. Fuzzification is necessary and desirable at an early stage of the uncertainty theory. As a result, the fuzzifier may be defined as a mapping from an observable crisp data space to a fuzzy data space in a given discourse universe. Triangular membership functions are the most widely employed in the fuzzification process because they are easily implemented by embedded controllers. In this case, a triangular neutrosophic fuzzy set is employed in the fuzzification procedure.

Step 2: The Neutrosophic DEA model is converted form the traditional DEA model by taking neutrosophic inputs-outputs as given in equation (10).

Step 3: Applying the possibility mean function ($\widetilde{\Im}$) in the Neu-DEA model
The Neu-CCR models will be

$$\begin{aligned} \max_{u,v}\ \theta &= \widetilde{\Im}\Big(\sum_{k=1}^{r} u_k \widehat{y_{ko}}\Big), \\ \text{subject to } \widetilde{\Im}\Big(\sum_{i=1}^{m} v_i \widehat{x_{io}}\Big) &= \widetilde{\Im}(1), \\ \widetilde{\Im}\Big(\sum_{k=1}^{r} u_k \widehat{y_{kj}}\Big) &\le \widetilde{\Im}\Big(\sum_{i=1}^{m} v_i \widehat{x_{ij}}\Big), \quad j = 1, 2, \cdots n, \\ \text{and } u_k \ge 0,\ \forall k\ ; v_i &\ge 0,\ \forall i. \end{aligned} \tag{11}$$

Step 4: The above Neu-DEA model is turned into a corresponding crisp LP model with the help of the modified Khatter method.
Using Lemma 1, The equation (11) become

$$\begin{aligned} \max_{u,v}\ \theta = \sum_{k=1}^{r} \Bigg[& \delta\Big(\frac{y_{ko}^L + 4y_{ko}^M + y_{ko}^U}{6}\Big)\Big(\bigwedge_{k=1}^{r} \phi_{y_{ko}}\Big)^2 \\ & + \frac{1-\delta}{2}\Bigg(\frac{2\Big[y_{ko}^L + y_{ko}^M + y_{ko}^U\Big] - \Big[y_{ko}^L - 2y_{ko}^M + y_{ko}^U\Big]\Big(\bigvee_{k=1}^{r} \varphi_{y_{ko}}\Big) - \Big[y_{ko}^L + 4y_{ko}^M + y_{ko}^U\Big]\Big(\bigvee_{k=1}^{r} \varphi_{y_{ko}}\Big)^2}{6} \\ & + \frac{2\Big[y_{ko}^L + y_{ko}^M + y_{ko}^U\Big] - \Big[y_{ko}^L - 2y_{ko}^M + y_{ko}^U\Big]\Big(\bigvee_{k=1}^{r} \psi_{y_{ko}}\Big) - \Big[y_{ko}^L + 4y_{ko}^M + y_{ko}^U\Big]\Big(\bigvee_{k=1}^{r} \psi_{y_{ko}}\Big)^2}{6}\Bigg)\Bigg] u_k, \\ \text{subject to } \sum_{i=1}^{m} \Bigg[& \delta\Big(\frac{x_{io}^L + 4x_{io}^M + x_{io}^U}{6}\Big)\Big(\bigwedge_{k=1}^{r} \phi_{x_{io}}\Big)^2 \\ & + \frac{1-\delta}{2}\Bigg(\frac{2\Big[x_{io}^L + x_{io}^M + x_{io}^U\Big] - \Big[x_{io}^L - 2x_{io}^M + x_{io}^U\Big]\Big(\bigvee_{k=1}^{r} \varphi_{x_{io}}\Big) - \Big[x_{io}^L + 4x_{io}^M + x_{io}^U\Big]\Big(\bigvee_{i=1}^{m} \varphi_{x_{io}}\Big)^2}{6} \\ & + \frac{2\Big[x_{io}^L + x_{io}^M + x_{io}^U\Big] - \Big[x_{io}^L - 2x_{io}^M + x_{io}^U\Big]\Big(\bigvee_{k=1}^{r} \psi_{x_{io}}\Big) - \Big[x_{io}^L + 4x_{io}^M + x_{io}^U\Big]\Big(\bigvee_{i=1}^{m} \psi_{x_{io}}\Big)^2}{6}\Bigg)\Bigg] v_i = 1, \\ \sum_{k=1}^{r} \Bigg[& \delta\Big(\frac{y_{kj}^L + 4y_{kj}^M + y_{kj}^U}{6}\Big)\Big(\bigwedge_{k=1}^{r} \phi_{y_{kj}}\Big)^2 \end{aligned} \tag{12}$$

$$
\begin{aligned}
&+\frac{1-\delta}{2}\Bigg(\frac{2\Big[y^L_{kj}+y^M_{kj}+y^U_{kj}\Big]-\Big[y^L_{kj}-2y^M_{kj}+y^U_{kj}\Big]\Big(\bigvee_{k=1}^{r}\varphi_{y_{kj}}\Big)-\Big[y^L_{kj}+4y^M_{kj}+y^U_{kj}\Big]\Big(\bigvee_{k=1}^{r}\varphi_{y_{kj}}\Big)^2}{6}\\
&\quad+\frac{2\Big[y^L_{kj}+y^M_{kj}+y^U_{kj}\Big]-\Big[y^L_{kj}-2y^M_{kj}+y^U_{kj}\Big]\Big(\bigvee_{k=1}^{r}\psi_{y_{kj}}\Big)-\Big[y^L_{kj}+4y^M_{kj}+y^U_{kj}\Big]\Big(\bigvee_{k=1}^{r}\psi_{y_{kj}}\Big)^2}{6}\Bigg)\Bigg]u_k\\
&\leq\sum_{i=1}^{m}\Bigg[\delta\Big(\frac{x^L_{ij}+4x^M_{ij}+x^U_{ij}}{6}\Big)\Big(\bigwedge_{k=1}^{r}\phi_{x_{ij}}\Big)^2\\
&+\frac{1-\delta}{2}\Bigg(\frac{2\Big[x^L_{ij}+x^M_{ij}+x^U_{ij}\Big]-\Big[x^L_{ij}-2x^M_{ij}+x^U_{ij}\Big]\Big(\bigvee_{k=1}^{r}\varphi_{x_{ij}}\Big)-\Big[x^L_{ij}+4x^M_{ij}+x^U_{ij}\Big]\Big(\bigvee_{i=1}^{m}\varphi_{x_{ij}}\Big)^2}{6}\\
&\quad+\frac{2\Big[x^L_{ij}+x^M_{ij}+x^U_{ij}\Big]-\Big[x^L_{ij}-2x^M_{ij}+x^U_{ij}\Big]\Big(\bigvee_{k=1}^{r}\psi_{x_{ij}}\Big)-\Big[x^L_{ij}+4x^M_{ij}+x^U_{ij}\Big]\Big(\bigvee_{i=1}^{m}\psi_{x_{ij}}\Big)^2}{6}\Bigg)\Bigg]v_i,\forall j
\end{aligned}
$$

and $u_k \geq 0,\ \forall k\ ; v_i \geq 0,\ \forall i$

This is the corresponding crisp DEA or CCR model of the proposed Neu-DEA model.

Step 5: The optimum solution (θ^*) of the crisp DEA model is obtained for each $\delta \in [0, 1]$ which reflects the DM's risk preference.

Step 6: The average of efficiency scores with different risk factor are considered for ranking the DMUs.

For the purpose of solving the proposed Neu-DEA model, MATLAB2013a software is utilized and evaluate the performance of each DMU. Figure 1 illustrates the flowchart for the solution technique of the proposed Neu-DEA model.

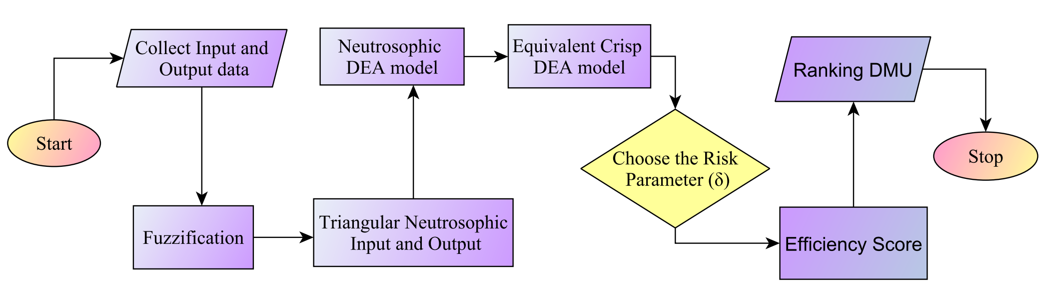

Figure 1: Flowchart for solving Neutrosophic DEA model

5. Numerical Example

This section examines the validity of the suggested approach and its applicability with a suitable example. The obtained results are compared to the technique previously presented. Consider the input-output data from the example [1] as shown in Table 1.

DMU	Input 1	Input 2	Output 1	Output 2
DMU 1	$\langle 3.5, 4.0, 4.5;$ 0.7,0.4,0.3$\rangle$	$\langle 1.9, 2.1, 2.3;$ 0.4,0.3,0.5$\rangle$	$\langle 2.4, 2.6, 2.8;$ 0.9,0.2,0.1$\rangle$	$\langle 3.8, 4.1, 4.4;$ 0.8,0.5,0.1$\rangle$
DMU 2	$\langle 2.9, 2.9, 2.9;$ 0.6,0.5,0.2$\rangle$	$\langle 1.4, 1.5, 1.6;$ 0.8,0.2,0.1$\rangle$	$\langle 2.2, 2.2, 2.2;$ 0.9,0.3,0.0$\rangle$	$\langle 3.3, 3.5, 3.7;$ 1.0,0.0,0.0$\rangle$
DMU 3	$\langle 4.4, 4.9, 5.4;$ 0.6,0.4,0.1$\rangle$	$\langle 2.2, 2.6, 3.6;$ 0.7,0.4,0.2$\rangle$	$\langle 2.7, 3.2, 3.7;$ 0.7,0.5,0.1 $\rangle$	$\langle 4.3, 5.1, 5.9;$ 0.7,0.5,0.1$\rangle$
DMU 4	$\langle 3.4, 4.1, 4.8;$ 0.4,0.3,0.2$\rangle$	$\langle 2.2, 2.3, 2.4;$ 1.0,0.0,0.0$\rangle$	$\langle 2.5, 2.9, 3.3;$ 0.7,0.5,0.1$\rangle$	$\langle 5.5, 5.7, 5.9;$ 0.4,0.2,0.1$\rangle$
DMU 5	$\langle 5.9, 6.5, 7.1;$ 0.7,0.4,0.3 $\rangle$	$\langle 3.6, 4.1, 4.6;$ 0.9,0.1,0.1$\rangle$	$\langle 4.4, 5.1, 5.8;$ 0.8,0.4,0.2$\rangle$	$\langle 6.5, 7.4, 8.3;$ 0.5,0.0,0.2$\rangle$

Table 1: *Data of the five DMUs used in the numerical example [1]*

DMUs	Efficiency Score					Mean
	$\delta = 0$	$\delta = 0.25$	$\delta = 0.5$	$\delta = 0.75$	$\delta = 1$	
DMU 1	0.8395	0.8709	0.9326	1.00	1.00	0.9286
DMU 2	1.0000	1.0000	1.0000	0.9554	0.6723	0.92554
DMU 3	0.7335	0.6853	0.6337	0.5449	0.3455	0.58858
DMU 4	0.9596	0.8565	0.7459	0.5914	0.3391	0.6985
DMU 5	0.9524	0.7796	0.5998	0.3881	0.1540	0.57478

Table 2: Efficiency Score of the DMUs

The steps given in Section 4 are implemented to solve the above example, and the DMU's efficiencies are determined with various risk factor, as shown in Table 2. Figure 2 depicts how the risk factor affects the efficiency of the DMUs. It has been observed that when the risk factor increases, most of the DMU's efficiencies are decreases. If a DMU has an efficiency score one, it is considered to be in the efficient group; otherwise, it is in the inefficient group according to Definition 9. The DMU 2 is efficient when the risk factors $\delta = 0, \delta = 0.25$, and $\delta = 0.5$ and other DMUs are in inefficient group whereas DMU 1 is efficient when risk factors $\delta = 0.75$, and $\delta = 1$ and other DMUs are in inefficient groups. The overall DMU's efficiency is obtained by considering average of the DMU's efficiency in various risk level and ranked them as : $DMU1 > DMU2 > DMU4 > DMU3 > DMU5$. Furthermore, as risk factors increase, the efficiency scores of DMUs 3, 4, and 5 decrease. The efficiency score of DMU 2 remains constant (i.e., one) until the risk parameter increases from 0 to 0.5 after that (increases from 0.5 to 1) the efficiency decreases. When the risk parameter is increased from 0 to 0.75, the efficiency score of DMU 1 increases after that (increases from 0.75 to 1), the efficiency score remain same (i.e, one) or it is efficient unit. The average DMU's efficiency with various risk levels is utilized to determine the overall efficiency score for complete ranking purpose (see Table 2). Table 3 compares the efficiency score obtain in proposed approach to the efficiency score obtain in the Abdelfattah parametric approach [1] and Edalatpanah ranking approach [9]. The efficiency scores in Abdelfattah parametric approach are obtained in interval form, which make it difficult to rank the DMUs and also difficult to categorize the DMUs into efficient and inefficient groups. Also, The Edalatpanah ranking approach converts the Neu-DEA model into a crisp LP model and ranks DMUs by DMU's efficiency. In comparison to earlier approaches, the proposed approach evaluates the efficiency score of the DMUs with a risk factor. The risk factor indicates how much risk is taken while evaluating the DMUs' efficiency in the Neu-DEA model. The decision maker ranked and classified the DMUs into efficient and inefficient groups, each with its own risk.

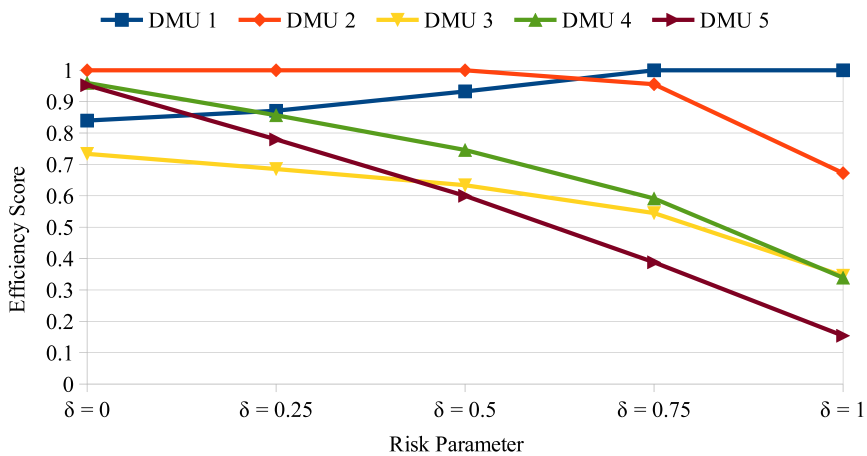

Figure 2: *Efficiency Score of the DMUs with risk parameter*

DMUs	Abdelfattah Parametric Approach		Edalatpanah Ranking Approach		Proposed Approach	
	Efficiency Score	Rank	Efficiency Score	Rank	Efficiency Score	Rank
DMU 1	[0.624, 1.000]	4	0.9183	2	0.9286	1
DMU 2	[0.836, 1.000]	2	1.0000	1	0.92554	2
DMU 3	[0.571, 1.000]	5	0.6302	5	0.58858	4
DMU 4	[0.855, 1.000]	1	0.7975	3	0.6985	3
DMU 5	[0.638, 1.000]	3	0.7180	4	0.57478	5

Table 3: *Comparative analysis of efficiency score*

6. Conclusion

The possibility mean for TNN is redefined, and this possibility mean function has the ability to compare both crisps real numbers and TNNs. The suggested method is linked to a risk factor that reflects how willing the DM is to take risks. The concept of the possibility mean approach is introduced by modifying Khatter's approach for the neutrosophic LP problem and used for the first time to solve the neutrosophic DEA models. This proposed approach has ability to convert directly the Neutrosophic DEA into its corresponding crisp DEA model with risk parameter. The decision-makers take a particular risk when determining the efficiency score of each DMU without utilizing any inappropriate evaluation values. An example has been presented here to demonstrate the proposed solution technique of Neu-DEA model. The recommended possibility mean approach is effective and practical for solving the Neu-DEA model, and can be widely applied in various disciplines based on the results. According to the comparative studies, the suggested approach offers a significant advantage over the existing approaches (see Table 3). It is suggested that further study should be done on estimating the DMUs' efficiency in various types of DEA models, such as BCC, SBM, Additive, and Super-efficiency model. Future study should concentrate on the applicability of our method in real-world scenarios.

Acknowledgements

The authors would like to express their gratitude to the Editor-in-Chief and the reviewers for their responses and insightful suggestions for improving the previous version of the manuscript.

References

[1] Abdelfattah, W. (2019). Data envelopment analysis with neutrosophic inputs and outputs. Expert Systems, 36(6), e12453. doi: 10.1111/exsy.12453.

[2] Abdelfattah, W. (2021). Neutrosophic Data Envelopment Analysis: An Application to Regional Hospitals in Tunisia. Neutrosophic Sets and Systems, 41, 89-105. doi: 10.5281/zenodo.4625686.

[3] Akram, M., Shah, S. M. U., Al-Shamiri, M. M. A., & Edalatpanah, S. A. (2023). Extended DEA method for solving multi-objective transportation problem with Fermatean fuzzy sets. AIMS Mathematics, 8(1), 924-961. doi: 10.3934/math.2023045.

[4] Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. European journal of operational research, 3(4), 339. doi: 10.1016/0377-2217(79)90229-7.

[5] Chaubey, V., Sharanappa, D. S., Mohanta, K. K., Mishra, V. N., & Mishra, L. N. (2022). Efficiency and Productivity Analysis of the Indian Agriculture Sector Based on the Malmquist-DEA. Universal Journal of Agricultural Research, 10(4), 331-343. doi: 10.13189/ujar.2022.100402.

[6] Deli, I., & Şubaş, Y. (2017). A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. International Journal of Machine Learning and Cybernetics, 8(4), 1309-1322. doi: 10.1007/s13042-016-0505-3.

[7] Edalatpanah, S. A. (2018). Neutrosophic perspective on DEA. Journal of applied research on industrial engineering, 5(4), 339-345. doi: 10.22105/jarie.2019.196020.1100.

[8] Edalatpanah, S. A., & Smarandache, F. (2019). Data envelopment analysis for simplified neutrosophic sets. Neutrosophic Sets and Systems, 29, 215-226. doi: 10.5281/zenodo.3514433.

[9] Edalatpanah, S. A. (2020). Data envelopment analysis based on triangular neutrosophic numbers. CAAI transactions on intelligence technology, 5(2), 94-98. doi: 10.1049/trit.2020.0016.

[10] Emrouznejad, A., Tavana, M., & Hatami-Marbini, A. (2014). The state of the art in fuzzy data envelopment analysis. Performance measurement with fuzzy data envelopment analysis, 1-45. doi: 10.1007/978-3-642-41372-8_1.

[11] Garai, T., Dalapati, S., Garg, H., & Roy, T. K. (2020). Possibility mean, variance and standard deviation of single-valued neutrosophic numbers and its applications to multi-attribute decision-making problems. Soft Computing, 24(24), 18795-18809. doi: 10.1007/s00500-020-05112-2.

[12] Kaffash, S., Azizi, R., Huang, Y., & Zhu, J. (2020). A survey of data envelopment analysis applications in the insurance industry 1993–2018. European journal of operational research, 284(3), 801-813. doi: 10.1016/j.ejor.2019.07.034.

[13] Kahraman, C., Otay, İ., Öztayşi, B., & Onar, S. Ç. (2019). An integrated AHP & DEA methodology with neutrosophic sets. In Fuzzy Multi-Criteria Decision-Making Using Neutrosophic Sets (pp. 623-645). Springer, Cham. doi: 10.1007/978-3-030-00045-$5_2$4.

[14] Khatter, K. (2020). Neutrosophic linear programming using possibilistic mean. Soft Computing, 24(22), 16847-16867. doi: 10.1007/s00500-020-04980-y.

[15] Mao, X., Guoxi, Z., Fallah, M., & Edalatpanah, S. A. (2020). A neutrosophic-based approach in data envelopment analysis with undesirable outputs. Mathematical problems in engineering, 2020. doi: 10.1155/2020/7626102.

[16] Mohanta, K. K., Sharanappa, D. S., & Aggarwal, A. (2021). Efficiency analysis in the management of covid-19 pandemic in India based on data envelopment analysis. Current Research in Behavioral Sciences, 2, 100063. doi: 10.1016/j.crbeha.2021.100063.

[17] Mohanta, K. K., Chaubey, V., Sharanappa, D. S., & Mishra, V. N. (2022). A modified novel method for solving the uncertainty linear programming problems based on triangular neutrosophic number. Transactions on Fuzzy Sets and Systems, 1(1), 155-169. doi: 10.30495/TFSS.2022.1956751.1022.

[18] Mohanta, K. K., Sharanappa, D. S., Dabke, D., Mishra, L. N., & Mishra, V. N. (2022). Data Envelopment Analysis on the Context of Spherical Fuzzy Inputs and Outputs. European Journal of Pure and Applied Mathematics, 15(3), 1158-1179. doi: 10.29020/nybg.ejpam.v15i3.4391.

[19] Mohanta, K. K., & Sharanappa, D. S. (2022). The Spherical Fuzzy Data Envelopment Analysis (SF-DEA): A Novel Approach for Efficiency Analysis. AIJR Abstracts, 52. doi: 10.21467/abstracts.138.

[20] Montazeri, F. Z. (2020). An overview of data envelopment analysis models in fuzzy stochastic environments. Journal of Fuzzy Extension and Applications, 1(4), 272-278. doi: 10.22105/jfea.2020.258330.1030.

[21] Rasoulzadeh, M., & Fallah, M. (2020). An overview of portfolio optimization using fuzzy data envelopment analysis models. Journal of fuzzy extension and applications, 1(3), 180-188. doi: 10.22105/jfea.2020.255034.1027.

[22] Sengupta, J. K. (1992). A fuzzy systems approach in data envelopment analysis. Computers & mathematics with applications, 24(8-9), 259-266. doi: 10.1016/0898-1221(92)90203-T.

[23] Smarandache, F. (1999). A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set, and Logic. Rehoboth: American Research Press. Retrieved from: gallup.unm.edu

[24] Taeb, Z., Hosseinzadeh Lotfi, F., & Abbasbandy, S. (2017). Determine the efficiency of time depended units by using data envelopment analysis. International journal of research in industrial engineering, 6(3), 193-201. doi: 10.22105/riej.2017.49156.

[25] Tapia, J. F. D. (2021). Evaluating negative emissions technologies using neutrosophic data envelopment analysis. Journal of Cleaner Production, 286, 125494. doi: 10.1016/j.jclepro.2020.125494.

[26] Ucal Sari, I., & Ak, U. (2022). Machine efficiency measurement in industry 4.0 using fuzzy data envelopment analysis. Journal of fuzzy extension and applications, 3(2), 177-191. doi: 10.22105/jfea.2022.326644.1199.

[27] Wan, S. P., Li, D. F., & Rui, Z. F. (2013). Possibility mean, variance and covariance of triangular intuitionistic fuzzy numbers. Journal of intelligent & fuzzy systems, 24(4), 847-858. doi: 10.3233/IFS-2012-0603.

[28] Yang, W., Cai, L., Edalatpanah, S. A., & Smarandache, F. (2020). Triangular single valued neutrosophic data envelopment analysis: application to hospital performance measurement. Symmetry, 12(4), 588. doi: 10.3390/sym12040588.

[29] Zadeh, L. A. (1978). Fuzzy sets as a basis for a theory of possibility. Fuzzy sets and systems, 1(1), 3-28. doi: 10.1016/0165-0114(78)90029-5.

[30] Zhou, W., & Xu, Z. (2020). An overview of the fuzzy data envelopment analysis research and its successful applications. International Journal of Fuzzy Systems, 22(4), 1037-1055. doi: 10.1007/s40815-020-00853-6.