On some quantitative indicators of a few features of electoral systems

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Abstract. Electoral systems can be analyzed by means of their numerous properties. Some quantitative indicators of a few technical features of electoral systems are considered. With respect to the effective number of parties in a electoral system, one can observe some known indicators (e.g., fractionalization of vote shares, the Laakso-Taagepera index, the Wildgen index, the Molinar index). With regard to the government stability, one can look at the indicator which is called the expectation of government stability. These indicators are examined from the empirical point of view, i.e., in relation of elections in different countries.

Keywords: electoral systems, government stability indicator, Laakso-Taagepera index, vote fractionalization, Wildgen index

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1. Introduction

Electoral systems can be analyzed by means of their numerous properties. For instance, electoral participation, number of parties [12] proportionality [18], party power and coalitions [20], government stability [9] can be observed. These features of electoral systems have certain influence on the electoral outcome and on the political structure of the country.
The number of parties in an electoral system is important, because it describes a basic variable of the electoral system. Several quantitative indicators of the effective number of parties that refer to the phase of electoral process before the transformation of votes into seats can be considered [4]. In this paper some indicators in relation to the empirical cases of elections in different countries are considered. Some indicators of effective number of parties are sensitive to the formal count n of parties [7]. The contribution of this paper to the reduction of this sensitivity is through modification of these indicators which is suggested by elimination of tiny parties. These modified indicators are calculated in several cases of elections in various countries and compared with corresponding indicators (the idea of not taking into account tiny parties has been used with Rae disproportionality indicator [19]). Some recent research about the effective number of parties can be seen in [8, 13, 22].

The government formation appears in electoral phase which is after the transformation of votes into seats. There are various approaches to this complex problem [9, 11, 21]. Numerous factors can influence the government stability (e.g., image of competence, government provided staff, media exposure, see [17, 10], and some recent research can be seen in [1, 3, 5]. With

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regard to the government stability, a special indicator which is called the expectation of government stability can be observed [4]. In the paper, this indicator is viewed from the empirical standpoint by a few examples of elections in several countries. The formula of the indicator of the government stability expectation uses the so-called minimal winning coalitions with distance threshold [4]. However, in certain practical situation the minimal winning coalitions with distance threshold can be formed with a very small possibility. This paper contributes by the modification of this indicator, by using the so-called politically feasible winning coalitions [15] that can better correspond real situation of government formation. In several empirical cases of various parliaments, the modified expectation of government stability is calculated and compared with the indicator of government stability expectation.

In Subsection 1.1, some well-known indicators of the effective number of parties are described.

1.1. The number of political parties

The formal number n of parties that go to elections can be just counted, but it does not seem to be the best way to describe complex relationships between the parties and their relative strength. The effective number of parties is used as an important parameter to illustrate competitive relation between the various actors of the political system. For example, if the total number of parties that participate in the election is $n \geq 3$ where two parties have approximately equal number of votes and almost all the votes together, then that system can be considered as a two-party system.

Therefore, with respect to the effective number of parties in the system, the votes each party obtains are taken into account by many proposed indicators. They refer to the pretransformation phase of the electoral process, i.e., before the transformation of votes into seats. The following notation is used:

- $n \geq 2$ - the total number of parties in the elections
- $v_i > 0$ - the number of votes that party i has got in the election ($i = 1, \ldots, n$), and $P = \sum_{i=1}^{n} v_i$
- $\omega_i = \frac{v_i}{P}$ - vote share of the party i ($i = 1, \ldots, n$), where $\sum_{i=1}^{n} \omega_i = 1$.

General indicator N_α for the effective number of parties in the electoral system, where vote shares ω_i is considered, is the following:

$$N_\alpha = (\sum_{i=1}^{n} \omega_i^\alpha)^{\frac{1}{1-\alpha}}, \tag{1}$$

where α is parameter.

Notice that for $\alpha = 2$, from (1) one gets the Laakso-Taagepera index [12]

$$N_2 = \frac{1}{\sum_{i=1}^{n} \omega_i^2}. \tag{2}$$

Notice that the vote share of the party i, $\omega_i = \frac{v_i}{P}$, can be viewed as the probability that one randomly selected voter votes for the party i. Thus, quantity in the denominator of (2), $\sum_{i=1}^{n} \omega_i^2$,

represents the probability that two randomly selected voters vote for the same party.

Reciprocally, the probability that two randomly selected voters do not vote for the same party is the following:

$$1 - \sum_{i=1}^{n} \omega_i^2 = F_2, \tag{3}$$

where F_2 is called the vote fractionalization [18]. It holds that $F_2 \in [0, 1]$.

From (2) and (3) it follows that

$$F_2 = 1 - \frac{1}{N_2}\,. \tag{4}$$

When parameter α tends to 1, then from (1) one obtains the so-called Wildgen index N_1 [24]:

$$N_1 = \lim_{\alpha \to 1} N_\alpha = e^{-\sum_{i=1}^{n} \omega_i \ln \omega_i}\,. \tag{5}$$

Notice that when parameter α tends to 0, then the general indicator N_α by (1) tends to n.

It can be seen that Laakso-Taagepera indicator N_2 is too sensitive to the share of votes of the largest party. The Wildgen indicator N_1 is more sensitive than the others to the total number of parties, even if they are tiny. In order to improve these imperfections, the Molinar indicator (denoted by NP) was introduced in the following form [16]:

$$NP = 1 + \frac{\sum_{i=1}^{n} \omega_i^2 - \omega_{[1]}^2}{(\sum_{i=1}^{n} \omega_i^2)^2} = 1 + N_2 - N_2^2 \cdot \omega_{[1]}^2, \tag{6}$$

where $\omega_{[1]}$ is the largest vote share. The Molinar indicator uses Laakso-Taagepera indicator N_2 and separates the largest party.

The following properties hold: $NP \leq N_2 \leq N_1$. Further, when $\omega_i = \dfrac{1}{n}$, $\forall i = 1, \ldots, n$, then $NP = N_2 = N_1 = n$.

In Section 2, some well-known indicators of the effective number of parties are examined by the cases of elections for the European Parliament in the EU member states, and modification of these indicators is suggested. In Section 3 the indicator of the expectation of government stability is considered in several empirical cases of parliaments after elections in certain countries in the EU and its modification is suggested. Finally, in Section 4 a few concluding remarks are given with respect to the suggested modified indicators which provide certain improvements compared to the original indicators.

2. Example for the EU states. Modified indicators of the number of parties.

Example 1. *In this example indicators N_2, N_1 and NP have been calculated in the cases of elections for the European Parliament in 2004, 2009, 2014, 2019 and 2024 in every member state of the European Union (EU). All member states of the EU use proportional electoral system in these elections [6].*

Corresponding values of these indicators per EU member state are given in Table 1. (Since 2020, the United Kingdom is not member of the EU.)

St.	F_2	N_2	N_1	NP	n
Ge	*0,791226*	*4,79*	*7,17*	*2,73*	*24*
	0,824429	*5,70*	*8,28*	*3,65*	*32*
	0,808839	*5,23*	*7,66*	*3,76*	*25*
	0,85836	*7,06*	*10,11*	*5,52*	*41*
	0,871574	*7,79*	*11,06*	*5,38*	*35*
Fr	*0,845679*	*6,48*	*8,12*	*3,97*	*11*
	0,857488	*7,02*	*9,82*	*5,34*	*34*
	0,848311	*6,59*	*8,17*	*4,90*	*12*
	0,846433	*6,51*	*8,38*	*4,22*	*12*
	0,837767	*6,16*	*8,94*	*3,40*	*37*
It	*0,833528*	*6,01*	*9,59*	*3,52*	*25*
	0,782984	*4,61*	*6,52*	*2,97*	*16*
	0,752989	*4,05*	*5,51*	*2,36*	*12*
	0,787954	*4,72*	*6,37*	*3,11*	*18*
	0,823983	*5,68*	*7,40*	*4,01*	*15*
(U)	*0,833457*	*6,00*	*8,61*	*4,58*	*32*
	0,847336	*6,55*	*9,47*	*4,34*	*31*
	0,80597	*5,15*	*7,46*	*4,28*	*30*
	0,824739	*5,71*	*8,13*	*3,74*	*23*
Sp	*0,631421*	*2,71*	*3,56*	*2,30*	*31*
	0,659211	*2,93*	*4,18*	*2,37*	*35*
	0,844335	*6,42*	*9,77*	*4,48*	*39*
	0,81255	*5,33*	*7,45*	*3,2*	*32*
	0,770479	*4,35*	*6,56*	*3,10*	*34*
Pl	*0,866993*	*7,52*	*9,58*	*5,24*	*21*
	0,70595	*3,40*	*4,61*	*2,12*	*12*
	0,773091	*4,41*	*5,90*	*3,40*	*12*
	0,638856	*2,77*	*3,48*	*2,19*	*9*
	0,708468	*3,43*	*4,18*	*2,81*	*11*
Ro	*0,829278*	*5,86*	*7,976*	*4,01*	*14*
	0,77561	*4,46*	*5,31*	*3,54*	*8*
	0,800973	*5,02*	*7,36*	*2,46*	*16*
	0,813011	*5,35*	*6,84*	*4,26*	*14*
	0,720669	*3,58*	*5,67*	*1,56*	*13*
Ne	*0,844502*	*6,43*	*8,14*	*4,96*	*15*
	0,871897	*7,81*	*8,93*	*6,36*	*17*
	0,888711	*8,99*	*10,16*	*8,05*	*19*
	0,887935	*8,92*	*10,54*	*7,04*	*16*
	0,88511	*8,70*	*10,94*	*6,33*	*15*
Be	*0,881528*	*8,44*	*9,90*	*7,28*	*22*
	0,906863	*10,74*	*12,64*	*9,34*	*30*
	0,896797	*9,69*	*11,19*	*7,97*	*15*
	0,907875	*10,85*	*11,90*	*9,49*	*18*
	0,903734	*10,39*	*11,67*	*9,03*	*18*
Gr	*0,686474*	*3,19*	*4,49*	*2,31*	*21*
	0,745303	*3,93*	*5,88*	*2,86*	*27*
	0,851304	*6,73*	*11,04*	*4,53*	*40*
	0,818273	*5,52*	*10,22*	*3,18*	*40*
	0,857895	*7,04*	*10,71*	*4,04*	*31*
Cz	*0,831157*	*5,92*	*8,52*	*3,76*	*31*
	0,819901	*5,55*	*8,88*	*3,50*	*33*
	0,892622	*9,31*	*12,28*	*8,06*	*38*
	0,879694	*8,31*	*11,31*	*6,21*	*39*
	0,846182	*6,50*	*9,14*	*4,61*	*30*
Sw	*0,848514*	*6,60*	*7,70*	*4,97*	*15*
	0,85808	*7,05*	*8,62*	*5,09*	*15*
	0,864653	*7,39*	*8,71*	*5,19*	*14*
	0,854204	*6,86*	*8,02*	*5,26*	*22*
	0,848483	*6,60*	*7,81*	*4,93*	*20*

St.	F_2	N_2	N_1	NP	n
Pr	*0,653222*	*2,88*	*3,76*	*2,10*	*13*
	0,769197	*4,33*	*5,53*	*3,17*	*13*
	0,76603	*4,27*	*5,89*	*3,16*	*16*
	0,790327	*4,77*	*6,98*	*2,84*	*17*
	0,767989	*4,31*	*5,87*	*3,32*	*17*
Hu	*0,647964*	*2,86*	*3,64*	*2,03*	*8*
	0,626287	*2,68*	*3,72*	*1,40*	*8*
	0,684297	*3,17*	*4,40*	*1,51*	*8*
	0,677473	*3,10*	*4,62*	*1,44*	*9*
	0,698806	*3,32*	*4,58*	*2,12*	*11*
Au	*0,741729*	*3,87*	*4,41*	*3,21*	*6*
	0,794289	*4,86*	*5,53*	*3,74*	*8*
	0,801197	*5,03*	*5,81*	*4,49*	*9*
	0,766852	*4,29*	*4,87*	*3,09*	*7*
	0,797473	*4,94*	*5,53*	*4,37*	*7*
Bu					
	0,853409	*6,82*	*8,28*	*5,05*	*14*
	0,8225	*5,63*	*7,84*	*3,70*	*22*
	0,815498	*5,42*	*7,78*	*3,75*	*23*
	0,862094	*7,25*	*10,12*	*5,14*	*31*
De	*0,816956*	*5,46*	*8,14*	*3,28*	*9*
	0,840611	*6,27*	*6,94*	*5,46*	*9*
	0,83283	*5,98*	*6,77*	*4,45*	*8*
	0,84957	*6,65*	*7,86*	*5,21*	*10*
	0,887333	*8,88*	*9,8*	*7,48*	*11*
Fi	*0,819971*	*5,55*	*7,74*	*4,82*	*14*
	0,845158	*6,65*	*7,55*	*5,21*	*14*
	0,853799	*6,84*	*7,88*	*5,45*	*15*
	0,861045	*7,20*	*8,36*	*5,96*	*18*
	0,848415	*6,60*	*7,69*	*4,92*	*14*
Sk	*0,86318*	*7,31*	*10,77*	*6,75*	*17*
	0,829075	*5,85*	*8,30*	*3,34*	*16*
	0,891787	*9,24*	*13,69*	*5,28*	*29*
	0,890687	*9,15*	*12,52*	*6,76*	*31*
	0,828227	*5,82*	*8,04*	*4,20*	*23*
Ir	*0,786302*	*4,68*	*5,96*	*3,77*	*7*
	0,807732	*5,19*	*6,06*	*3,90*	*8*
	0,817256	*5,47*	*6,47*	*4,98*	*11*
	0,826356	*5,76*	*7,01*	*3,86*	*13*
	0,869713	*7,68*	*9,98*	*6,13*	*16*
Cr					
	0,779832	*4,54*	*8,25*	*3,31*	*28*
	0,72322	*3,61*	*5,47*	*2,37*	*25*
	0,88493	*8,69*	*13,10*	*5,79*	*33*
	0,791779	*4,80*	*7,54*	*2,96*	*25*
Li	*0,842445*	*6,35*	*8,14*	*3,68*	*12*
	0,853568	*6,83*	*9,09*	*4,46*	*15*
	0,862893	*7,29*	*8,07*	*6,66*	*10*
	0,892238	*9,28*	*11,65*	*6,91*	*16*
	0,88685	*8,84*	*11,21*	*6,29*	*15*
La	*0,840644*	*6,28*	*8,97*	*3,71*	*16*
	0,864835	*7,40*	*9,83*	*5,04*	*17*
	0,731497	*3,72*	*5,59*	*1,72*	*14*
	0,844416	*6,43*	*8,15*	*4,55*	*16*
	0,852599	*6,78*	*9,16*	*4,82*	*16*
Sn	*0,83317*	*5,99*	*7,21*	*5,0*	*13*
	0,837013	*6,14*	*7,42*	*4,46*	*12*
	0,87335	*7,90*	*10,36*	*5,07*	*16*
	0,848132	*6,58*	*8,54*	*4,60*	*14*
	0,82428	*5,69*	*7,43*	*3,66*	*11*

Table 1a: *Indicators per the EU state in 5 electoral years.*

State	F_2	N_2	N_1	NP	n
Estonia	*0,793738*	*4,85*	*7,64*	*2,67*	*11*
	0,792391	*4,82*	*5,90*	*4,24*	*12*
	0,81064	*5,28*	*5,78*	*4,63*	*9*
	0,822843	*5,65*	*6,57*	*4,45*	*10*
	0,840226	*6,26*	*7,05*	*5,45*	*10*
Cyprus	*0,789211*	*4,74*	*5,59*	*3,95*	*10*
	0,724208	*3,62*	*4,48*	*2,95*	*9*
	0,75992	*4,17*	*5,59*	*2,69*	*11*
	0,800278	*5,01*	*6,35*	*3,90*	*14*
	0,828763	*5,84*	*7,11*	*4,75*	*14*
Luxembourg	*0,761747*	*4,20*	*4,92*	*2,77*	*7*
	0,79358	*4,84*	*5,62*	*3,54*	*8*
	0,81064	*4,73*	*6,14*	*2,56*	*9*
	0,822843	*6,25*	*7,13*	*5,45*	*10*
	0,835228	*6,07*	*7,34*	*5,14*	*13*
Malta	*0,598492*	*2,49*	*2,88*	*2,04*	*8*
	0,535367	*2,15*	*2,46*	*1,76*	*10*
	0,553163	*2,24*	*2,60*	*1,81*	*7*
	0,560188	*2,27*	*2,78*	*1,75*	*9*
	0,614569	*2,59*	*3,27*	*2,22*	*9*

Table 1b: *Indicators per member states of the EU in 5 electoral years.*

In Table 1a, 1b extreme values of corresponding indicators can be noticed and they are given in Table 2.

$\min N_2 = 2,15$	*Malta,*	*2009*	$\max N_2 = 10,85$	*Belgium,*	*2019*
$\min F_2 = 0,535367$			$\max F_2 = 0,907875$		
$\min N_1 = 2,46$	*Malta,*	*2009*	$\max N_1 = 13,69$	*Slovakia,*	*2014*
$\min NP = 1,40$	*Hungary,*	*2009*	$\max NP = 9,49$	*Belgium,*	*2019*
($\min n = 6$	*Austria,*	*2004)*	*(*$\max n = 41$	*Germany,*	*2019)*

Table 2: *Extreme values of indicators from Table 1.*

In order to summarize multitude data in Table 1, let us observe the average values of indicators per states in EU that are given in Table 3. Average values of the indicator N_2 of each EU member state are illustrated in Figure 1.

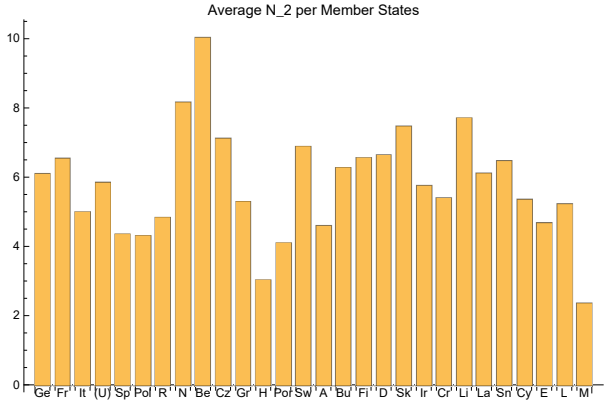

Figure 1: *Average values of the indicator N_2 of each EU member state.*

In order to summarize a lot of data in Figure 1, the average values of indicator N_2 per states in EU are rounded and put together by integer values in the groups: 2, 3, ...,8, ≥ 9. *These*

Indicator	$\bar{F}_2$	$\bar{N}_2$	$\bar{N}_1$	$\bar{NP}$	$\bar{n}$
Germany	0, 830886	6, 11	8, 86	4, 21	31, 4
France	0, 847136	6, 55	8, 69	4, 37	21, 2
Italy	0, 796288	5, 01	7, 08	3, 19	17, 2
(UK)	0, 827876	5, 85	8, 42	4, 23	29
Spain	0, 743599	4, 35	6, 30	3, 09	34, 2
Poland	0, 738672	4, 31	5, 55	3, 15	13
Romania	0, 787908	4, 85	6, 63	3, 17	13
Netherlands	0, 875631	8, 17	9, 74	6, 55	16, 4
Belgium	0, 893723	10, 02	11, 46	8, 62	20, 6
Czechia	0, 853911	7, 12	10, 03	5, 23	34, 2
Greece	0, 79185	5, 28	8, 47	3, 38	31, 8
Hungary	0, 666965	3, 03	4, 19	1.7	8, 8
Portugal	0, 749353	4, 11	5, 61	2, 92	15, 2
Sweden	0, 854787	6, 9	8, 17	5, 09	17, 2
Austria	0, 780308	4, 60	5, 23	3, 78	7, 4
Bulgaria	0, 838375	6, 28	8, 51	4, 40	22, 5
Finland	0, 845678	6, 57	7, 84	5, 27	15
Denmark	0, 84546	6, 65	7, 90	5, 18	9, 4
Slovakia	0, 860591	7, 47	10, 66	5, 27	23, 2
Ireland	0, 821472	5, 76	7, 10	4, 53	11
Croatia	0, 79494	5, 41	8, 59	3, 61	27, 75
Lithuania	0, 867599	7, 72	9, 63	5, 6	13, 6
Latvia	0, 826798	6, 12	8, 34	3, 97	15, 8
Slovenia	0, 843189	6, 46	8, 19	4, 56	13, 2
Estonia	0, 811968	5, 37	6, 59	4, 29	10, 4
Cyprus	0, 780476	4, 68	5, 82	3, 65	11, 6
Luxembourg	0, 804808	5, 22	6, 23	3, 89	9, 4
Malta	0, 572356	2, 35	2, 80	1, 92	8,6

Table 3: *Average values of indicators per the EU member state.*

groups are the following: $2 \equiv \{MAL\}$, $3 \equiv \{HUN\}$,
$4 \equiv \{SPA,\ POL,\ POR\}$, $5 \equiv \{ITA,\ ROM,\ GRE,\ AUS,\ CRO,\ EST,\ CYP,\ LUX\ \}$,
$6 \equiv \{GER,\ (UK),\ BUL,\ IRE,\ LAT,\ SLOVEN\}$,
$7 \equiv \{FRA,\ CZE,\ SWE,\ FIN,\ DEN,\ SLOVAK\}$, $8 \equiv \{NET,\ LYT\}$, $9 \equiv \{BEL\}$,
and they are illustrated in Figure 2.

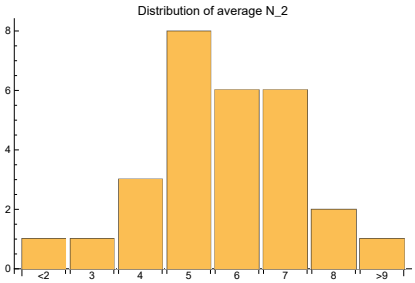

Figure 2: *Average values of the indicator N_2 of each EU member state, rounded and grouped at integer values: 2,3, . . . ,8, ≥ 9.*

With respect to the indicators of the effective number of parties, each EU member state has its specific values of indicators, that also depend on the year in which the elections are held.

2.1. Modified indicators of the number of parties

In order to reduce a sensitivity of indicators N_2, N_1, NP to the formal count n of parties, these indicators are modified by eliminating "tiny" parties. Therefore, parties that are above the threshold of $0,5\%$ (i.e., the set $I' = \{i : \omega_i > 0,005\}$, where $|I'| = n_M$) are taken into account. Then the corresponding modified vote shares are determined by the expression:

$$\omega'_i = \frac{v_i}{\sum_{i\in I'} v_i}\,, \qquad i \in I'\,. \tag{7}$$

Thus, the corresponding modified indicators are obtained as follows. The modified Laakso-Taagepera index has the form (from (2)):

$$MN_2 = \frac{1}{\sum\limits_{i\in I'} \omega_i'^2}\,. \tag{8}$$

In connection with the indicator MN_2, the modified fractionalization can be obtained from (4):

$$MF_2 = 1 - \sum_{i\in I'} (\omega'_i)^2 = 1 - \frac{1}{MN_2}\,. \tag{9}$$

In addition, the modified Wildgen index is defined by the following formula (from (5)):

$$MN_1 = e^{-\sum\limits_{i\in I'} \omega'_i \cdot \ln\omega'_i} \tag{10}$$

The modified Molinar index has the following form (from (6)):

$$MNP = 1 + \frac{\sum_{i\in I'}(\omega'_i)^2 - (\omega'_{[1]})^2}{(\sum_{i\in I'} \omega_i'^2)^2} = 1 + MN_2 - MN_2^2 \cdot (\omega'_{[1]})^2, \tag{11}$$

where $\omega'_{[1]}$ is the largest vote share.

Example 2. *In this example modified indicators MN_2, MN_1 and MNP have been calculated in the cases of elections for the European Parliament in 2004, 2009, 2014, 2019 and 2024 in member states of the European Union (EU), as in Example 1.*

Some results about the indicators and the modified indicators of the effective numbers of parties are given for certain cases of EU member states, in Table 4.

Portugal (2024)	$F_2 = 0,767988$ $MF_2 = 0,762028$	$N_2 = 4,31$ $MN_2 = 4,20$	$N_1 = 5,87$ $MN_1 = 5,46$	$NP = 3,32$ $MNP = 3,26$	n=17 $n_M = 9$
Spain (2024)	$F_2 = 0,770479$ $MF_2 = 0,760097$	$N_2 = 4,35$ $MN_2 = 4,17$	$N_1 = 6,56$ $MN_1 = 5,77$	$NP = 3,10$ $MNP = 3,01$	n=34 $n_M = 11$
Croatia (2024)	$F_2 = 0,791779$ $MF_2 = 0,782055$	$N_2 = 4,80$ $MN_2 = 4,59$	$N_1 = 7,54$ $MN_1 = 6,70$	$NP = 2,96$ $MNP = 2,87$	n=25 $n_M = 14$
Belgium (2024)	$F_2 = 0,903734$ $MF_2 = 0,902692$	$N_2 = 10,39$ $MN_2 = 10,28$	$N_1 = 11,67$ $MN_1 = 11,34$	$NP = 9,03$ $MNP = 8,94$	n=18 $n_M = 13$
Germany (2024)	$F_2 = 0,871574$ $MF_2 = 0,861605$	$N_2 = 7,79$ $MN_2 = 7,23$	$N_1 = 11,06$ $MN_1 = 9,28$	$NP = 5,38$ $MNP = 5,06$	n=35 $n_M = 15$

Table 4: *Comparison of indicators and modified indicators.*

Table 4 shows that the modified indicators have smaller values than the original indicators. The modified indicator MN_1 has got the largest decrease in comparison with the modified indicators MN_2 and MNP.

3. Indicator of the expectation of government stability

After transformation of votes into seats, a government can be formed if it has support of more than 50% of the representatives. The winning coalition is a coalition that has more than 50% of the seats in the parliament. Therefore, the winning coalitions (i.e., a parliamentary majority) are neccessary for government formation in most cases. Greater fractionalization (i.e., greater the effective number of parties) leads to more difficult formation of a parliamentary majority. Political differences between parties makes some winning coalitions almost impossible. There are different approaches within public choice theory regarding the minimal winning coalition [23] and the minimal-connected-winning coalition [2]. For example, the subset of minimal winning coalitions, denoted by C_{dM}, that have political distance which is lower than a given threshold d can be taken into account [4].

With respect to this, one can consider government stability, i.e., how stable a government is or how long it will last. In analysis of government stability one can use the historical approach, or approach based on measuring the expectation for a stable government.

The indicator called the expectation of government stability is defined as follows ([4]):

$$ES = \frac{1}{|C_{dM}|} \sum_{C \in C_{dM}} \frac{w(C)\sigma(C)}{|C|}, \tag{12}$$

where:

C_{dM} is the subset of *minimal winning coalitions* whose *political distance* is lower than a given threshold d,

$\sigma(C)$ is the share of total seats that a minimal winning coalition $C \in C_{dM}$ holds,

$w(C)$ is a weight associated with the coalition $C \in C_{dM}$.

One can see by (12) that the indicator ES has values in $[0, 1]$ and increases when the number of seats $s(C)$ of minimal winning coalition C increases, and when there exist 'large parties' that are frequently contained in the set of C_{dM}. On the other side, indicator ES decreases when the number of coalitions $|C_{dM}|$ increases, and when the number of parties $|C|$ that form a minimal winning coalition C increases.

This indicator is presented by a few cases of elections in certain countries as follows.

Example 3. *In this example indicator ES is calculated in several empirical cases of parliaments after elections in certain countries in the EU. In addition, the indicator ES in the cases of the EU Parliament after elections in 2014 and 2024 is calculated.*

Some obtained results about the values of indicator ES are given in Table 5a, 5b and 5c.

From Table 5a, 5b, 5c, it can be seen that empirical values of this indicator are in accordance with its theoretical properties.

Remark 1. *Notice that if there is only one minimal winning coalition with given threshold, i.e., if* $|C_{dM}| = 1$, *then* $ES = \frac{\sigma(C)}{|C|}$.

3.1. Modified indicator of the expectation of government stability

The indicator ES (12) takes into account the set C_{dM} of minimal winning coalitions with distance threshold d. However, a formally measured political distance between the parties in a minimal winning coalition does not have to significantly affect government stability.

Thus, instead of the set C_{dM}, it can be reasonable to take into account the set of politically feasible winning coalitions, which depends on practical political and social relationships between parties [15]. The set of politically feasible winning coalitions could be determined by political experts (let us denote it by C_P).

Country, Year	Total seats, Quota	C_{dM}	ES
Croatia, 2015	$S = 151$, $q = 76$	$\{\{59, 15, 8\}, \{56, 15, 8\}\}$	$0,089$
Croatia, 2020	$S = 151$, $q = 76$	$\{\{61, 3, 12\}, \{12, 61, 3\}\}$	$0,084$
Croatia, 2024	$S = 151$, $q = 76$	$\{\{61, 12, 4\}, \{10, 42, 5, 11, 4, 4\}\}$	$0,058$
Spain, 2016	$S = 350$, $q = 176$	$\{\{137, 32, 17\}, \{85, 71, 17, 7\}\}$	$0,075$
Slovenia, 2022	$S = 90$, $q = 46$	$\{\{41, 7\}, \{41, 5\}\}$	$0,131$
Austria, 2019	$S = 183$, $q = 92$	$\{\{71, 26\}, \{71, 31\}\}$	$0,136$
Euro. Parl., 2014	$S = 751$, $q = 376$	$\{\{215, 74, 46, 39, 16\}, \{189, 70, 52, 50, 16\}\}$	$0,051$
Euro. Parl., 2024	$S = 720$, $q = 361$	$\{\{188, 77, 136\}, \{188, 78, 84, 25\}\}$	$0,077$

Table 5a: *Values of ES when $|C_{dM}| = 2$.*

Country, Year	Total seats, Quota	C_{dM}	ES
Croatia, 2020	$S = 151$, $q = 76$	$\{\{10, 2, 61, 3\}, \{2, 61, 3, 10\}, \{61, 3, 10, 2\}\}$	$0,042$
Croatia, 2024	$S = 151$, $q = 76$	$\{\{61, 12, 4\}, \{61, 5, 4, 4, 2\}, \{10, 42, 5, 9, 2, 4, 4\}\}$	$0,034$
Slovenia, 2018	$S = 90$, $q = 46$	$\{\{25, 10, 7, 4\}, \{25, 13, 10\}, \{13, 10, 10, 9, 4\}, \{13, 10, 10, 9, 5\}, \{13, 10, 10, 7, 5, 4\}\}$	$0,023$
Austria, 2013	$S = 183$, $q = 92$	$\{\{52, 47\}, \{47, 40, 11\}, \{47, 40, 9\}\}$	$0,067$

Table 5b: *Values of ES when $|C_{dM}| > 2$.*

Country, Year	Total seats, Quota	C_{dM}	ES
Hungary, 2018=2014	$S = 199$, $q = 100$	$\{\{133\}\}$	$0,668$
Austria, 2017	$S = 183$, $q = 92$	$\{\{62, 51\}\}$	$0,309$
Croatia, 2020	$S = 151$, $q = 76$	$\{\{12, 61, 3\}\}$	$0,168$
Croatia, 2024	$S = 151$, $q = 76$	$\{\{61, 12, 4\}\}$	$0,170$

Table 5c: *Values of ES when $|C_{dM}| = 1$.*

In addition, a feasible winning coalition does not have to be a minimal winning coalition. In that case, one can look at the number of parties that are critical in a feasible winning coalition C, denoted by $|C_{crit}|$. (By definition, the party i is critical in the coalition C, if when it exits the coalition C, the coalition $C \setminus \{i\}$ becomes the non-winning.) Here, based on the formula (12), the modified expectation of government stability can be proposed by the following formula:

$$ES_M = \frac{1}{|C_P|} \sum_{C \in C_P} \frac{w(C)\sigma(C)}{|C_{crit}|}, \tag{13}$$

where:

C_P is the set of *feasible winning coalitions*,

$\sigma(C)$ is the share of total seats that a coalition $C \in C_P$ holds,

$w(C)$ is a weight associated with a coalition $C \in C_P$,

$|C_{crit}|$ is the number of parties that are critical in the coalition $C \in C_P$.

Notice that if a feasible winning coalition C is minimal, then it holds $|C_{crit}| = |C|$.
The modified indicator (13) is illustrated by a few examples.

Example 4. *a) The data from results of elections for the Croatian Parliament in 2015 are used. The electoral system in Croatia is considered in [14]. There are $S = 151$ members of Croatian Parliament, so quota $q = 76$ represents the majority votes. Let us assume that there are two feasible winning coalitions, i.e., $C_P = \{\{59, 15, 8, 2, 2, 1\}, \{56, 15, 8, 3, 3, 1, 1, 1\}\}$.*

The calculated value of the modified indicator - the modified expectation of government stability is given in Table 6. In comparison with the corresponding indicator $ES = 0,089$ from Table 5a, one gets $ES_M = 0,1450$.

b) The data from results of elections for the Croatian Parliament in 2024 are used. Assume that there are two feasible winning coalitions, i.e.,
$C_P = \{\{61, 12, 4, 1\}, \{10, 42, 5, 10, 4, 12\}\}$.

The calculated value of the modified indicator - the modified expectation of government stability is given in Table 6. In comparison with the corresponding indicator $ES = 0,058$ *from Table 5a, one gets* $ES_M = 0,076$.

Country, Year	Total seats, Quota	C_P	ES_M
Croatia, 2015	$S = 151$, $q = 76$	$\{\{59, 15, 8, 2, 2, 1\}, \{56, 15, 8, 3, 3, 1, 1, 1\}\}$	0,1450
Croatia, 2024	$S = 151$, $q = 76$	$\{\{61, 12, 4, 1\}, \{10, 42, 5, 10, 4, 12\}\}$	0,076
Spain, 2016	$S = 351$, $q = 176$	$\{\{137, 32, 17, 7, 1\}, \{85, 71, 17, 7, 1\}\}$	0,1049

Table 6: *Values of the modified indicator* ES_M.

c) The Congress of Deputies in Spain after elections in June 2016 is observed. It consists of 350 members. So quota $q = 176$ *represents the majority votes. Let us suppose hypothetical situation, that there are two feasible winning coalitions, i.e.,* $C_P = \{\{137, 32, 17, 7, 1\}, \{85, 71, 17, 7, 1\}\}$.

The corresponding value of the modified indicator - the modified expectation of government stability is given in Table 6. In comparison with the corresponding indicator $ES = 0,075$ *from Table 5a, one gets* $ES_M = 0,1049$.

Remark 2. *In the special case when the government has been formed, where the government coalition* C_G *is winning, but not minimal winning coalition, then* $C_P = \{C_G\}$ *and from* (13) *it follows the formula:*

$$ES_M = \frac{\sigma(C_G)}{|C_{Gcrit}|},$$

where $|C_{Gcrit}|$ *is number of parties in the government coalition* C_G *that are critical.*

Example 5. *a) Given the data for Croatia in 2011:* $S = 151$, $q = 76$, $C_G = \{60, 14, 4, 2\}$, *for the modified expectation of government stability one obtains* $ES_M = \frac{0,530}{2} = 0,265$. *Notice that if the set of minimal winning coalitions is* $C_{dM} = \{\{60, 14, 4\}\}$, *then for the expectation of government stability one obtains* $ES = \frac{0,517}{3} = 0,172$.

b) Given the data for Croatia in 2024: $S = 151$, $q = 76$, $C_G = \{61, 12, 4, 1\}$, *one obtains* $ES_M = \frac{78/151}{3} = 0,172$. *Notice that if the set of minimal winning coalitions is* $C_{dM} = \{\{61, 12, 4\}\}$, *then one obtains* $ES = \frac{77/151}{3} = 0,170$.

4. Conclusion

Electoral systems can be analyzed by means of their many features. With respect to the effective number of parties in a political system, in this paper some known quantitative indicators are considered: the Laakso-Taagepera index (N_2), fractionalization of vote shares (F_2), the Wildgen index (N_1), the Molinar index (NP). These indicators of the effective number of parties are examined from the empirical point of view in the cases of elections for the European Parliament in the EU member states. Each EU member state has its specific values of indicators.

Furthermore, a modification of these indicators is suggested, in order to decrease impact of tiny parties to the values of indicators. This is illustrated by some results about the indicators and the modified indicators of the effective numbers of parties for certain cases of the EU member states. The examples illustrate that the modified indicators have smaller values than the (original) indicators.

With regard to the government stability, in this paper the indicator of the expectation of government stability (ES) is observed. It takes into account the minimal winning coalitions

with distance threshold. Based on several examples of elections in different countries, one can see that empirical values of this indicator are in accordance with its theoretical properties. The question about government stability is too complex. A comparison of mutual values of the expectation of government stability in different cases can show in which cases the expectation of government stability is larger.

In order to extend the definition of this indicator, from the minimal winning coalitions to the politically feasible winning coalitions, a modification of the expectation of government stability is suggested. The modified expectation of government stability is illustrated by some empirical examples of elections. The results in these empirical cases confirm that the modified expectation of government stability (ES_M) is larger than the indicator ES.

With respect to future research, the effective number of parties can be studied as a parameter that changes in successive election cycles. In addition, the number of parliamentary parties can be considered as an indicator that refers to the phase after transformation of votes into seats. With regard to the (modified) expectation of government stability, the variability of this indicator can be studied in empirical cases of successive elections.

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