Group efficiency analysis in decision processes: a data envelopment analysis approach

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Abstract. Data envelopment analysis (DEA) is a powerful mathematical programming methodology for evaluating the relative efficiency of decision-making units (DMUs) with multiple outputs and multiple inputs. In the classic DEA, it has been implicitly assumed that all DMUs perform in a unique technology set and the traditional DEA cannot measure the relative performances of DMUs with dissimilar classes. In other words, if we have different groups of DMUs, the traditional DEA models cannot be applied to evaluate such cases. In this paper, it has been assumed that the DMUs do business in different groups. We are interested to evaluate the members of the groups. The main aim of this paper is proposing a DEA-based methodology to estimate the technical efficiency of DMUs along with different groups with different technologies. The proposed method is illustrated by an empirical example on banking industry.

Keywords: banking industry, data envelopment analysis, group efficiency, input, output

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1. Introduction

Discussion on how to evaluate the relative performances of the firms is very important and many researchers in the field of performance management and measurement have made substantial contribution in the last four decades. Following the pioneering work of [7], a body of literature has been developed to analyze the relative performances of homogeneous firms.

One of the most frequently used and representative methods of performance analysis is data envelopment analysis (DEA). In the last three decades, DEA, initiated by [5] and developed by [2] has been proven as an excellent data-oriented efficiency analysis method for comparing decision making units (DMUs) with multiple outputs and multiple inputs. DEA is a commonly used tool in performance evaluation of firms in the sense that DEA models do not estimate the exact production frontier and using the observed inputs/outputs data, an empirical production frontier is constructed to analysis of the firm's performances. Recently, many applications of DEA have been reported in the literature. For more information about the applications of DEA, see [6].

In traditional DEA, we have a set of DMUs that do business in the same technological environment and the DMUs use same inputs to produce same outputs. In real application in

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performance analysis of the firms, despite of the fact that the DMUs are homogeneous, they may perform in different groups with different technologies. So, there is a need to provide modified models and methods to evaluate the DMUs in such a cases. One way to evaluate the relative performances of the DMUs in different groups with same technology is classifying DMUs into different categories. In DEA, categorical DMUs are classified into controllable and non-controllable by decision maker. Then, DEA tries to solve this problem by a special algorithm. As mentioned earlier, the traditional DEA models such as "DEA with categorical DMUs" implicitly consider a same technology for all DMUs and in a special case, they didn't allow DMUs with different categories.

Discussion on the performance analysis with technological differences is an important subject that has been studied by DEA researchers. In all studies, making an envelope to the DMUs in the groups is proposed as a solution to this problem. In the DEA literature, such envelopment is referred to as meta-frontier.

The meta-frontier is defined as a common boundary that envelopes every group of DMUs. The idea of meta-frontier was first introduced by [9, 10, 15] have used the meta-frontier concept in their studies. O'Donnell [11] proposed a meta-frontier framework to study the firm level efficiencies. The meta-frontier constructed in all of the existing approaches is not a convex function. This paper has a different look at the group performance analysis. The method proposed in this paper is different from those that have been studied in previous works. As we know, empirical production frontier is our estimation related to the transformation of inputs to outputs and DEA overestimates the efficiencies of the DMUs, so, we need to evaluate the DMUs in a more solid condition. We assume that we have case in which there are K different groups and in each group a set of DMUs are doing business separately. The groups that the DMUs belong to are almost same and in this sense, some DMUs may change their group to tryout their performances. So, the DMUs can be transferred from one group to another one. To evaluate a specific DMU, a mixed integer linear programming problem is proposed and using the proposed model, we implicitly evaluate the DMU under consideration in each of the groups and the most favorable group is chosen to each DMU as the reference group.

The rest of this paper is organized as follows: In the next section (Section 2), performance analysis in the groups is given. Analyzing the problem of transferring a DMU from its current group to a specific group is given in Section 3. An application to a major Iranian bank is given in Section 4. Conclusions appear in Section 5.

2. Performance measurement in groups

2.1. Meta-technology

Suppose there are n DMUs to be evaluated in K different groups and the k-th group G_k consists of $DMU_{j1}, \ DMU_{j2}, ..., DMU_{jk}$. Each DMU_{jk} uses m_k inputs $(x_{1jk}, \ x_{2jk}, ..., x_{mjk}) \geq 0$ to produce s outputs $(y_{1jk}, \ y_{2jk}, ..., y_{sjk}) \geq 0$. An important point to be noted is that although each DMU_j faces to a different technology, but the groups are homogeneous in the sense that the DMUs in two different groups uses same inputs to generate same outputs and due to this similarity, in case of need, in order to improve their performances, they are allowed to be transferred from one group to another one. Battese et al. [3] argued that since a technology set is our knowledge related to the transformation of inputs to outputs, it is possible to conceptualize the existence of a meta-technology. O'Donnell et al. [11] have used the concept of meta-frontier to compute the technical efficiency of the DMUs that may be classified in to different groups with same technology.

Suppose $T_{G1}, \ T_{G2}, ..., T_{Gk}$ are K different technologies with K different natures. The meta-technology set T_M in our analysis is defined as follows:

$$T_M = \left\{ (x,y) : \exists k \in \{1,\ 2,...,K\} : (x,y) \in T_{Gk} \right\} \tag{1}$$

Indeed, T_M consists of all production possibilities (x,y) that belong to one of the technologies $T_{Gk},\ k = 1,\ 2,...,K$.

Consider the k-th technology set as follows:

$$T_{Gk} = \left\{ (x,y):\ x \geq \sum_{j \in j_k} \lambda_j^k x_j^k,\ y \leq \sum_{j \in j_k} \lambda_j^k y_j^k,\ \ \lambda^k = (\lambda_1^k,...,\lambda_{jk}^k) \in \Lambda_k \right\} \tag{2}$$

in which Λ_k is used to clarify the nature of T_{Gk}. Based on the definition of T_M an algebraic representation of the production set T_M is given as follows:

$$T_M = \left\{ (x,y): x \geq \sum_{k=1}^{K} \mu^k \Big(\sum_{j \in j_k} \lambda_j^k x_j^k \Big),\ y \leq \sum_{k=1}^{K} \mu^k \Big(\sum_{j \in j_k} \lambda_j^k y_j^k \Big),\ \sum_{k=1}^{K} \mu^k = 1, \right.$$
$$\left. \lambda^k = (\lambda_1^k,...,\lambda_{jk}^k) \in \Lambda_k,\ \mu^k \in \{0,1\} \right\} \tag{3}$$

As we stated before, the meta-technology T_M is the union of the technology sets, $T_{Gk},\ k = 1,\ 2,...,K$. The boundary points of the metatechnology T_M is the metafrontier and this frontier plays the main role in our analysis. A DMU under evaluation would be assessed to the part of this frontier to maximize its performance.

2.2. Technical efficiency measurement

Now, we come to evaluate the relative efficiency of a specific DMU_o To describe the DEA efficiency measurement of the DMU_o, we use the directional distance function as follows:

$$DF(x_o, y_o) = Max\left\{ 1\delta + 1\rho : \Big(x_o - \delta d^x,\ y_o + \rho d^y\Big) \in T_M,\ 0 \leq \delta \leq 1,\ \rho \geq 0 \right\} \tag{4}$$

in which $\delta = (\delta_1,\ \delta_2,...,\delta_m)$, $\rho = (\rho_1,\ \rho_2,...,\rho_s)$ and $1 = (1,\ 1,...,1)$ is used to summation. $d^x = (d_1^x,\ d_2^x,...,d_m^x)$ and $d^x = (d_1^y,\ d_2^y,...,d_s^y)$ are user-defined values that are used to show the directional distance. Based on the definition of the meta-technology T_M model in eq.(4) becomes as the following form:

$$e_o^* = Max \sum_{i=1}^{m} \delta_i^o + \sum_{r=1}^{s} \rho_r^0$$

$$\sum_{k=1}^{K} \mu^k \Big(\sum_{j \in j_k} \lambda_j^k x_{ij}^k \Big) \leq x - \delta_i^o d_{io}^x,\qquad i = 1,\ 2,...,m\ \ k = 1,\ 2,...,K$$

$$\sum_{k=1}^{K} \mu^k \Big(\sum_{j \in j_k} \lambda_j^k y_{rj}^k \Big) \geq y + \rho_r^o d_{ro}^y,\qquad r = 1,\ 2,...,s\ \ k = 1,\ 2,...,K \tag{5}$$

$$\sum_{k=1}^{K} \mu^k = 1,$$

$$\lambda^k = (\lambda_1^k,...,\lambda_{jk}^k) \in \Lambda_k,\qquad\qquad k = 1,\ 2,...,K$$

$$0 \leq \delta_i \geq,\qquad\qquad\qquad\qquad i = 1,\ 2,...,m$$

$$\rho_r \geq 0,\qquad\qquad\qquad\qquad r = 1,\ 2,...,s$$

In given model, each DMU_o is evaluated with respect to each of the groups and its efficiency score is obtained. When we maximize $\delta_i^o + \rho_r^o$, we try to evaluate the DMU_o to those frontier that DMU_o has maximum distance to the frontier. This shows the inefficiency of DMU_o. The binary variable μ_k is used to show if G_k is considered as the reference group to DMU_o or not. $\mu_k = 1$ shows that the k-th group is the reference group to DMU_o and hence the reference set of DMU_o is selected among DMUs in G_k. It should be noted that if E_o be the reference set of DMU_o, all members of E_o belong to a same group G and this is impossible to E_o to have members from two different G_k.

Model given by eq.(5) is a mixed integer nonlinear programming problem. In order to linearize the above model, we use the following changes of variables:

$$\overline{\lambda}_j^k = \mu^k \lambda_j^k, \quad k = 1,\ 2, ..., K \quad j = 1,\ 2, ..., j_k \tag{6}$$

If $\mu_k = 0$ we must have $\overline{\lambda}_j^k = 0$ and when $\mu_k = 1$ we must have $\overline{\lambda}_j^k = \lambda_j^k$ for all $j = 1,\ 2, ..., j_k$. Supose M be a large positive number and consider the following constraints:

$$0 \leq \overline{\lambda}_{jk}^k \leq \mu^k M, \quad k = 1,\ 2, ..., K \quad \forall j \in j_k \tag{7}$$

Clearly $\mu^k = 0$ forces $\overline{\lambda}_j^k = 0$ for all $j = 1,\ 2, ..., j_k$ and if $\mu_k = 1$ then $\overline{\lambda}_j^k \geq 0$ for all $j = 1,\ 2, ..., j_k$, and this is what we want. With these changes of variables, model in eq.(5) becomes of the following form:

$$
\begin{aligned}
&e_o^* = Max \sum_{i=1}^{m} \delta_i^o + \sum_{r=1}^{s} \rho_r^0 \\
&\sum_{k=1}^{K} \sum_{j \in j_k} \overline{\lambda}_{jk}^k x_{ij}^k \leq x - \delta_i^o d_{io}^x, \quad i = 1,\ 2, ..., m \ \ k = 1,\ 2, ..., K \\
&\sum_{k=1}^{K} \sum_{j \in j_k} \overline{\lambda}_{jk}^k y_{rj}^k \geq y + \rho_r^o d_{ro}^y, \quad r = 1,\ 2, ..., s \ \ k = 1,\ 2, ..., K \\
&\sum_{k=1}^{K} \mu^k = 1, \\
&0 \leq \overline{\lambda}_{jk}^k \geq \mu^k M, \quad k = 1,\ 2, ..., K \quad j \in j_k \\
&\overline{\lambda}^k = (\overline{\lambda}_1^k, ..., \overline{\lambda}_{jk}^k) \in \overline{\Lambda}_k, \quad k = 1,\ 2, ..., K \\
&\mu^k \in \{0,1\}, \quad k = 1,\ 2, ..., K \\
&0 \leq \delta_i \geq, \quad i = 1,\ 2, ..., m \\
&\rho_r \geq 0, \quad r = 1,\ 2, ..., s
\end{aligned} \tag{8}
$$

Note that the form of $\overline{\Lambda}_k$ depends of the form of Λ_k and μ^k. Model (8) is a mixed integer linear programming problem. It is easy to show the feasibility and the boundedness of this problem. In model (6), DMU_o is assessed to all of the groups and in the objective function, we look for the maximum reduction in the inputs and maximum expansion in the outputs, while remaining in some technology set T_{Gi}.

Although the individual technologies $T_{G1},\ T_{G2}, ..., T_{Gk}$ are convex sets, we should not expect the new meta-technology be a convex set. Despite of this, the DMU under evaluation, is assessed to the part of the meta-technology that is a convex function and the corresponded technology is a convex set. This guaranties that the local optimal solution obtained from model (8) is a global optimal solution. This is the proof of the following theorem.

Theorem 1. *Each local optimal solution to the mathematical programming problem (8) is a global optimal solution to this problem.*

Definition 1. *DMU_o is said to be full efficient if and only if the objective function value of model (8) is zero and all slack variables are zero.*

Theorem 2. *The new score e_o^* of DMU_o in the whole group is greater than or equal to its old score in its own group.*

Proof: Suppose DMU_o belongs to G_k. To calculate the efficiency score of DMU_o in its own group, using model (8), we set $\mu^k = 1$ and for each $j \neq k$, $\mu_j = 0$. With these values to μ^j, model (8) reduces to the following form:

$$
\begin{aligned}
&z_k = Max \sum_{i=1}^{m} \delta_i^o + \sum_{r=1}^{s} \rho_r^0 \\
&\sum_{j \in j_k} \overline{\lambda}_{jk}^k x_{ij}^k \leq x - \delta_i^o d_{io}^x, \qquad i = 1,\ 2, ..., m \ \ k = 1,\ 2, ..., K \\
&\sum_{j \in j_k} \overline{\lambda}_{jk}^k y_{rj}^k \geq y + \rho_r^o d_{ro}^y, \qquad r = 1,\ 2, ..., s \ \ k = 1,\ 2, ..., K \\
&0 \leq \overline{\lambda}_{jk}^k \geq \mu^k M, \qquad\qquad k = 1,\ 2, ..., K \quad j \in j_k \\
&\overline{\lambda}^k = (\overline{\lambda}_1^k, ..., \overline{\lambda}_{jk}^k) \in \overline{\Lambda}_k, \qquad k = 1,\ 2, ..., K \\
&\mu^k \in \{0, 1\}, \qquad\qquad\qquad k = 1,\ 2, ..., K \\
&0 \leq \delta_i \geq, \qquad\qquad\qquad i = 1,\ 2, ..., m \\
&\rho_r \geq 0, \qquad\qquad\qquad\quad r = 1,\ 2, ..., s
\end{aligned}
\tag{9}
$$

If $(\delta_i^{o*},\ \rho_r^{o*},\ \overline{\lambda}_k^*)$ is the optimal solution the model (9) with optimal value of z^*, then $(\delta_i^{o*},\ \rho_r^{o*},\ \overline{\lambda}_k^*,\ \mu^k = 1)$ is a feasible solution to model (8) and this completes the proof.

Theorem 2 shows that a DMU may be efficient in its own group, but, when we come to evaluate it with whole group, it may be inefficient.

3. The transfer between the groups

In this section, the effect of transferring a specific DMU from its current group to another one is considered. Suppose we are interested to transfer a specific $DMU_o \in G_A$ from its current group to another group G_B. Clearly, this transmission may affect on the performance of DMU_o and DMUs in G_B. The impact of the presence or absence of a DMU in a technology is shown in Figure 1. After including DMU_o to G_B, the performance of DMUs in G_B can easily be calculated by a classic DEA model. However, to determine the best group among G_A and G_B to DMU_o, the following model is proposed.
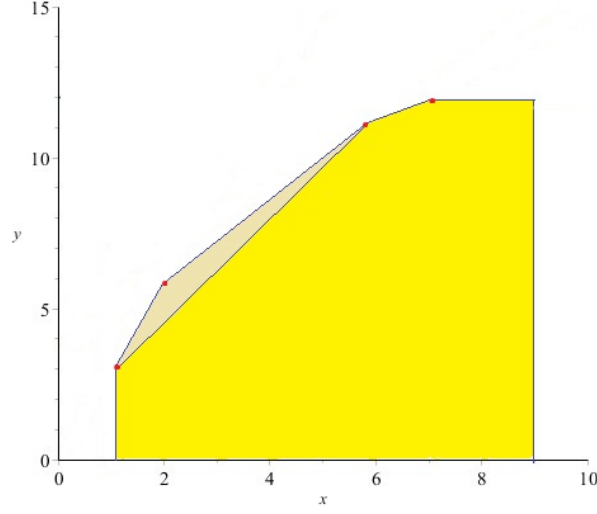
Figure 1: *The impact of the presence and absence of a DMU*

$$Max \sum_{i=1}^{m} \delta_i + \sum_{r=1}^{s} \rho_r$$

$$\sum_{k \in A, B} \sum_{j \in j_k} \lambda_{jk}^k x_{ij}^k + d_i^- = (1 - \delta_i) x_{io}^A, \qquad i = 1, \ 2, ..., m$$

$$\sum_{k \in A, B} \sum_{j \in j_k} \lambda_{jk}^k y_{rj}^k - d_r^+ = (1 + \rho_r) y_{ro}^A, \qquad r = 1, \ 2, ..., s$$

$$\sum_{j \in j_k}^{K} = \mu^k, \quad k \in \{A, \ B\} \tag{10}$$

$$\mu^A + \mu^B = 1$$

$$\forall j \in j_A, \ \ 0 \le \lambda_j^A \le \mu^A M$$

$$\forall j \in j_B, \ \ 0 \le \lambda_j^B \le \mu^B M$$

$$\mu^A, \ \mu^B \in \{0, \ 1\}$$

$$\delta_i, \ \rho_r^B \ge 0, \qquad i = 1, \ 2, ..., m \quad r = 1, \ 2, ..., s$$

Note that in model given by eq.(10), we assumed $(d^x, d^y) = (x_o, x_o)$. In this model we assumed that groups G_A and G_B has two different technologies. In this model, if $\mu^A = 1$ then $\mu^B = 0$ and G_A would be determined as the reference group to DMU_o.

At the end of this section, we illustrate the proposed approach with a small-scale example consisting eight DMUs in two groups. DMUs 1-4 belong to the first group and DMUs 5-8 are in second group. The DMUs use one input to produce one output. The data are listed in Table 1 and the two technology sets are depicted in Figure 2. The first three columns show the input and output data. The technical efficiency of the DMUs in their group is given in the fourth column of Table 1. As we can see, DMUs 1, 5 and 7 are efficient in their groups.

DMUs	input	output	DMUs TE in groups	meta frontier TE	μ^1	μ^2	group
DMU1	1	2	0.000	0.500	0	1	
DMU2	2	1	3.000	3.333	0	1	group 1
DMU3	2	3	0.333	0.500	0	1	
DMU4	4	3	1.667	1.667	1	0	
DMU5	1	3	0.000	0.000	0	1	
DMU6	2	4	0.125	0.125	0	1	group 2
DMU7	4	7	0.000	0.143	1	0	
DMU8	5	1	6.200	9.000	1	0	

Table 1: *Data and results for eight DMUs*
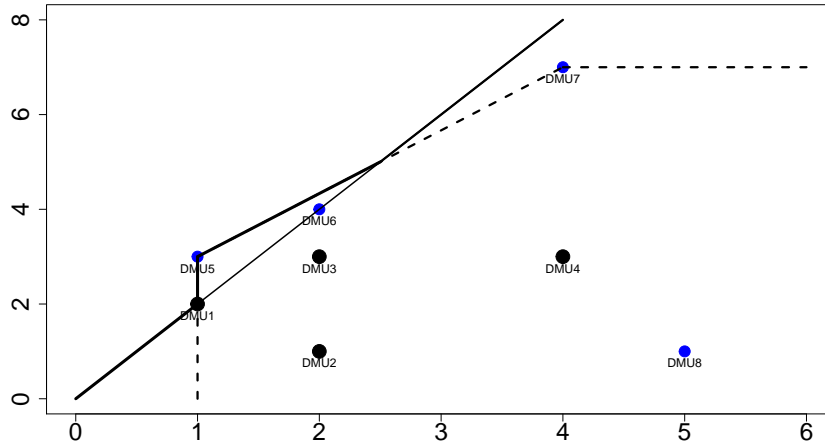
Figure 2: *The meta-frontier*

We applied our proposed model (8) to this data set. The results are given in columns 5, 6 and 7. As the results show, only one unit #5 prevail as efficient. It is interesting to note that DMUs #1 and #7 that were efficient in their group are now in-efficient. Moreover, the reference group to $DMU_1 \in G_1$ is G_2 and the reference group to $DMU_7 \in G_2$ is G_1. Finally, comparing the columns 4 and 5, we can see that the new inefficiency score of all DMUs are greater than or equal to the previous one.

4. An application to Iranian banking industry

In this section, we apply the model developed in this paper for the purpose of evaluating the technical efficiency in a major Iranian bank. In performance analysis in banking sector, DEA is one of the most frequently used tool to evaluate the relative efficiency of bank branches. In the last two decades, a lot of research studies have been conducted to evaluate the bank efficiency using DEA. In what follows, we briefly review some of these works.

Silva et al. [14] made a comparison of DEA and stochastic frontier approach (SFA) to investigate the efficiency of Chinese local commercial and rural banks. [13] made a survey to 80 published DEA applications in the bank branches of 24 countries. They studied the design of DEA models in these papers. [4] reviewed the recent developments related to empirical

methodological advances of bank performance analysis. They selected 103 researches and analyzed them based on their efficiency measures, input–output approaches and methodologies. Ahn and Le [1] investigated the application of DEA in measuring bank efficiency They discussed some pitfalls which have been occurred in choosing inputs, outputs, weights and other factors. Recently [12] make a survey to the application of DEA in the banking industry.

In what follows, we focus on the performance evaluation of 92 bank branches of an Iranian commercial bank. According to Iranian banking law, based on the operation volume, the branches are divided into six degrees 1–6. Since the branches in the third, fourth and fifth degrees are the most homogeneous branches, our performance evaluation is focused on these branches. We collect the data from operations in 2017. Seven variables have been used as inputs and outputs in this analysis.

Inputs include number of staff, non-operational costs and assets. Outputs are deposits, profit, charges and loans (all monetary variables are stated in 10 thousands of current Iranian Rials). The data set for these 92 branches are given in Table 2. Note that the first thirty branches belong to G_4 (branches with degree 4), the second 36 branches belong to G_3 and the last 26 branches belong to G_5. After calculating the Pearson correlation coefficients of each two variables, it has been indicated that we cannot ignore none of these variables.

We first evaluated the branches in their own groups and we saw that of 92 branches, 45 branches are efficient. Now, we make the conditions more rigorous and model 7 has been used to this data set. The relative efficiency of the branches along with the projection points and the reference groups are given in Table 3. As the second column of Table 3 shows, the number of efficient branches has reduced from 45 to 28. The membership values μ^1, μ^2 and μ^3 are given in columns 3-5, respectively.

Consider branch 1. This branch is inefficient in both cases and $\mu^1 = 1$ and $\mu^2 = \mu^3 = 0$ means that its frontier projection point belongs to the boundary of technology G_4. In other words, the most solid group to this branch is G_4.

Now consider branch 2. This branch was efficient in its original group, while, when it has been compared to G_5, it prevail as inefficient. This means that the most serious competitors to this branch belong to G_5. As we saw in this application, of 92 branches, 17 branches are efficient in their own group and when we extend their evaluation context, they would be determined as inefficient. This means that these 17 branches were overestimated in their own group. The data for this analysis are derived from operations during the first 6 months of 2018.

j	k	staff	costs	assets	deposits	profit	charge	loans	TE
1	4	17.67	32.15	3204.07	295207.1	60272.84	8091.01	114515.8	3.06
2	4	13.26	27.77	1693.66	273151.3	55083.47	1064.62	231965.2	0.00
3	4	11.96	29.26	2241.59	329613.6	75425.89	8929.29	142206.2	0.00
4	4	10.36	15.88	3380.67	257394.6	77502.37	8232.96	195372.2	0.00
5	4	11.31	12.14	3748.06	342367.3	56423.84	7791.50	217331.8	0.00
6	4	10.14	21.08	2738.06	330547.5	41158.48	8775.26	227504.5	0.00
7	4	12.28	21.38	3369.30	261761.8	74911.95	9393.61	299683.4	0.00
8	4	14.43	29.60	2725.98	297084.0	68539.69	7829.84	115560.6	2.58
9	4	12.84	31.31	2348.56	285941.6	53048.49	8823.33	279750.8	1.28
10	4	13.41	12.80	2689.27	297157.2	41075.47	8215.23	262026.7	0.00
11	4	17.81	15.77	2469.29	275714.9	77014.07	1026.15	134198.1	0.00
12	4	16.45	13.71	2642.67	320751.1	48507.34	8206.47	231765.0	0.00
13	4	17.93	14.54	3571.45	344216.7	74733.73	8204.02	242432.0	0.00
14	4	13.73	25.34	2547.62	325392.9	67022.31	8121.02	222130.3	0.00
15	4	12.40	31.41	1762.66	298896.8	67658.31	1057.76	124088.0	0.00
16	4	16.21	33.40	2322.38	299345.4	59837.03	9112.26	131226.4	2.26
17	4	12.87	29.05	2785.44	327187.4	58887.30	8804.32	151428.0	1.05
18	4	13.90	26.79	2590.02	265506.5	49964.90	7941.57	231802.6	1.84
19	4	10.33	34.67	3365.09	280549.0	53860.29	1043.09	167049.9	0.00
20	4	11.85	20.46	1889.08	330483.4	64912.87	7682.65	177879.6	0.00
21	4	17.41	26.84	2422.96	258813.7	57878.96	1075.00	233516.8	1.39
22	4	10.80	19.69	2213.23	287846.0	76741.84	1091.55	280376.4	0.00
23	4	12.05	24.36	2336.70	311613.8	76902.98	8860.29	304056.5	0.00
24	4	15.44	23.75	2042.31	323177.3	77361.98	9258.39	239353.9	0.00
25	4	16.05	24.64	2417.98	301945.6	59873.65	8063.77	195359.7	1.42
26	4	14.42	18.27	2201.93	312154.0	51484.73	1046.29	148193.5	0.00
27	4	10.48	23.00	2061.26	262576.7	52080.45	8165.35	125564.4	0.00
28	4	16.09	11.04	2432.86	306486.7	60224.01	7982.80	127022.2	0.00
29	4	17.89	27.82	3230.53	259216.6	61998.96	1063.56	175721.0	1.97
30	4	15.37	33.23	2444.65	343557.5	42072.82	9312.11	120647.0	0.00
31	3	10.81	16.38	1899.61	423833.4	43752.56	9734.46	149745.0	0.00
32	3	17.08	25.51	3084.53	352108.8	63259.99	8537.17	287840.2	2.49
33	3	10.12	29.22	1784.00	404359.6	69058.50	9187.99	306496.5	0.00
34	3	10.26	21.29	3043.68	377625.4	78507.03	9069.75	297462.4	0.00
35	3	11.76	22.08	2202.98	413313.7	71972.41	1025.22	129124.3	0.00
36	3	12.86	26.57	1939.69	396614.0	43814.81	7719.83	161312.9	2.10
37	3	12.43	30.42	3165.39	446841.3	40048.83	1047.89	129437.1	3.28
38	3	12.05	20.87	3575.59	387431.0	70326.85	8329.52	151071.4	1.91
39	3	17.61	34.65	2530.99	399086.0	71580.55	8498.93	203561.7	2.46
40	3	10.43	29.84	1693.94	444012.3	51197.85	8901.52	167444.4	0.00
41	3	16.53	22.97	3246.32	352017.3	79128.39	1088.40	106213.7	4.23
42	3	12.81	26.35	3648.67	427962.6	76403.70	8838.28	244440.3	1.24
43	3	11.59	12.78	3218.10	352969.5	78353.22	1093.12	156264.1	0.00
44	3	10.51	28.77	2405.27	381479.8	65797.91	1052.70	187689.4	0.00
45	3	12.99	18.66	1622.84	437978.8	50560.63	9498.72	195722.5	0.00
46	4	17.89	30.04	2512.88	381827.8	40636.01	9158.30	308273.3	2.45

Table 2a: *Data for 92 bank branches*

j	k	staff	costs	assets	deposits	profit	charge	loans	TE
47	3	10.04	28.56	3783.93	381479.8	56359.14	8998.29	136913.4	2.02
48	3	16.21	34.37	3383.34	375711.8	65141.15	8844.16	215016.9	3.01
49	3	16.47	10.45	2069.40	373706.8	60574.36	10704.12	247537.2	0.00
50	3	15.79	34.27	1724.34	356860.6	59265.72	9695.79	178305.0	1.11
51	3	10.68	28.26	2355.09	412627.0	54349.80	8802.39	230820.3	0.00
52	3	11.06	10.80	1814.25	351348.9	47168.19	1028.18	278862.4	0.00
53	3	15.01	21.21	2606.02	445376.4	52157.35	1045.48	179431.1	2.11
54	3	13.24	23.39	2608.13	377579.6	55129.86	9651.89	309931.2	1.11
55	3	14.44	23.29	3165.95	384336.4	57035.43	10129.89	106495.3	3.79
56	3	11.45	34.19	1639.54	404240.5	59484.24	10669.73	147011.0	0.00
57	3	14.23	10.37	3664.32	357867.7	41782.28	1087.91	251272.2	0.00
58	3	16.45	31.33	2090.32	435570.8	68117.31	10867.34	225877.9	1.06
59	3	12.10	34.49	3546.04	403608.8	77946.71	10058.96	264760.7	1.12
60	3	16.93	23.34	2231.13	442684.7	62267.53	9181.91	224657.9	1.40
61	3	10.92	28.21	3383.97	389564.2	68957.18	8987.72	154581.1	1.73
62	3	15.91	15.34	2524.32	424025.7	41972.72	10823.86	226753.7	1.80
63	3	17.23	30.34	2599.43	391798.2	52660.30	7936.34	130231.6	4.18
64	3	14.36	30.05	3186.94	362573.6	67826.78	9075.52	289316.7	1.75
65	3	11.17	26.69	3202.31	447216.7	46954.56	7879.30	260062.3	0.00
66	3	15.85	24.66	3272.08	362930.7	68493.30	7523.71	243652.0	2.55
67	5	13.06	23.41	2741.43	242251.4	55098.12	9991.24	277798.9	0.00
68	5	14.77	32.71	2092.07	187180.7	49996.64	9530.98	149857.6	2.23
69	5	17.19	28.80	2003.28	221175.3	53054.60	7927.37	141580.6	2.30
70	5	16.91	28.24	1969.87	193324.1	44301.89	9700.39	296580.2	1.03
71	5	11.11	29.85	2314.73	157269.5	61463.06	10473.83	131520.4	0.00
72	5	12.74	14.90	3107.27	150073.2	43285.01	10517.94	298019.2	0.00
73	5	12.97	19.40	2235.27	197141.9	58374.58	10681.59	108178.2	0.00
74	5	17.28	12.11	1927.68	235155.8	75436.87	9804.32	267357.1	0.00
75	5	13.73	19.44	2726.76	230358.3	44647.36	7932.07	153617.7	1.82
76	5	15.64	17.60	1668.88	208302.6	59763.79	9566.97	209523.9	0.00
77	5	17.78	22.42	1975.91	159558.4	43877.07	10445.10	157421.5	2.22
78	5	16.00	25.14	3540.29	206251.7	48806.42	8816.60	234348.9	2.27
79	5	10.59	19.05	3589.63	222200.7	72793.97	9856.01	258304.2	0.00
80	5	14.09	20.41	3381.16	166580.7	75825.07	8378.87	151503.0	1.92
81	5	11.39	12.35	2664.92	192756.4	72448.50	10627.54	152754.3	0.00
82	5	15.69	17.98	2350.17	181421.9	64041.26	9091.22	232866.1	0.00
83	5	17.76	13.14	1999.42	226284.1	67583.85	9552.13	190705.0	0.00
84	5	15.50	27.41	2996.86	162002.9	57709.28	10242.58	167982.1	2.47
85	5	16.37	34.30	3560.99	185331.3	53069.25	7933.45	267851.3	2.78
86	5	11.42	27.42	1891.96	243304.2	58447.83	8901.20	293020.4	0.00
87	5	17.40	34.94	1623.47	233425.4	54676.96	8801.54	260487.7	0.00
88	5	14.01	27.05	2287.70	195860.2	43471.79	10449.26	188271.3	1.71
89	5	15.40	23.60	1518.53	159375.3	47613.76	9938.15	216762.4	0.00
90	5	13.92	30.93	1907.54	227922.9	78413.04	7940.40	135899.8	0.00
91	5	10.30	22.58	2964.57	157956.2	46695.76	7917.11	249132.5	1.05
92	5	16.37	13.49	2193.43	239632.9	46816.61	8914.98	305852.1	0.00

Table 2b: *Data for 92 bank branches (continued)*

j	k	staff	costs	assets	deposits	profit	charge	loans	TE	μ^1	μ^2	μ^3
1	4	17.7	32.1	2994.4	625257.1	95321.9	16259.5	485136.0	6.01	1	0	0
2	4	13.3	27.8	1693.7	273151.3	55083.5	10646.2	231965.2	0.00	0	1	0
3	4	12.0	29.3	2131.5	455560.2	75425.9	10907.7	348715.8	2.11	1	0	0
4	4	10.4	15.9	3380.7	257394.6	77502.4	8233.0	195372.2	0.00	0	1	0
5	4	11.3	12.1	2212.1	360163.0	56423.8	10516.1	261370.6	1.01	1	0	0
6	4	10.1	21.1	2738.1	372588.1	73509.4	90328.0	292174.0	1.23	1	0	0
7	4	12.3	21.4	2727.6	432133.0	74912.0	11139.2	339090.5	1.16	1	0	0
8	4	14.4	29.6	2467.0	524995.4	82331.9	13241.4	405099.9	4.26	1	0	0
9	4	12.8	31.3	2228.0	488594.6	79927.5	11731.7	373782.6	1.93	1	0	0
10	4	13.1	12.8	2150.3	416428.1	55905.0	12186.2	330515.2	1.73	1	0	0
11	4	17.8	15.8	2469.3	275714.9	77014.1	10261.5	134198.1	0.00	0	1	0
12	4	14.0	13.7	2303.4	446071.4	59884.6	13053.7	354042.8	2.02	1	0	0
13	4	17.9	14.5	2754.5	483839.7	74733.7	14105.7	340724.7	1.76	1	0	0
14	4	13.7	25.3	2329.5	487465.0	74561.7	12632.2	377982.4	1.95	1	0	0
15	4	12.4	31.4	1762.7	298896.8	67658.3	10577.6	124088.0	0.00	0	1	0
16	4	13.4	33.4	2322.4	511714.9	84247.0	12190.6	390944.8	3.61	1	0	0
17	4	12.9	29.0	2216.9	479509.4	76965.1	11777.9	368275.1	2.75	1	0	0
18	4	13.9	26.8	2364.9	498249.2	76992.0	12771.9	385581.4	2.78	1	0	0
19	4	10.3	34.7	3365.1	280549.0	53860.3	10430.9	167049.9	0.00	0	1	0
20	4	11.8	20.5	1889.1	330483.4	64912.9	7682.7	177879.6	0.00	0	1	0
21	4	14.3	26.8	2423.0	508554.7	78141.2	13115.4	393989.8	2.40	1	0	0
22	4	10.8	19.7	2213.2	287846.0	76741.8	10915.5	280376.4	0.00	0	1	0
23	4	12.1	24.4	2336.7	311613.8	76903.0	8860.3	304056.5	0.00	0	1	0
24	4	15.4	23.7	2042.3	323177.3	77362.0	9258.4	239353.9	0.00	0	1	0
25	4	14.7	14.4	2418.0	468266.9	62864.3	13703.2	371659.1	2.70	1	0	0
26	4	13.4	13.1	2201.9	426426.0	57247.2	12478.8	338450.4	2.31	1	0	0
27	4	10.5	23.0	1800.9	387645.2	61799.6	9596.8	298132.2	2.34	1	0	0
28	4	15.9	11.0	2106.8	386239.2	60224.0	11116.2	267071.2	1.90	1	0	0
29	4	17.9	27.8	3001.7	612814.4	90175.0	16517.9	478658.4	4.17	1	0	0
30	4	14.1	33.2	2444.7	532545.9	86328.6	12928.2	408176.9	4.45	1	0	0
31	3	10.8	16.4	1899.6	423833.4	43752.6	9734.5	149745.0	0.00	1	0	0
32	3	17.1	16.7	2801.7	542584.2	72841.3	15878.0	430644.1	2.49	1	0	0
33	3	10.1	29.2	1784.0	404359.6	69058.5	9188.0	306496.5	0.00	1	0	0
34	3	10.3	21.3	3043.7	377625.4	78507.0	9069.8	297462.4	0.00	1	0	0
35	3	11.8	22.1	2203.0	413313.7	71972.4	10252.2	129124.3	0.00	1	0	0
36	3	11.2	26.6	1939.7	423193.7	68747.0	10247.6	324220.7	2.10	1	0	0
37	3	12.4	30.4	3003.9	475569.2	90433.7	11127.7	367855.0	3.28	1	0	0
38	3	12.1	20.9	2477.2	423030.3	70326.9	10984.3	331074.5	1.91	1	0	0
39	3	14.6	34.6	2531.0	552143.6	89681.7	13372.4	423025.6	2.46	1	0	0
40	3	10.4	29.8	1693.9	444012.3	51197.9	8901.5	167444.1	0.00	1	0	0
41	3	16.5	23.0	2756.2	554553.1	79662.6	15294.7	435047.0	4.23	1	0	0
42	3	12.8	26.4	2401.5	466981.3	76403.7	11700.7	361310.5	1.24	1	0	0
43	3	11.6	12.8	3218.1	352969.5	78353.2	10931.2	156264.1	0.00	1	0	0
44	3	10.5	28.8	2405.3	381479.8	65797.9	10527.0	187689.4	0.00	1	0	0
45	3	13.0	18.7	1622.8	437978.8	50560.6	9498.7	195722.5	0.00	1	0	0
46	3	14.7	30.0	2512.9	534406.9	83729.6	13492.8	412438.5	2.45	1	0	0

Table 3a: *Meta TE results and the projection points*

j	k	staff	costs	assets	deposits	profit	charge	loans	TE	μ^1	μ^2	μ^3
47	3	10.0	28.6	1834.9	399622.4	68979.9	9105.9	303491.1	2.02	1	0	0
48	3	16.2	34.4	2778.0	594449.5	93973.7	14858.9	457959.6	3.01	1	0	0
49	3	16.5	10.5	2069.4	373706.8	60574.4	10704.1	247537.2	0.00	1	0	0
50	3	10.5	24.7	1724.3	382041.5	59265.7	9695.8	253669.4	1.11	1	0	0
51	3	10.7	28.3	1866.9	415829.4	69447.2	9729.7	316725.1	0.00	1	0	0
52	3	11.1	10.8	1814.3	351348.9	47168.2	10281.8	278862.4	0.00	1	0	0
53	3	15.0	14.7	2463.0	476979.3	64033.9	13958.2	378574.1	2.11	1	0	0
54	3	13.2	12.9	2172.0	420623.6	56468.2	12309.0	333845.1	1.11	1	0	0
55	3	14.4	23.3	2428.4	498238.8	73905.0	13324.0	388589.1	3.79	1	0	0
56	3	11.4	34.2	1639.5	404240.5	59484.2	10669.7	147011.0	0.00	1	0	0
57	3	14.2	10.4	3664.3	357867.7	41782.3	10879.1	251272.2	0.00	1	0	0
58	3	12.3	23.7	2090.3	440563.9	68117.3	11286.2	340902.1	1.06	1	0	0
59	3	12.1	34.5	2130.6	481659.9	81989.4	10992.9	365353.3	1.12	1	0	0
60	3	13.5	16.8	2231.1	443142.9	62267.5	12471.0	349005.1	1.40	1	0	0
61	3	10.9	28.2	1904.2	422196.8	70087.5	9954.5	321988.5	1.73	1	0	0
62	3	15.4	15.0	2524.3	488861.0	65629.0	14305.9	388004.5	1.80	1	0	0
63	3	15.2	30.3	2599.4	550464.6	85708.9	13994.3	425355.8	4.18	1	0	0
64	3	14.4	30.1	2459.0	525004.7	82726.9	13171.1	404722.3	1.75	1	0	0
65	3	11.2	26.7	3202.3	447216.7	46954.6	7879.3	260062.3	0.00	1	0	0
66	3	15.9	16.1	2605.2	506499.6	68493.3	14733.2	401519.0	2.55	1	0	0
67	5	13.1	23.4	2210.6	460570.7	69978.6	12019.3	357587.3	1.86	0	1	0
68	5	11.9	32.7	2092.1	469266.8	79088.5	10851.8	356726.1	3.80	1	0	0
69	5	11.5	28.8	2003.3	441386.9	72664.5	10515.9	337218.8	3.40	0	1	0
70	5	13.1	28.2	1969.9	481641.3	70041.2	10720.9	296580.2	2.40	1	0	0
71	5	11.1	29.8	2154.5	422526.5	71405.4	10473.8	283336.1	3.07	0	1	0
72	5	12.7	14.9	2387.1	416182.5	61925.2	11747.9	329829.3	2.66	1	0	0
73	5	13.0	19.4	2172.0	441119.9	64362.9	11988.0	345085.9	3.68	1	0	0
74	5	17.3	12.1	1927.7	235155.8	75436.9	9804.3	267357.1	0.00	0	0	1
75	5	13.7	19.4	2291.1	462079.7	66646.3	12696.3	362239.9	3.62	1	0	0
76	5	15.6	17.6	1668.9	208302.6	59763.8	9567.0	209523.9	0.00	0	0	1
77	5	11.6	2.2	1975.9	416401.5	64370.4	10669.2	322216.4	3.49	1	0	0
78	5	16.0	25.1	3540.3	551439.4	94027.6	14544.3	434649.9	4.10	1	0	0
79	5	10.6	19.1	3589.6	222200.7	72794.0	9856.0	258304.2	0.00	0	1	0
80	5	14.1	20.4	2801.0	477359.5	75825.1	12907.0	376044.2	4.06	1	0	0
81	5	11.4	12.4	2664.9	192756.4	72448.5	10627.5	152754.3	0.00	0	0	1
82	5	15.4	17.1	2350.2	498588.5	64041.3	13425.0	342366.8	2.76	0	0	1
83	5	17.8	12.4	1981.2	241684.3	77531.2	10076.5	274779.6	0.00	0	0	1
84	5	15.5	27.4	2621.2	545008.5	82548.2	14269.3	423398.8	4.83	0	1	0
85	5	16.4	34.3	3561.0	600722.3	105918.2	14816.8	466736.0	4.85	1	0	0
86	5	10.9	27.4	1892.0	417551.3	68893.3	9920.7	318859.1	1.15	1	0	0
87	5	95.9	20.4	1623.5	349172.6	54677.0	8801.5	260487.7	1.36	1	0	0
88	5	13.4	27.0	2287.7	485561.8	75857.9	12298.7	374955.2	3.44	1	0	0
89	5	15.4	23.6	1518.5	159375.3	47613.8	9938.2	216762.4	0.00	0	0	1
90	5	13.9	30.9	1907.5	227922.9	78413.0	7940.4	135899.8	0.00	0	0	1
91	5	10.3	22.6	1771.0	381080.0	60724.5	9439.3	293110.8	2.48	1	0	0
92	5	13.4	13.1	2193.4	424781.2	57026.4	12430.7	337144.9	1.70	1	0	0

Table 3b: *Meta TE results and the projection points (continued)*

5. Conclusion

A main assumption in performance measurement using tools such as DEA is that all firms are in a single technology set. However, in many real applications, we confront cases in which different groups of firms perform in different but similar environments. The differences in the nature of the DMUs and the region that they do business have led to different technology sets and in this case, we confront groups of DMUs that technologically are different but, this difference is not a significant difference. So, a DMU can be transferred between the groups to select best reference group.

In this paper, we have developed a DEA based procedure and a directional distance function model to evaluate DMUs in different groups. The proposed model has been used to evaluate the relative efficiency of 92 bank branches of an Iranian major bank. As the results showed, we saw that when we extended the evaluation context, the efficiency classification of the DMUs may be altered. A DMU may be efficient in its current group and when we assess this unit in a larger group, it may be inefficient. This shows the impact and importance of group efficiency analysis.

References

[1] Ahn, H., and Le, M. H. (2014). An insight into the specification of the input–output set for DEA–based bank efficiency measurement. *Management Review Quarterly*, 64(1), 3–37. doi: 10.1007/s11301-013-0098-9

[2] Banker, R. D., Charnes, A. and Cooper, W. W. (1984). Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis. *Management Science*, 30(9), 1078–1092. doi: 10.1287/mnsc.30.9.1078

[3] Battese, G. E., Rao, D. S. P. and O'Donnell, C. J. (2004). A Metafrontier Production Function for Estimation of Technical Efficiencies and Technology Gaps for Firms Operating Under Different Technologies. *Journal of Productivity Analysis*, 21(1), 91–103. doi: 10.1023/b:prod.0000012454.06094.29

[4] Bhatia, V., Basu, S., Mitra, S. K. and Dash, P. (2018). A review of bank efficiency and productivity. *OPSEARCH*, 55(3–4), 557–600. doi: 10.1007/s12597-018-0332-2

[5] Charnes, A., Cooper, W. W. and Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2(6), 429–444. doi: 10.1016/0377-2217(78)90138

[6] Emrouznejad, A., Parker, B. R. and Tavares, G. (2008). Evaluation of research in efficiency and productivity: A survey and analysis of the first 30 years of scholarly literature in DEA. *Socio-Economic Planning Sciences*, 42(3), 151–157. doi: 10.1016/j.seps.2007.07.002

[7] Farrell, M. J. (1957). The Measurement of Productive Efficiency. *Journal of the Royal Statistical Society*, Series A (General), 120(3), 253–290. doi: 10.2307/2343100

[8] Hayami, Y. and Ruttan, V. W. (1971). *Agricultural development: An international perspective.* Baltimore: Johns Hopkins University Press.

[9] Lovell, K. C. A. and Pastor, J. T. (1997). Target setting: An application to a bank branch network. *European Journal of Operational Research*, 98(2), 290–299. doi: 10.1016/s0377-2217(96)00348-7

[10] McMillan, M. L. and Chan, W. H. (2006). University Efficiency: A Comparison and Consolidation of Results from Stochastic and Non–stochastic Methods. *Education Economics*, 14(1), 1–30. doi: 10.1080/09645290500481857

[11] O'Donnell, C. J., Rao, D. S. P. and Battese, G. E. (2008). Metafrontier frameworks for the study of firm–level efficiencies and technology ratios. *Empirical Economics*, 34(2), 231–255. doi: 10.1007/s00181-007-0119-4

[12] Paradi, J. C., Sherman, H. D. and Tam, F. K. (2018). *Data Envelopment Analysis in the Financial Services Industry.* International Book Series in Operations Research & Management Science: Springer. doi: 10.1007/978-3-319-69725-3

[13] Paradi, J. C. and Zhu, H. (2013). A survey on bank branch efficiency and performance research with data envelopment analysis. *Omega*, 41(1), 61–79. doi: 10.1016/j.omega.2011.08.010

[14] Silva, T. C., Tabak, B. M., Cajueiro, D. O. and Dias, M. V. B. (2017). A comparison of DEA and SFA using micro– and macro–level perspectives: Efficiency of Chinese local banks. *Physica A: Statistical Mechanics and its Applications*, 469, 216–223. doi: 10.1016/j.physa.2016.11.041

[15] Worthington, A. C. and Lee, B. L. (2008). Efficiency, technology and productivity change in Australian universities, 1998–2003. *Economics of Education Review*, 27(3), 285–298. doi: 10.1016/j.econedurev.2006.09.012