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Improving the productivity of the copper mining process in the Chilean copper industry

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Abstract. This paper presents a linear programming model used for decision making in the mining process of copper concentration from sulphide minerals. The developed model enables the decision maker to select the types of ore to be used in the mix to maximize the metallurgical recovery and the copper grade at the end of the process. The model is of the mixture model of minerals with added economic variables such as processing costs, electric power and others. The process has four sub-processes that are crushing the ore, crushing the crushed ore, flotation of the ground ore to obtain copper concentrate and drying, in which the water is extracted. The model uses a set of variables whose size varies according to the number of lots of minerals and the number of planning days considered. The model may be considered a considerable problem when a long period of time is planned, but has only been implemented with 3.000 variables and 2.000 constraints. The developed model is being implemented in the National mining company, which buys ore from small producers to produce copper concentrate and then melt and refine it to obtain high grade copper. The generated model produces savings of the order of thousand dollars per day, when compared to the current methods of allocating minerals, which represents millions of dollars per year. It also produces a benefit due to the fact that lower operating costs are obtained, with estimate savings of the order of 5% of the current cost.

Keywords: linear programming, mining, productivity, mix minerals

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1. Introduction

The aim of this paper is to use mathematical methods for the management of the processes of a mining company, improving the productivity by optimizing the production of copper concentrate. We present an application of the optimal mineral mixture and explain how the development of the model allowed those responsible for the process to realize that the way of managing the process was erroneous. This is because they sought to optimize only by managing the metallurgical recovery parameters and did not consider the option of mixing the minerals. In addition, they simultaneously sought to optimize two variables of final copper concentrate ore and metallurgical recovery. Several runs of models showed that optimize metallurgical recovery produces minimum final copper law and vice versa. In the second chapter, a brief review of the importance of mining in the country is presented, while in the third chapter, the type of company is explained, in which the application was made, which is a state company, with a notable role of promotion for the small mining businessman.

In chapter three, a bibliographic review is carried out searching the main applications of operations research of the crushing, grinding and flotation processes. Chapter four shows the

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model of mineral mixture initially used following the logic of the process managers and then another model that maximizes the copper law. Finally, the final model that was implemented in the company was developed, consisting of maximizing the product of the final copper law multiplied by metallurgical recovery, which gives the final tons of copper. Chapter five presents the final conclusions of the work, including the benefit of using this type of models, and the figure of annual savings that is of the order of one million dollars.

2. Importance of mining

2.1. Mining in Chile

Mining is one of the priority sectors in Chile representing 9% of the GDP in 2015 according to sources from the Central Bank of Chile, the 55% of total exports and 49% represents copper mining. Chile is known to be a mining country, because the institutional framework allows for mining investment due to macroeconomic and political stability, among other factors, and also because it has great geological potential as it is part of the main copper reserves in the world. In 2015, Chile represented 30% of world copper production, leading the list with the production of 5,764 metric tons. Currently copper leaves a series of positive milestones for the sector, since in 2017 it closed with a value of US $ 2.79 per pound, higher than forecast price (US $ 2.40 per pound), where the highest demand was from China, the main consumer of the "red mineral" in the world. The metal price is projected to be US $ 3.06 per pound for 2018 and US $ 3.11 per pound for 2019 thus it will continue to be the country's main commodity.

2.2. National mining company (ENAMI)

ENAMI is a Chilean State-run company founded in 1960, created with the purpose of promoting the development of the mining sector at small and medium scale, providing the services required to access the market of refined metals. Its objectives are the recognition of mining resources, technical and credit assistance, purchase, processing and marketing, in conditions of competitiveness. It is present in eight regions of the country, with five production plants. ENAMI, concentrates its task in the management of three areas that are production, commercialization and support to the mining sector with financing of tools that promote sustainable development of this sector. Regarding production, it starts with the benefit of minerals, which aims to add value to the production of small and medium-sized mining, with profit and smelting plants. On the other hand, commercialization enables closing the cycle of development and exploitation, which includes the purchase of minerals and mining products under market conditions and the sale of products in globalized markets. On the other hand, with regard to promotion, it includes the financing of tools that contribute to the sustainable development of the sector, supporting the implementation of viable projects.

2.2.1. Production processes of copper concentrate

The production process of copper concentrate is shown in the Figure 1.

3. Review of the literature

3.1. The use of OR in crushing processes

Due to the importance of crushing plants for mining companies, Svedensten and Magnus [17], presented a method for the modeling and optimization of crushing plants, where the modeling is represented with the different production units, rock materials and economy of the plant. A

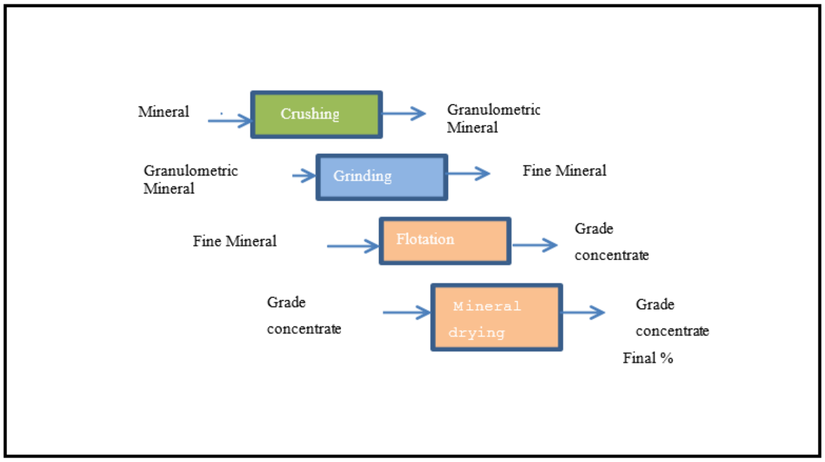

Figure 1: *The process of the production the copper concentrate*

model that considers customer demand is included, thus an optimization problem and a genetic evolutionary algorithm are formulated. In the crushing process Itävou et al. [9] present an effective way to produce models of dynamic processes based on established models of steady state in order to make a simulator that allows for developing control methods to make the most of the capacity of the crushers. The modeling for this control design is related to the dependence between the input and output of a crusher. On the other hand, Asbjörnsson et al. [1] created a wear function for an existing model that is related to the size of the particle, with the purpose of obtaining the transitory consequences of wear of a crusher. Here the modeling is carried out according to the function of Swebrec and the Correlation model is implemented in a simulation software with simulated events. According to Gang et al. [5] the geometry of the chamber of a crusher is a key factor for its performance, since its design integrates the quality of the product, as well as the crushing efficiency. Then, a population balance model is made combining the empirical model to predict the shape of the particle with the size distribution model, having a size reduction model and a scale prediction model as restrictions.

3.2. The use of OR in grinding processes

In the milling circuit, Mendez et al. [11] and [12] propose a linear model from a classification of the mill allowing for the rapid increase in the buoyancy index, where the particles are defined by size and chemical composition, introducing a parameter that represents a class of particles that delivers material to another one. Regarding the particle size distribution of industrial discharges from ball mills, Gharehgheshlagh et al. [6] propose a perfect mixing model for the investigation of their level of precision and thus simulate the particle size distribution.

3.3. The use of the OR in the flotation process

In the flotation process there is uncertainty in the design of the circuit, which is why Jamett et al., [10] analyze the effect of stochastic uncertainty in circuit design and analyze various strategies to improve the flotation circuit using variables such as the price of copper and the ore grade, resulting in the problem of nonlinear optimization with two-stage mixed stochastic integers (TS - MINLP). The first stage refers to the design and the second, to the operational stage. On the other hand, Montenegro et al. [13] propose a methodology to analyze and/or design processes in which it is necessary to have several stages to achieve the separation objectives and where it is not possible to define the operating conditions with exactitude. Another method for the design of flotation circuits is presented by Cisternas et al. [3] based on a mixed-integer linear programming model (MILP), using the development of two superstructures hierarchized by the procedure and the tasks to be developed and then modeling said superstructures in order to maximize the benefits. Subsequently, Cisternas et al. [4] propose the optimal selection of the circuit through an objective function where the values of the structural and operative variables can be determined.

The separation of minerals by foam that are carried out in the flotation circuit is done through modeling and experience, so Hu et al., [8] expose an optimization methodology using a genetic algorithm with the modeling of pulp and foam in each flotation cell to determine the optimal design. The separation efficiency and the selectivity index are important for flotation, which is why Salmani Nuri et al. [17] propose an optimization of both factors (separation efficiency and selectivity index), through the artificial hybrid neural network (ANN) and a genetic algorithm (GA), selecting as input variables the dose of the reagent, pH, feed rate, among others, that must be optimized to achieve the desired values, using the MLP structure for ANN modeling. On the other hand, Pirouzan et al. [16] pose a problem of optimization of the configuration of the flotation circuit using metallurgical parameters such as the yield and the content of the mineral in addition to the genetic algorithm. According to Nakhaei et al. [15] flotation is a multivariate process, so its optimization guarantees the metallurgical performance of the process through the ore grade and recovery of the concentrate.

Due to the dynamics of the flotation circuits Bouchard et al. [2] present a framework for the simulation of these circuits, emphasizing water, solid and gas flows and the effect they have on the level of the pulp and the outflow rates. This is done aiming to use a dynamic simulator in a non-linear control strategy that is based on models to maintain critical process variables. Currently, there are new economic and environmental challenges, where Gruzdeva et al. [7] emphasize thatthey propose a deterministic framework of bio-objective mathematical programming, combined with experimental design and regression analysis, in order to optimize the performance of the flotation and determine the optimum conditions of operation satisfying the needs of the process, maximizing the degree of concentration and recovery. Although the study of the sulfide mining processes is very time consuming, only five publications cover more than one process, either crushing and grinding or milling and flotation, or covering the three processes (crushing, grinding and flotation). The drying process is not contemplated in the bibliographic analysis, as it has not yielded results, but it does mention the process carried out in this stage, where copper concentrate with a permitted humidity of 12 to 14% is obtained. On the other hand, the vast majority of publications refer to improving the flotation process, 35 studies of which are found, while the rest of the publications are related to the crushing and / or grinding process.

Taking into account that the studies have objectives which may be economic, social or technological, among others, the vast majority of publications aim to improve the process with respect to the economic and technological axis. Regarding the economic objective, there are 7 publications, while the technological axis has an impact of 40% of the total of studies. Publications that contemplate two axes make up 35% of a total of 52 publications. Therefore, with this information the vast majority of the research seeks to improve some process (crushing, grinding or flotation) in the technological field related to the production of copper concentrate to increase the copper law and in turn increase the recovery of the concentrate coppermade.

4. Mix of minerals

The problem studied in this work is to improve the productivity of the copper concentrate production process. Specifically, it seeks to improve the way in which the company decides to process the different lots of ore and this is improving the selection of minerals. The lots come from small producers of copper ore in the form of rocks and are stored after obtaining samples to study the ore grade of each of them. Subsequently, the company processes the batches by order of arrival. In an initiative of the company to improve productivity, the process and production obtained was studied and it was concluded that it may be improved by using other ways of deciding which lots to process, for example mixing lots from different mines in the same litter.

It is evident that at the end of a long period of time all the minerals from different copper mines must be processed, but the way in which the lots are combined affects improving the value of the variables that are considered critical in the process. These variables are the law of final copper concentrate and metallurgical recovery. The law of final copper concentrate depends on the copper law upon entry and metallurgical recovery. The production process consists of three stages: crushing, grinding and flotation. Only the flotation process has the possibility of improving productivity through the control of process parameters such as the amount of foam, the amount of reagents, the amount of air consumed and other parameters. These parameters affect metallurgical recovery and are the only variable that is "managed" during the process, in order to maximize recovery. Minerals can only yield the amount of copper they carry and this amount is given by the ore grade on entry. Therefore it is key to be able to control the mixture of input minerals. For this reason, Model 1 is proposed, which maximizes copper recovery.

Model 1: Maximizing copper recovery

$$\max \sum_{i=1}^{n} R_i x_i$$

Copper concentrate grade constraint

$$\frac{\sum_{i=1}^{n} L_i x_i}{\sum_{i=1}^{n} x_i} \geq \alpha. \tag{1}$$

Batch mineral availability

$$x_i \leq d_i, i = 1, \ldots, n. \tag{2}$$

Mineral processing capacity of the plant in a day:

$$\sum_{i=1}^{n} x_i \leq Cap. \tag{3}$$

Restrictions of non-negativity

$$x_i \geq, \; i = 1, \ldots, n. \tag{4}$$

Parameters:

L_i : concentrate grade of the batch i,

R_i : metallurgical recovery of the batch i,

α : minimum concentrate grade of the final production,

Cap : capacity of the plant in a day,

d_i : availability of the batch i.

Decision variables:

x_i : quantity of batch i to produce.

lot	Available	Copper %	Recovery %	lot	Available	Copper %	Recovery %
1	715	0.97	85.0	12	703	1.11	79.8
2	713	0.97	81.0	13	719	1.10	86.0
3	712	1.01	84.6	14	709	0.95	73.5
4	711	0.97	83.1	15	697	1.06	72.4
5	707	0.97	83.9	16	622	1.33	85.7
6	724	0.98	88.1	17	703	1.06	72.6
7	722	1.05	85.3	18	705	1.01	77.9
8	707	1.14	80.5	19	684	0.98	81.3
9	712	1.13	80.4	20	697	0.89	78.2
10	705	1.13	83.7	21	704	0.91	86.2
11	495	1.05	78.9	**Total**	**14568**		

Table 1: *Characteristics of copper law and metallurgical recovery of different minerals for one day, from 21 different mining suppliers*

The following table shows mineral availability data for the different lots with different ore grades and metallurgical recovery. The law is known from the study of the mineral at the purchase and the recovery is known from the characteristics of the mine from which the ore comes.

First, we solve model 1 with the data of Table 1, but without considering restriction (1). The results are recovery $R = 83.83$ and concentrate grade $\alpha = 26.883$.
Copper concentrate grade constraint

$$\frac{\sum_{i=1}^{n} L_i x_i}{\sum_{i=1}^{n} x_i} \geq \alpha_k,\ \alpha_k = \alpha + k \cdot step;\ k = 1, \ldots, 6,\ step = 0.125.$$

k	Recovery	Concentrate grade
0	83.83	26.883
1	83.76	27.008
2	83.54	27.133
3	83.28	27.258
4	82.52	27.383
5	81.77	27.508
6	80.62	27.633

Table 2: *Results of Model 1*

Figure 2 shows the recovery values for different values of copper grade, which are modified in 0.125% intervals in each section. The recovery value decreases, which reflects the thesis that both variables are opposed. The idea of the model is to represent the way of managing the current production process, which consists of maximizing the metallurgical recovery given a level of copper grade of the mineral. These points were obtained by solving several optimization models, and represent the Pareto possibilities frontier, that is, the different combinations of recovery points and copper law that deliver the optimum of both variables. It was found that the company does not operate regularly on this border, losing optimality, which results in the loss of tons of fine copper. For example, point A corresponds to any production shift with copper grade values of 27.20% and metallurgical recovery of 82.0%. It is observed that this point is far from the efficient border, because for a given value of copper grade it can be improved up to point B, with a metallurgical recovery of 83.25, increasing the total fine copper tons. By way of contrast we tested the model that could be considered opposite to Model 1, called Model 2. That is, to maximize the copper grade subject to a given value of metallurgical recovery and

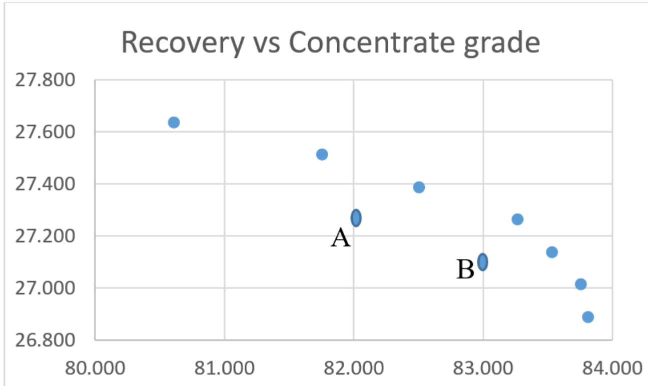

Figure 2: *Max concentrate grade vs metallurgical recovery Model 1*

also complying with the mineral availability restriction of each lot and a restriction of daily plant capacity.

Model 2: Maximizing the copper concentrate grade

$$\max \sum_{i=1}^{n} L_i x_i.$$

Metallurgical recovery

$$\frac{\sum_{i=1}^{n} R x_i}{\sum_{i=1}^{n} x_i} \geq R. \tag{5}$$

Batch mineral availability

$$x_i \leq d_i,\ i_1, \ldots, n. \tag{6}$$

Mineral processing capacity of the plant in a day

$$\sum_{i=1}^{n} x_i \leq Cap\,. \tag{7}$$

Restrictions of non-negativity

$$x_i \geq 0 \ i = 1, \ldots, n. \tag{8}$$

Parameters:

R : minimum recovery,

R_k : metallurgical recovery in iteration k.

Metallurgical recovery

$$\frac{\sum_{i=1}^{n} R x_i}{\sum_{i=1}^{n} x_i} \geq R_k,\ R_k = R + k \cdot step;\ k = 1, \ldots, 7,\ step = 0.5;\ R = 80.62.$$

Model 2 shows the same inverse relationship between copper law and recovery as Model 1 and a similar Pareto type border, which confirms the inverse relationship already explained. Finally, a model that maximizes the product of the final copper law by metallurgical recovery is proposed, which multiplied by the amount of processed ore delivers the tons of fine copper obtained. This will be named Model 3.

Concentrate grade	Recovery
27.633	80.62
27.590	81.12
27.528	81.62
27.448	82.12
27.368	82.62
27.285	83.12
27.090	83.62

Table 3: *Results of Model 2*

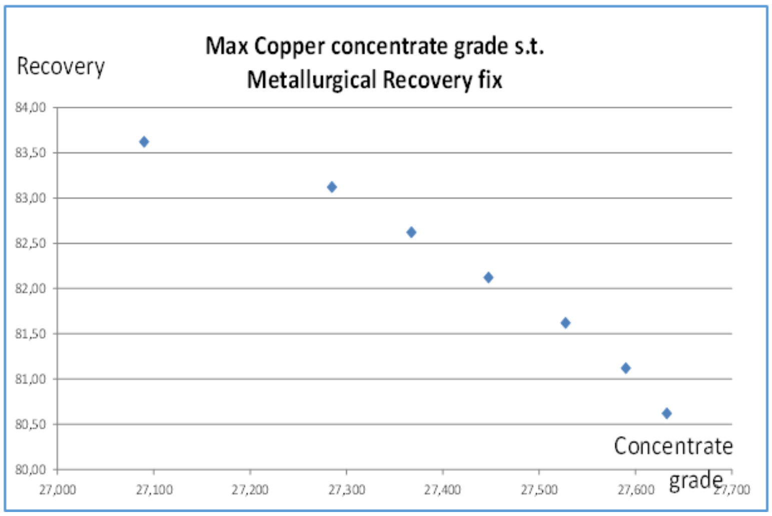

Figure 3: *Max concentrate grade vs metallurgical recovery Model 2*

Model 3: Maximizing the final tons

$$\max Tons = \sum_{i=1}^{n} R_i L_i x_i.$$

Metallurgical recovery

$$\frac{\sum_{i=1}^{n} R x_i}{\sum_{i=1}^{n} x_i} \geq R_k. \tag{9}$$

Batch mineral availability

$$x_i \leq d_i,\ i = 1, \ldots, n.$$

Mineral processing capacity of the plant in a day:

$$\sum_{i=1}^{n} x_i \leq Cap.$$

Restrictions of non-negativity

$$x_i \geq 0,\ i = 1, \ldots, n.$$

Solving this problem

$$R_k = R + k \cdot step,\ k = 1, \ldots, 7,\ step = 0.5;\ R = 80.62.$$

Concentrate grade	Recovery	Ton copper
27.633	80.62	89.10
27.590	81.12	89.52
27.528	81.62	89.87
27.448	82.12	90.15
27.368	82.62	90.44
27.285	**83.12**	**90.71**
27.090	83.62	90.61

Table 4: *Results of Model 3*

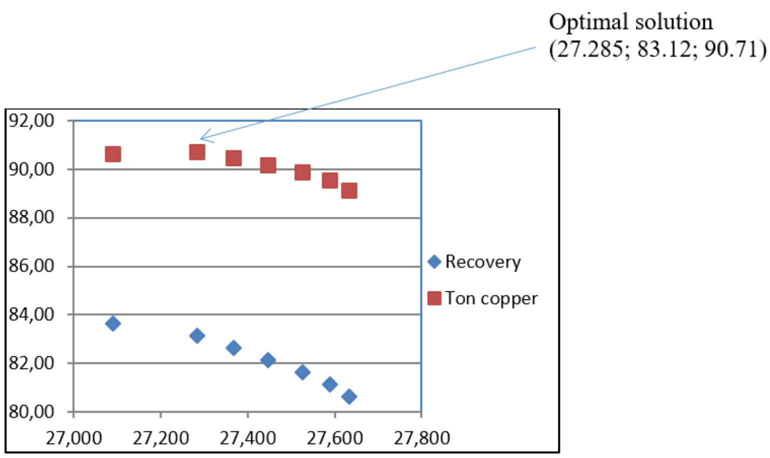

Figure 4: *Max concentrate grade vs metallurgical recovery Model 3*

Finally, the model that will be used in the process will be Model 3 with restrictions (2) and (3). This expression results in the maximization of the tons of copper at the end of the process. To this model of mineral mixture, economic variables and restrictions will be added. That is to say, revenues and costs, in order to represent the economic decision for the company, incorporating all the other concepts, such operational expenses, maintenance costs, cost of electrical energy, costs of the processes of crushing, grinding, flotation and drying, inventory costs in the different stages of the process will be added. It also incorporates the capacities of the processes that are a limitation to the desirable production quotas. This process and its variables are shown in Figure 5.

Model 4: Integral model considering inventory and sales price of the grade concentrate Subscript

- Daily shifts for the sulphide process, $t = 1, 2, \ldots, T$
- Batch of mineral, $j = 1, 2, \ldots, J$

Decision variables
For $t = 1, \ldots, T$ and for $j = 1, 2, \ldots, J$:

x_{jt}^{C} = Flow of tons of ore obtained in the crushing process of lot j at time t.

x_{jt}^{CI} = Flow of tons of ore obtained in the crushing process that passes to inventory of lot j at time t.

x_{jt}^{CM} = Flow of tons of ore that enters the grinding process from the crushing process of lot j at time t.

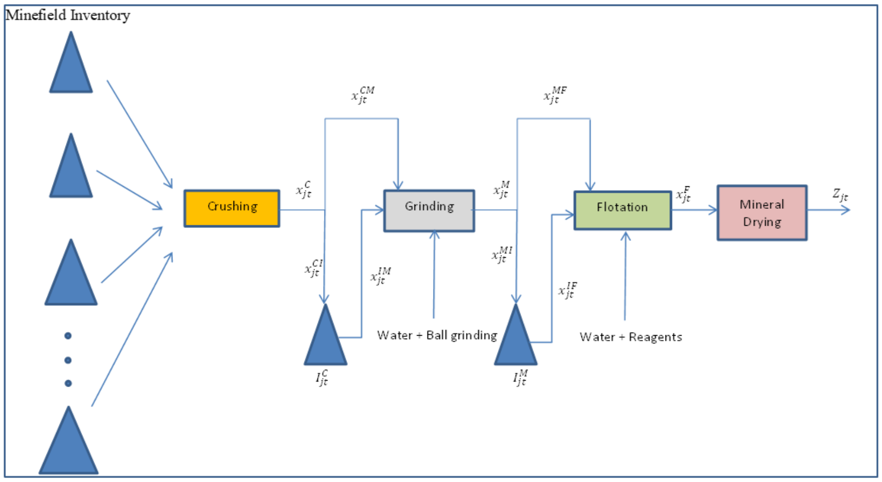

Figure 5: *Diagram of the process with its material flows and variable names*

x_{jt}^{IM} = Flow of tons of ore that enters the grinding process from the inventory of lot j at time t.

x_{jt}^{M} = Flow of tons of ore obtained in the milling process of lot j at time t.

x_{jt}^{MI} = Flow of tons of ore obtained in the grinding process that passes to inventory of lot j at time t.

x_{jt}^{MF} = Flow of tons of ore entering the flotation process from the milling process of lot j at time t.

x_{jt}^{IF} = Flow of tons of ore that enter the flotation process from inventory of lot j at time t.

x_{jt}^{F} = Flow of tons of ore obtained in the flotation process of lot j at time t.

Z_{jt} =Flow of tons of ore obtained in the drying process of lot j at time t.

I_{jt}^{C} =Inventory by tons of ore obtained in the crushing process of lot j at time t.

I_{jt}^{M} = Inventory for tons of ore obtained in the milling process of lot j at time t.

Parameters
For $t = 1, \dots, T$ and for $j = 1, 2, \dots, J$:

P_{jt} : price of a ton of copper concentrate on day j on shift t.

C_j^C : cost of the crushing process that depends on the ore (per ton) of day j.

C_j^M : cost of the milling process that depends on the mineral (per ton) of the day j.

C_j^F : cost of the flotation process that depends on the mineral (per ton) of day j.

C_j^S : cost of the drying process that depends on the mineral (per ton) of day j.

C^{IC} : unit inventory cost per ton after the crushing process.

C^{IM} : unit inventory cost per ton after the grinding process.

E^j : energy cost of mining sulfide processes.

M^j : maintenance cost of mining sulfide processes.

CAP^{jt} : processing capacity of incoming minerals (in tons) to the crushing process on day j, in turn t.

$CAPm^{jt}$: processing capacity of incoming minerals (in tons) to the grinding process on day j, on shift t.

$CAPf^{jt}$: processing capacity of incoming minerals (in tons) to the flotation process on day j, in turn t.

β : humidity rate of the drying process.

$dispch_{jt}$: availability of incoming minerals (in tons) to the crushing process on day j, in turn t.

$dispcm_{jt}$: availability of incoming minerals (in tons) to the grinding process on day j, on shift t.

$dispmf_{jt}$: availability of incoming minerals (in tons) to the flotation process on day j, on shift t.

Constraints

$$x_{jt}^{C} \leq CAP_{jt} \tag{10}$$

$$\sum_{t=1}^{T} x_{jt}^{C} \leq \sum_{t=1}^{T} CAP_{jt}, \forall j \tag{11}$$

$$\sum_{t=1}^{T} x_{jt}^{CM} + \sum_{t=1}^{T} x_{jt}^{IM} \leq \sum_{t=1}^{T} CAPm_{jt}, \forall j \tag{12}$$

$$\sum_{t=1}^{T} x_{jt}^{MF} + \sum_{t=1}^{T} x_{jt}^{IF} \leq \sum_{t=1}^{T} CAPf_{jt}, \forall j \tag{13}$$

$$x_{jt}^{C} \leq dispch_{jt} \tag{14}$$

$$x_{jt}^{CM} \leq dispcm_{jt} \tag{15}$$

$$x_{jt}^{MF} \leq dispmf_{jt} \tag{16}$$

$$x_{jt}^{IM} \leq I_{j,t-1}^{C} \tag{17}$$

$$x_{jt}^{IF} \leq I_{j,t-1}^{M} \tag{18}$$

Constraints (10) to (18) correspond to capacity restrictions of the flow variables. The law of conservation of mass is applied to the constraints that follow, analyzing the physical systems. Constraints (19) and (20) equalize the amounts entered into the drying process with the quantities withdrawn from the same process by applying the loss factor by evaporation of moisture alpha at the aggregate and individual level, respectively. Restrictions (21) and (22) correspond to typical inventory conservation restrictions for the crushing and milling process respectively.

Restrictions (23), (24), (25) and (26) correspond to the equations of flow inflows to the crushing, milling, flotation and drying processes, respectively.

$$\sum_{t=1}^{T} Z_{jt} = \beta \cdot \sum_{t=1}^{T} x_{jt}^{F} \, \forall j \tag{19}$$

$$Z_{jt} = \beta \cdot x_{jt} \tag{20}$$

$$I_{jt}^{C} = x_{j,t}^{CI} - x_{j,t}^{IM} + I_{j,t-1}^{C} \tag{21}$$

$$I_{jt}^{M} = x_{j,t}^{MI} - x_{j,t}^{IF} + I_{j,t-1}^{M} \tag{22}$$

$$x_{jt}^{C} = x_{j,t}^{CM} + x_{j,t}^{CI} \tag{23}$$

$$x_{jt}^{M} = x_{j,t}^{MF} + x_{j,t}^{MI} \tag{24}$$

$$x_{jt}^{M} = x_{j,t}^{CM} + x_{j,t}^{IM} \tag{25}$$

$$x_{jt}^{F} = x_{j,t}^{MF} + x_{j,t}^{IF} \tag{26}$$

Objective function

$$\max B = \sum_{t=1}^{T}\sum_{j=1}^{J}\left(\sum_{t=1}^{T}\sum_{j=1}^{J} P_{jt} \cdot Z_{jt} - Cost\right) \tag{27}$$

$$\begin{aligned} Cost = \sum_{t=1}^{T}\sum_{j=1}^{J}\Bigg(&\sum_{t=1}^{T}\Big(C_t^{C} + E_t + M_t\Big)x_{jt}^{C} L_j R_j + \sum_{t=1}^{T}\Big(C_t^{M} + E_t + M_t\Big)x_{jt}^{M} \\ &+ \sum_{t=1}^{T}\Big(C_t^{F} + E_t + M_t\Big)x_{jt}^{F} + \sum_{t=1}^{T}\Big(C_t^{S} + E_t + M_t\Big)Z_{jt} + C^{IC} \cdot I_{jt}^{C} + C^{IM} \cdot I_{jt}^{M}\Bigg) \end{aligned} \tag{28}$$

The cost function is given by six terms. The first corresponds to the cost of crushing, the second, to the cost of grinding, the third, to the cost of flotation and the fourth, to the cost of drying. The fifth term corresponds to the cost of inventory after crushing and the sixth, to the cost of inventory after flotation. The results obtained using Model 4 are shown in Table 5.

Inst.	Days	#variables	#constraints	Optimal solution (Profit USD)	Income (USD)	Costs (USD)
1	10	360	430	700575	918958	218383
2	20	860	860	1342572	1761014	418442
3	40	1720	1720	2639313	3461898	822585
4	60	2580	2580	3970266	5207721	1237455
5	70	3010	3010	4608494	6044835	1436341
6	90	3870	3870	6011311	7884958	1873647
$\overline{X}$	48	1740	2078	3212089	4213231	1001142

Table 5: *Results obtained*

Six instances corresponding to different time periods were solved, as shown in Table 5. Comparing the method of administration using the mineral mixture model, with the previous method, called "random solution", there are savings of the order of 66.457 dollars per day, which in an annual projection delivers a total saving of around 25 million dollars.

5. Conclusions

The productivity of the flotation process to produce copper concentrate was studied and the managers were convinced to use a mix of minerals to obtain more tons of fine copper at the end of the process. In addition, the relationship between the metallurgical recovery and the copper law of the final concentrate was studied, finding an inverse relationship, which shows that when a combination of minerals that maximizes metallurgical recovery is chosen, this mixture of minerals minimizes the obtained copper grade. The dilemma raised is to choose a mixture of minerals that maximizes the amount of final copper obtained. It was found that this term corresponds to the product of the copper law of the incoming ore multiplied by the metallurgical recovery, which enabled building an objective function. Additionally, the mineral mixture model was developed adding economic variables of income and costs, which produces a total saving of around 25 million dollars per year.

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