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Utilizing a new approach for solving fully fuzzy linear programming problems

Hamiden Abdelwahed Khalifa*

Department of Operations Research, Institute of Statistical Studies and Research, Cairo University, Giza, Egypt
Present Address: Mathematics Department, College of Science and Arts Al-Badaya, Qassim University, Saudi Arabia
E-mail: ⟨hamidenkhalifa@cu.edu.eg, Ha.Ahmed@qu.edu.sa⟩

Abstract. This paper deals with fully fuzzy linear programming (FFLP) problem in which all parameters and variables are characterized by $L - R$ fuzzy numbers. By a proposed approach, the FFLP problem is converted into the triple objective functions, and hence a single objective using the weighting method. Through this approach the problem is not transformed into the crisp linear programming problem (LPP) that is enable for obtaining fuzzy optimal solution and the corresponding fuzzy optimal solution which is more realistic to the real world problems. Then a numerical example is taken to the utility and clarify the practically and the efficiency of the approach.

Keywords: fully fuzzy linear programming, fuzzy optimal solution, $L - R$ fuzzy numbers, software GAMS, weighting method

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1. Introduction

Linear programming (LP) is a branch of mathematical programming designed to solve optimization problem for which all the constraints and objective functions are linear functions. LP is an approach for finding the answer fittest from a range of possible answers. LP problems have enormous applications as in arising in business, government, industry, hospitals, libraries, etc. It is clear that LP is in two forms: Classical LP and Fuzzy LP. Bector and Chandra [2] classified the fuzzy LP problems into four categories:

- Type I. LP with fuzzy inequalities and crisp objective function,
- Type II. LP with crisp inequalities and fuzzy objective function,
- Type III. LP with fuzzy inequalities and fuzzy objective function and
- Type IV. LP with fuzzy parameters.

In many scientific areas, such as system analysis and operations research, a model has to be set up using data which is only approximately known. Fuzzy sets theory, introduced by Zadeh [30], makes this possible. Dubois and Prade [10] extended the use of algebraic operations on real numbers to fuzzy numbers by the use a fuzzification principle.

*Corresponding author.

Since the pioneer work on fuzzy LP by Tanaka et al. [28] and Zimmermann [33]. Several kinds of fuzzy LP problems of satisfying different constraints were appeared in the literature so far and with them corresponding approaches of resolution have proposed too. Tanaka et al. [29] formulated a fuzzy linear programming (FLP) problem to obtain a reasonable solution under consideration of the ambiguity of parameters. Zhao et al. [31] studied the complete solution set for fuzzy linear programming problems included fuzzy and non-fuzzy equality and inequality constraints. The basic concepts of fuzzy decision making in fuzzy environment were first proposed by Bellman and Zadeh [3] The first formulation of fuzzy LP problems was proposed by Zimmermann [33], Shaocheng [27], Buckley [4, 5] and others considered situation where all parameters are fuzzy. Li and Shi [18] studied fuzzy LP problems with interval- value fuzzy coefficients, where corresponding auxiliary models in different criteria are obtained. Zhong et al. [32] studied LP problem with fuzzy random variable coefficients and fuzzy pseudorandom variables. Maleki et al. [19] proposed a new method for solving LP problem with fuzzy variables based on the concept of comparison of fuzzy numbers. The references [6, 7, 8, 21, 24, 28] studied fuzzy mathematical programming with fuzzy number coefficients. Ishibuchi and Tanaka [15] investigated mathematical programming problem with interval objective function coefficients.

The fuzzy LP problems in which all the parameters as well as variables are represented by fuzzy numbers is known as FFLP problems. FFLP has many different applications in sciences and engineering, and various methods have been proposed for solving it. FFLP problems can be divided into two categories: FFLP with equality constraints and with equality constraints. Kumar et al. [17] proposed a new method for solving fully fuzzy LP problem with equality constraints and determined the optimal fuzzy solution for it. Hashemi et al. [13] and Allahviranloo et al. [1] proposed different methods for FFLP problems with inequality constraints. Sahaya Sudha and Karpagamani [23] discussed FFLP problem with trapezoidal fuzzy numbers with the help of linear system and ranking function. Rajarajeswari and Sahaya Sudha [22] proposed a new method for solving FFLP problem. Based on Lexicography method, Shamooshaki et al. [26] proposed a new method for solving FFLP problem. Hosseinzadeh and Edalatpanah [14] proposed a new method for solving FFLP based on the $L-R$ fuzzy numbers and the Lexicography method. Das [8] developed a modified algorithm to find the fuzzy optimal solution for the FFLP problem with equality constraints. Through the MOLP problem and extended L-R fuzzy numbers, Gong et al. [12] and Ezzati et al. [11] solved FFLP problems. Das et al. [9] used the ranking function for solving FFLP problem with mixed constraints so as to overcome limitations.

The rest of the paper is organized as follows. In Section 2, some preliminaries need in the paper are introduced. In Section 3, a fully fuzzy linear programming problem is formulated. In Section 4, a method for solving the FFLP problem is proposed. In Section 5, a numerical example is given for illustration. Finally, some concluding remarks are reported in Section 6.

2. Preliminaries

In order to discuss our problem conveniently, basic concepts and results related to fuzzy numbers, L-R fuzzy numbers and their arithmetic operations are recalled (Kaufmann and Gupta [16], Bellman and Zadeh [3], Sakawa [25] and Hosseinzadeh and Edalatpanah [14]).

Definition 1. *A fuzzy number $\tilde{a}$ is mapping $\mu_{\tilde{a}} : R \to [0,\ 1]$ with following properties:*

i) $\mu_{\tilde{a}}(x)$ *is an upper semi–continuous membership function,*
ii) $\tilde{a}$ *is a convex fuzzy set, i.e.* $\mu_{\tilde{a}}(\lambda x + (1-\lambda)y) \geq min\{\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)\}$ *for all* $x, y \in R$ *and* $0 \leq \lambda \leq 1$,
iii) $\tilde{a}$ *is normal, i.e.* $\exists x_0 \in R$ *for which* $\mu_{\tilde{a}}(x_0) = 1$,
iv) $supp(\tilde{a}) = \{x \in R : \mu_{\tilde{a}} > 0\}$ *is the support of the* $\tilde{a}$ *and its closure* $cl(supp(\tilde{a}))$ *is compact set.*

Definition 2. *A triangular fuzzy number can be represented completely by a triplet* $\tilde{A} = (a_1, a_2, a_3)$ *and has membership:*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & if \;\; x < a_1, \\ \dfrac{x - a_1}{a_2 - a_1} & if \;\; a_1 \leq x \leq a_2, \\ \dfrac{a_3 - x}{a_3 - a_2} & if \;\; a_2 \leq x \leq a_3, \\ 0 & if \;\; x > a_3. \end{cases}$$

Definition 3. *A fuzzy number* $\tilde{B}$ *is said to be* $L - R$ *type if*

$$\mu_{\tilde{B}}(x) = \begin{cases} L\left(\dfrac{m - x}{\alpha}\right) & if \;\; x \leq m \;\; and \;\; \alpha > 0, \\ R\left(\dfrac{x - m}{\beta}\right) & if \;\; x \geq m \;\; and \;\; \beta > 0, \end{cases}$$

where m is the mean value of $\tilde{B}$, α and β are left and right spreads, respectively, and a function $L(\cdot)$ is a left shape function satisfying

$i)$ $L(x) = L(-x)$,

$ii)$ $L(0) = 1$,

$iii)$ $L(x)$ *is non decreasing on*$[0, \infty)$.

Similarly, a right shape function $R(\cdot)$ is defined as $L(\cdot)$. Symbolically, a $L-R$ fuzzy number $\tilde{B}$ can be written as $\tilde{B} = (b, \alpha, \beta)_{LR}$.

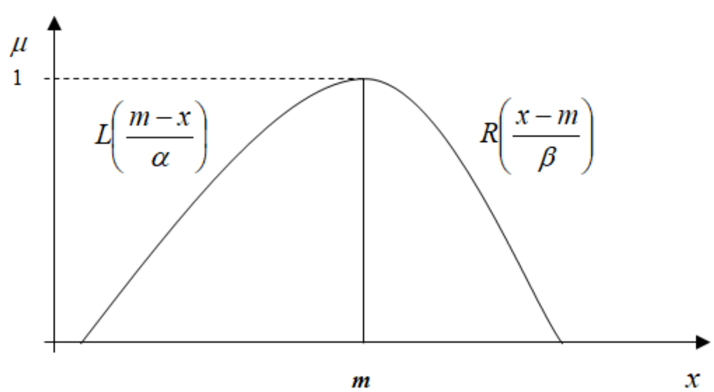

Figure 1: *L–R fuzzy number*

The formulas of addition, subtraction, opposite, multiplication and order relation for $\tilde{A} = (a, \gamma, \delta)_{LR}$ and $\tilde{B} = (b, \alpha, \beta)_{LR}$ are

- addition $\tilde{A} \oplus \tilde{B} = (a + b, \gamma + \alpha, \delta + \beta)_{LR}$
- opposite $-\tilde{A} = (-a, \gamma, \delta)_{LR}$
- substraction $\tilde{A}(-)\tilde{B} = (a - b, \gamma + \beta, \delta + \alpha)_{LR}$

- multiplication $\tilde{A} \otimes \tilde{B}$:

  if $\tilde{A} > 0$ and $\tilde{B} > 0$ then $\tilde{A} \otimes \tilde{B} \cong (ab,\ a\alpha + b\gamma,\ a\beta + b\delta)_{LR}$,
  if $\tilde{A} < 0$ and $\tilde{B} > 0$ then $\tilde{A} \otimes \tilde{B} \cong (ab, -b\delta - a\beta, -b\gamma - a\alpha)_{LR}$,
  if $\tilde{A} < 0$ and $\tilde{B} < 0$ then $\tilde{A} \otimes \tilde{B} \cong (ab,\ b\gamma - a\beta,\ b\delta - a\alpha)_{LR}$

- inverse $\tilde{A}^{-1} \cong (a^{-1}, \delta a^{-2}, \gamma a^{-2})_{LR}$

- order relation $\tilde{A}(<)\tilde{B}\ \ iff\ \ a < b,\ \ or\ \ a = b\ \ and\ \ (\gamma + \delta) > (\alpha + \beta)\ \ or\ \ a = b,\ (\gamma + \delta) = (\alpha + \beta)\ \ and\ \ (2a - \gamma + \delta) < (2b - \alpha + \beta)$

3. Problem formulation and solution concepts

A fully fuzzy linear programming problem can be formulated as follows

$$\begin{aligned} &min\tilde{Z} = \tilde{C}^T \otimes \tilde{X} \\ &s.t.\ \ \tilde{A} \otimes \tilde{X} = \tilde{B}, \\ &\qquad \tilde{X} \geq \tilde{0}, \end{aligned} \tag{1}$$

where $\tilde{C}^T = (\tilde{c}_j)_{1\times n}$, $\tilde{X} = (\tilde{x}_j)_{n\times 1}$, $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, $\tilde{B} = (\tilde{b}_i)_{m\times 1}$ and $\tilde{c}_j,\ \tilde{x}_j,\ \tilde{a}_{ij},\ \tilde{b}_i \in F_0(R)$. Through this paper $F_0(R)$ is the set of all $L - R$ fuzzy numbers on R.

Definition 4. *The $\tilde{x}^*$ which satisfies the conditions in problem (1) is called a fuzzy optimization solution [32].*

The FFLP problem (1) may be written as

$$\begin{aligned} min\tilde{Z} &= \sum_{j=1}^{n} (c_j,\ \alpha_j,\ \beta_j)_{LR} \otimes (x_j,\ \zeta_j,\ \xi_j)_{LR} \\ s.t.\ \ &\sum_{j=1}^{n} (a_{ij},\ \varepsilon_{ij},\ \phi_{ij})_{LR} \otimes (x_j,\ \zeta_j,\ \xi_j)_{LR} = (b_i,\ \mu_i,\ v_i)_{LR}, \\ &(x_j,\ \zeta_j,\ \xi_j)_{LR} \geq \tilde{0}. \end{aligned} \tag{2}$$

Using the arithmetic operations of $L - R$ fuzzy numbers, problem (2) may be rewritten as

$$\begin{aligned} min\tilde{Z} &= \sum_{j=1}^{n} (c_j x_j,\ c_j\zeta_j + \alpha_j x_j,\ c_j\xi_j + \beta_j x_j)_{LR} \\ s.t.\ \ &\sum_{j=1}^{n} (a_{ij}x_j,\ a_{ij}\zeta_j + \varepsilon_{ij}x_j,\ a_{ij}\xi_j + \phi_{ij}x_j)_{LR} = (b_i,\ \mu_i,\ v_i)_{LR}, \\ &(x_j,\ \zeta_j,\ \xi_j)_{LR} \geq \tilde{0}. \end{aligned} \tag{3}$$

Also, from problem (3) we have the following problem

$$min\tilde{Z} = \sum_{j=1}^{n}\left((c_jx_j)^L,\ (c_jx_j)^C,\ (c_jx_j)^U\right)$$

$$s.t.\quad x \in X = \begin{cases} \sum_{j=1}^{n} a_{ij}x_j = b_i, \quad i = 1,2,...,m \\ \sum_{j=1}^{n}(a_{ij}\zeta_j + \varepsilon ijx_j,\ a_{ij}\xi_j + \phi_{ij}x_j) = \mu_i + v_i, \quad \forall i \\ \sum_{j=1}^{n}\left(2a_{ij}x_j - (a_{ij}\zeta_j + \varepsilon ijx_j) + (a_{ij}\xi_j + \phi_{ij}x_j)\right) = 2b_i - \mu_i + v_i \quad \forall i \\ x_j,\ \zeta_j,\ \xi_j \geq 0 \quad \forall j, \end{cases} \tag{4}$$

where $(c_jx_j)^C = c_jx_j,\ (c_jx_j)^L = c_jx_j - (a_{ij}\zeta_j + \varepsilon_{ij}x_j)$ and $(c_jx_j)^U = c_jx_j + (c_j\xi_j + \beta_jx_j)$. From problem (4) we have the following MOLP problem

$$\begin{aligned} &minf_1 = (c_jx_j)^C \\ &minf_2 = (c_jx_j)^U - (c_jx_j)^L \\ &minf_3 = (c_jx_j)^U + (c_jx_j)^L \\ &s.t.\quad x \in X. \end{aligned} \tag{5}$$

Definition 5. *(Pareto optimal solution) $x^0 \in X$ is said to be Pereto optimal solution to problem (5) if and only if there does not exist another $x \in X$ such that $f_1(x) \leq f_1(x^0),\ f_2(x) \geq Z_2(x^0)$ and $f_3(x) \leq f_3(x^0)$, and $f_1(x) \neq f_1(x^0),\ f_2(x) \geq f_2(x^0)$ or $f_3(x) \neq f_3(x^0)$.*

Problem (5) can be treated using the weighting method [20] as

$$\begin{aligned} &minE = (w_1f_1 - w_2f_2 + w_3f_3) \\ &s.t.\quad x \in X,\quad w_i \geq 0,\quad \sum_{i=1}^{3} w_i = 1. \end{aligned} \tag{6}$$

4. Proposed approach

The steps of the proposed approach for solving the FFLP problem (1) can be summarized as:

Step 1: Consider the FFLP problem (1)

Step 2: Convert the problem (1) into problem (4)

Step 3: Transform the problem (4) into the MOLP problem (5), and then into problem (6)

Step 4: Solve the problem (6) to obtain the efficient solution

Step 5: Referring to problem (1) for determining the fuzzy optimal solution and the corresponding fuzzy optimum value

Step 6: Stop

5. Numerical example

Consider the following FFLP problem

$$\begin{aligned} & min\tilde{Z} = (2,1,1)_{LR} \otimes \tilde{x}_1 \oplus (3,1,1)_{LR} \otimes \tilde{x}_2 \\ & s.t.\ \ (1,1,1)_{LR} \otimes \tilde{x}_1 \oplus (2,1,1)_{LR} \otimes \tilde{x}_2 = (10,8,14)_{LR} \\ & \quad (2,1,1)_{LR} \otimes \tilde{x}_1 \oplus (1,1,1)_{LR} \otimes \tilde{x}_2 = (8,7,13)_{LR} \\ & \quad \tilde{x}_1,\ \tilde{x}_2 \geq 0, \quad \tilde{x}_1,\ \tilde{x}_2 \in F_0(R). \end{aligned} \tag{7}$$

Based on the proposed approach, the above FFLP problem can be written as

$$\begin{aligned} & minf_1 = 2x_1 + 3x_2 \\ & minf_2 = 2x_1 + 2x_2 + 2\xi_1 + 3\xi_2 + 2\zeta_1 + 3\zeta_2 \\ & minf_3 = 4x_1 + 6x_2 + 2\xi_1 + 3\xi_2 - 2\zeta_1 - 3\zeta_2 \\ & s.t.\ \ x_1 + 2x_2 = 10 \\ & \quad 2x_1 + 2x_2 + \xi_1 + 2\xi_2 + \zeta_1 + 2\zeta_2 = 22 \\ & \quad 2x_1 + 4x_2 - \xi_1 - 2\xi_2 + \zeta_1 + 2\zeta_2 = 26 \\ & \quad 2x_1 + x_2 = 8 \\ & \quad 2x_1 + 2x_2 + 2\xi_1 + \xi_2 + 2\zeta_1 + \zeta_2 = 20 \\ & \quad 4x_1 + 2x_2 - 2\xi_1 - \xi_2 + 2\zeta_1 + \zeta_2 = 22 \\ & \quad x_1,\ x_2,\ \xi_1,\ \xi_2,\ \zeta_1,\ \zeta_2 \geq 0 \\ & \quad x_1 - 2\zeta_1 \geq 0, x_1 + 2\zeta_1 \geq 0, x_1 + 2\xi_2 \geq 0 \\ & \quad x_2 - 3\zeta_2 \geq 0, x_2 + 3\zeta_2 \geq 0, x_2 + 3\xi_2 \geq 0. \end{aligned} \tag{8}$$

Using the weighting method, the problem (8) becomes

$$\begin{aligned} & minE = 0.5f_1 + 0.2f_2 + 0.3f_3 = 1.8x_1 + 2.9x_2 + 0.2\xi_1 + 0.3\xi_2 - \zeta_1 - 1.5\zeta_2 \\ & s.t.\ \ x_1 + 2x_2 = 10 \\ & \quad 2x_1 + 2x_2 + \xi_1 + 2\xi_2 + \zeta_1 + 2\zeta_2 = 22 \\ & \quad 2x_1 + 4x_2 - \xi_1 - 2\xi_2 + \zeta_1 + 2\zeta_2 = 26 \\ & \quad 2x_1 + x_2 = 8 \\ & \quad 2x_1 + 2x_2 + 2\xi_1 + \xi_2 + 2\zeta_1 + \zeta_2 = 20 \\ & \quad 4x_1 + 2x_2 - 2\xi_1 - \xi_2 + 2\zeta_1 + \zeta_2 = 22 \\ & \quad x_1,\ x_2,\ \xi_1,\ \xi_2,\ \zeta_1,\ \zeta_2 \geq 0 \\ & \quad x_1 - 2\zeta_1 \geq 0, x_1 + 2\zeta_1 \geq 0, x_1 + 2\xi_2 \geq 0 \\ & \quad x_2 - 3\zeta_2 \geq 0, x_2 + 3\zeta_2 \geq 0, x_2 + 3\xi_2 \geq 0. \end{aligned} \tag{9}$$

Using GAMS Software, the solution of FFLP problem (7) and thus the solution of problem (9) is given in Table 1.

Fuzzy optimal solution	Fuzzy optimum value
$\tilde{x}_1^* = (2,0,2)_{LR}$	$\tilde{Z}^* = (16,9,19)_{LR}$
$\tilde{x}_2^* = (4,1,3)_{LR}$	$E = 15$

Table 1: *The fuzzy solution of the FFLP problem*

6. Conclusions

In this paper, an approach for obtaining fuzzy optimal solution for FFLP has introduced. The proposed approach based on converting the FFPL problem into triple objective function and hence single objective using the weighting method. The advantage of this approach is more flexible, realistic to the real world problem, useful for the future study and can be extended in generalized fuzzy numbers.

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