WITTGENSTEIN'S TURN FROM MODELS TO MODELING

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Abstract

We present a solution to the difference between Wittgenstein's use of model in TLP and modeling in PI, RFM and OC by means of analysis of the difference in word choices and concept descriptions of the phenomena in his remarks. The method we used is conceptual analysis of Wittgenstein's treatments of scientific models and modeling. Additionally, textual analysis and textual comparison are used in the study of his major remarks on the subject. Given the discovered difference between model and modeling and the continuity of dependence of both concepts on the concept of picture, with the difference between picture and picturing implied, we suggest that Wittgenstein tried to incorporate his old concept of model into the new concept of modeling. The explicated difference and incorporation contribute to the clarification of the issues of Wittgenstein's indirect and direct influence on philosophy of science (especially on Toulmin, Hanson, Kuhn and Feyerabend).

KEYWORDS: model, modeling, philosophy of science, Philosophical Investigations, science, Tractatus Logico–Philosophicus, Wittgenstein

Introduction: Wittgenstein and philosophy of science

In her recent paper "Wittgenstein and Philosophy of Science", V. Kindi writes:

"In the course of the twentieth century, Wittgenstein's work has influenced the development of philosophy of science as a discipline at decisive points: first in the beginning, when contemporary philosophy of science was founded, and

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then around mid-century, when it took a turn away (or even against) its previous practice. In both cases he was creatively read and misread." (Kindi 2017, 599)

Some elements of the mentioned "reading and misreading" will be explicated below without any attempt whatsoever to predict the future influence of Wittgenstein's thoughts since, as P. M. S. Hacker points out, "In the absence of a crystal ball, one cannot say what Wittgenstein's role in 21st century philosophy will be. But one can attempt to articulate his philosophical legacy to the 21st century" (Hacker 2018).

The topic of this paper is a small number of Wittgenstein's remarks on scientific models and his change from models to modeling, namely, from the early ideas of TLP to the later ones of PI and RFM. Elements of this research include the introductory and the core part. Due to their complexity both will be presented via tables. The first part describes Wittgenstein's relation to sciences and philosophy of science, and the second focuses on his application of the ideas of scientific models and modeling in his early and later works (regarding the remark by an anonymous reviewer concerning the difference between models and modeling, we can only point to introductory texts such as Frigg and Hartmann (2012), which will be referred to later on; however, this does not fully apply to Wittgenstein, because for him a model is static and logical, while modeling is morphological and dynamic, the former being given by logical structure, and the latter being performed by grammatical practice).

The first part is important for clarifying Wittgenstein's relation to sciences and for clearing the ground for his anti–scientific remarks. The second one has three elements: Wittgenstein's application of the concept of scientific model in TLP, the application of the concept of scientific modeling in PI, RFM, ROC and OC, and the explication of the turn from model to modeling. Our most important remarks will be given in small letters in brackets, (a), (b), (c) …, in order to make it easier to follow the line of reasoning and the arguments.

1.   *Wittgenstein's relation to sciences and philosophy of science*

The goal here is to show the major influences of Wittgenstein's philosophy in its relation to sciences (as shown in Table 1, see Garver 1989) and (anti) scientism (see Beale and Kidd 2017) in a simplified manner.

Table 1:

The complex network of scientific influences on Wittgenstein's philosophy and its influence on the early and late philosophy of science of the 20th century.

Tablica 1:

Složena mreža znanstvenih utjecaja na Wittgensteinovu filozofiju i njezin utjecaj na ranu i kasnu filozofiju znanosti 20. st.

His scientific and engineering education	→	**Wittgenstein's philosophy in relation to sciences** ↙ ↘		→	His generally recognized influence on logical positivism with TLP
His scientific and engineering experience and results (inventions)	→			→	His criticism of scientism (of the Vienna Circle) and logical positivism
His experience and results in logic and mathematics (truth tables invention)	→	**The application of the idea of scientific models in TLP**	**The application of the idea of scientific modeling in PI, RFM and OC**	→	His interpretations of and objections to various scientific theories
His embrace of scientific models (Hertz, Boltzmann) and modeling (Goethe, Spengler)	→			→	His generally unrecognized influence on later philosophy of science (Toulmin, Hanson, Kuhn, and Feyerabend)

In the middle column of Table 1, the issue of Wittgenstein's philosophy in relation to sciences is explicated with two elements: the application of the idea of scientific model in TLP and scientific modeling in PI and his later works. In the left column are relevant influences that led to the application of one or the other, and in the right column are the results in terms of his reactions to scientific theories, various philosophies of science, and direct or indirect influence on two major movements in the philosophy of science of the 20th century (Eccles 1963, Mays 1980, Biggs 2001, Hamilton 2001, Sterrett 2005, Kindi 2017).

In this section, the subject of the research is simply differentiated from similar issues that are not of our concern at the moment, but of which we are completely aware, and which have their own internal problems that we consider here as resolved. Therefore, in this introductory part we only mention some major authors that discuss these similar issues in order to individuate the issue that is of our concern here.

2. *Wittgenstein's application of scientific models and modeling in his philosophy*

Given that the hypothesis has been made clear, we can mention some elements that are not of major importance for this research, such as historical influences on Wittgenstein's application of the idea of scientific model in TLP (namely of Hertz and Boltzmann; see Gargani 1980, Wilson 1989, Myrvold 1990, Kjaergaard 2002, Preston 2008, 2017), and scientific modeling in his later works (namely of Goethe and Spengler, see Cahill 2001, Schulte 2002, Klagge 2003, 2011). Of course, as suggested by the reviewers, we should mention that Goethe and Spengler cannot be considered as proper scientific influences, while Hertz and Boltzmann by all means can. However, Goethe and Spengler did impact Wittgenstein after Hertz and Boltzmann, whose influence was early and basic. Furthermore, Wittgenstein's influence on the schools of philosophy of science is not of major interest for us at this point either (namely on the *Vienna Circle* and *Logical Positivism* by TLP and on Toulmin, Hanson, Kuhn, and Feyerabend by PI and his later works, Maudgil 1989, Kindi 2017).

These elements have their own interpretative relevance, but hereafter they will be mentioned insofar as they contribute to the central analysis. The second part of the research is explicated by three questions:

(a) How should we describe Wittgenstein's application of the idea of scientific model in TLP?

(b) How should we describe his application (perhaps the invention) of the idea of scientific modeling in PI and his later works?

(c) Which were the crucial descriptions of the turn from the application of scientific model to the application of scientific modeling?

The question (c) is the core of the research. The answer could help to understand various influences on Wittgenstein on this matter in terms of criticism (of Ramsey and Sraffa, see: Ferber 1990). Also, it could help to understand his influence on philosophies of science of the 20th century. It could also help to clarify how his views advanced in relation to the major standpoints of philosophies of science of the time. These elements are exegetically important at best (as shown in Table 2). However, the main goal of the research is to understand Wittgenstein's turn from models to modeling, i.e. their application in his early and later philosophy, and the major descriptions, analyses, and arguments of this turn (the research question (c)) because this understanding seems to help some of the present–day dilemmas and debates on the nature of models in contemporary philosophy of science.

Table 2:

Comparison of scientific heuristics, Wittgenstein's heuristics, and heuristics in philosophy of science in 1918/1922, 1945/1953 and 1980s.

Tablica 2:

Poredba znanstvene heuristike, Wittgensteinove heuristike i heuristike filozofije znanosti 1918./1922., 1945./1953. i 1980–ih.

Sciences (1918, 1922)	→	Sciences (1945, 1953)	→	Science (1980s)
Scientific models		Scientific modeling		4 level modeling
Wittgenstein TLP (1918, 1922)		**Wittgenstein PI (1945, 1953)**		**Interpreting Wittgenstein**
Model (influence: Hertz, Boltzmann) (criticized by: Ramsey)		**Modeling** (influence: Goethe, Spengler) (sparked by: James, Sraffa)		Interpreting Wittgenstein as a philosopher of science implementing scientific models and modeling in his early and later philosophy.
➢ Case (Fall)	→	➢ Form of life (Lebensform)	→	
➢ Picture (Bild)		➢ Surveyable representation (übersichlitsche Darstellung)		
➢ Logic (Logik)		➢ Grammar (Grammatik)		
➢ Proposition (Satz)		➢ Language–game (Sprachspiel)		
Philosophy of science (1918)	→	**Philosophy of science (1945)**	→	**Philosophy of science (1980s)**
Theory		Model		Modeling

3. *The difference between the use of models and modeling in Wittgenstein's philosophy*

Concerning the central question (c), there is a wider debate about the continuity and discontinuity of Wittgenstein's philosophy, especially between TLP and PI. Whatever the answer to question (c) may be, it should not contradict one of these two general theses (of continuity or of discontinuity) if they are textually and analytically justified. However, the answer cannot be simply explicated or deduced from either of these two theses, because it has its own merit, that is to say, the actual use of the words "model" and "modeling" (as shown in Table 3; as suggested by the reviewers, here we emphasize that Wittgenstein's distinction between models and modeling isn't purely linguistic, but is also presented in practice; in several places Wittgenstein provides practical examples of models and modeling, model being something given by the nature of logic, while modeling being something performed by the nature of scientific or engineering actions).

Table 3:

Mentioning of "model", "modeling" and similar concepts in Wittgenstein's published works.

Tablica 3:

Spominjanje „modela", „modeliranja" i sličnoznačnih pojmova u Wittgensteinovim objavljenim djelima.

Work	Model	Modeling, to model	Similar expressions
	Section / page	Section / page	
NB	12, 20, 30	——	picture
TLP	2.12, 4.01, 4.463	——	picture, prototype
PR	34, 68, 71, 217, 230	10 (making models, preparing a model)	picture
PG	I:18, 58, 65, 127, 131, App. 6	I:7 (model it), 82, II:7, 40	picture, drawing, representation, painted, imagined
BB	6, 33, 34, 117, 122, 163	4, 68 (I model), 70 (to model)	Picture, scheme, mould, prototype, pattern, paradigm
PI, PPF	131 (Vorbild), 141, 156, 191–192 (Vorbild), 293 (Muster), 302, II (PPF):196	512 (to be modelled)	picture, drawing, pattern, paradigm
RFM	I:24, 40, 55, 73, 78, 123, 124, App. 1:6, III:4, 22–24, 31, 33, 39, 41, 44, 85, IV:31, VII:11, 20, 48–49,	III:85 (gemodelt, modelled), IV:30 (umgemodelt, remodeled, transmogrified, transformed)	picture, pattern, paradigm
CV	59 (Vorlage)	16 (modelled)	picture
Z	94, 106, 201, 331	245 (model it), 480 (to model), 624 (to model)	picture, to draw
RPP I	429, 431, 588, 593	260 (to model)	picture, image, picture
RPP II	66, 188, 226, 368, 556	14 (models)	picture, drawing, painting
LWPP I	161, 290, 442, 447, 451, 495, 500, 622, 633, 672	——	picture
ROC	Metaphors are present, but not the actual expression or various synonyms (e.g. ROC I 32).		picture
OC	Metaphors are present, but not the actual expression or various synonyms (e.g. OC 94–99 and 152).		picture

Given that here are the major mentions of "model" and "modeling" in Wittgenstein's published works (and it can be hypothesized that a similar situation would occur in the *Nachlass*, in *Letters*, and in various lectures and notes), it can be said that these terms were used quite scarcely, however, in important places and contexts. Moreover, it can be said that these words were used in parallel or closely connected to other similar words, among which "picture" and "to picture" stand out (there are others which became popular decades after the completion and publication of TPL and PI, such as paradigm, pattern, prototype, representation, imagination, and visualization).

> "Thinking is comparable in every way to drawing pictures." (…) "If one conceives of propositions as instructions for making models [*um Modelle zu bilden*], their picture–like [*Bildhaftigkeit*] quality becomes even clearer." (BT 21)

Exactly this (BT 21) is what makes modeling, as practiced by scientists, engineers, and artists, related to a picture (picturing). However, modeling formally in logic or mathematics, modeling from a sketch, or from a small sculpture are all different versions of modeling and picturing. Given that a picture is a primordial sign, while a model is a more precise sign, an obvious ambiguity of a model seems to be inherited from the hidden ambiguity of its ancestor, a picture. This is no surprise since, on the face of it, they seem to share a family resemblance, a language–game and a form of life. As long as this stands for the relation of a model to a picture, it seems to stand for the relation of a model to all words and expressions used as synonyms for a picture *ceteris paribus* (as shown in Table 3).

Consequently, the best way to answer question (c) would be to single out a few important uses of "model" according to some criterion, such as their relation to other similar words, i.e. "picture", and the major standpoints of TLP and PI, to try to analyze these uses, and to find the crucial description that separates modeling from models. Since Wittgenstein in his later philosophy still provided quotes from Hertz and Boltzmann (in his remarks, lectures, and discussions) besides Goethe and Spengler (they did not appear in his early works), in general context of his early philosophy partly being contrasted to, and in the same time being incorporated in his later one (e.g. logic in philosophical grammar, reference in use, etc.), it is possible to research the hypothesis that his early concept, application, and use of the scientific "model" became incorporated in his later concept, application, and use of "the activity of modeling" (perhaps by the constant influence of James; see Haack 1982, Goodman 2002, Bakhurst and Misak 2017).

Concerning question (c), perhaps the main reason for this change in continuity and continuity in change was a more general one:

(c1) putting emphasis on human practice, activity, and human action in general (sometimes labeled as the "anthropological" or "pragmatist" standpoint) and putting emphasis on modeling as an activity rather than on models (even as dynamic) was merely a consequence of this change or a shift of aspect concerning his main philosophical interests (e.g., language, meaning, logic, grammar, method, etc.).

However, whether (c1) really is the case depends on various interpretations of the whole opus, and these include very complicated arguments which could eventually create a vicious circle.

More promising reasons may include a more particular move:

(c2) his shift not only in understanding the scientific practices of his time (his first–hand scientific experience included) and the realities of philosophy of science, but also in his inner reflexive remodeling of the very concept of scientific model that he adopted in his early philosophy.

Possibility (c2) is quite hard to analyze with sufficient precision, since his perhaps most promising early remarks on "models" and later ones on the "activity of modeling" are not clear. In other words, he seemed to use these concepts not only interchangeably but he had also used them earlier as sufficiently clear and everyday terms (which they were not; neither generally speaking in philosophy, nor in mainstream philosophy of science during the 1930s and 1940s).

It can also be suggested:

(c3) that the concepts of models and modeling were introduced into his philosophy and into philosophy of science slowly, gradually, and indirectly via similar concepts of metaphor, analogy, paradigm, pattern, and the like (from the 1950s onwards), and therefore it is not clear why he was using models and modeling so lightly and commonly, as if everyone should understand them directly and clearly.

Concerning (c3), the analogy between the pair "model–modeling" and the pair "picture–picturing" (representing, drawing, molding, painting and similar) can help, but then the analogy between these concepts, their uses, and applications in various examples should be established, which is not easy.

Whichever path of explication one chooses, and apart from (c3) compared to (c1) and (c2), due to the scarcity of Wittgenstein's remarks, one needs to have an established clear connection with the most dominant concept under this topic, which is the concept of picture. Possibilities (c1) to (c3) may have different goals because (c1) and (c2) interpret the actual difference, if there is such, between model and modeling, while (c3) tends to interpret the presupposed use of both concepts in relation to much more used concepts such as picture. Therefore, they are not inconsistent.

4. *Models and modeling in TLP, PI and RFM*

There is a section in PI which could be interpreted in a way that it could serve as the background and a "rough ground" of our hypothesis that there is a turn or expansion from models to modeling in Wittgenstein's philosophy. In PI 108 Wittgenstein finishes the section by saying the following:

> "The preconception [*Vorurteil*] of crystalline purity [*the idea of TLP*] can only be removed by turning our whole inquiry around [*drechen*]. (One might say: the inquiry must be turned around [*gedrecht werden*], but on the pivotal [*Angelpunkt*] of our real need." (PI 108)

Now, *Angelpunkt* is translated as *pivot* (in the 4th edition of PI), which means central point, main point, or axis. However, *Angelpunkt* is a crucial point in terms of axis, but also the point of rotation, since *Angeln* are *hinges*. *Anglepunkt* is a technical term in mechanics. Namely, *Angelpunkt* is a fixed point around which a solid body can rotate under the effect of forces, and this is called a pivot point or point of rotation. When a body is observed in a three–dimensional space, the direction of rotational movement through a rotational axis can be indicated, and with the indication of the pivot point, the movement is restricted to rotations whose rotation axes extend through this point.

Given that the translation in PI is correct by emphasizing "the point of rotation" or "axis" rather than what the actual metaphor is in terms of, for instance, "hinges", or for that matter, a gyroscope or a planet, one can suppose that the axis is "our real need" and our everyday life with its "friction" and "rough ground" (PI 107). If PI 108 can be compared to OC 152, it is interesting that the concept of our everyday language and life is not real in itself; it is not a precondition of rotation of our everyday activities (or a model as in TLP), but rather it is an invisible axis that is created by the movement of everyday language–games and forms of life around it (or modeling as in PG, PI, and RFM).

Baker and Hacker suggest that "rotation" around *"Angelpunkt"* is a kind of "relocation" of the normative role of logic "within the practice of the use of ordinary words and sentences", and some, like Malcolm, suggested that it is about turning around of the whole point of view (Baker and Hacker, Vol. 1, Part II, 2nd ed. 2009:240–5). This turning around the *Angelpunkt* as turning around an axis, rather than hinges, is more probable if Wittgenstein used a mechanical technical term as a metaphor for this change (something like the aspect–change in case of a Necker's cube or the Duck–rabbit head).

An additional thing is that this "crystalline purity" is the purity of "logic" of TLP that was aiming at "seeing right into [*durchschauen*] phenomena" (PI 90) and that was wrong, while turning this whole (TLP) examination around

is what we should aim at, namely at "surveyability [Übersichtlichkeit] by the process of ordering" phenomena (PI 92, 122). Instead of a "logical analysis" of TLP, which was formal, we have turned around the whole examination and now we have a morphological method which results in a "surveyable representation" of PI which is grammatical. This section can be used in order to provide at least some context concerning the more obvious model/modeling sections.

Given that the scientific influence on Wittgenstein concerning models and modeling is sufficiently clear for the present purposes and that his influence on later philosophies of science is clear as well, and although much less is explicated in literature due to various causes, the explication of at least some of his uses of models and modeling is an exegetical procedure that needs to be done.

Here, three quotes are chosen from a long list (as shown in Table 3). These are chosen because of the following reasons: they are taken from his major works of early and later philosophy (TLP and PI) and from an important work that deals with mathematics, in which one would expect the most clear use of models and modeling; all of them contain both the concepts of models and/or modeling and the concept of picture, which is the most dominant among similar important concepts; they contain other similar concepts; and finally and most importantly, they at least suggest a difference between models and modeling.

Other references from other works do that only in part and much more weakly than the quoted ones. So, let us quote three instances from Wittgenstein's works (TLP, PI, and RFM) in which he uses "model" and relates it closely to "picture" (as shown in Table 4.1, concerning reviewers' objection to the factual/normative distinction between models and modeling, it should be mentioned that Wittgenstein never explicitly stated such distinction; however, connections to the concepts of picture, paradigms and the like are not necessary for the present exegesis, but they are still important if one approaches the same issue from Wittgenstein's point of view on the concept of picture).

Table 4.1:
Wittgenstein on models and modeling in TLP, PI, and RFM.
Tablica 4.1:
Wittgenstein o modelima i modeliranju u TLP, PI i RFM.

Model as a preconception that corresponds to	Criticism of model as a preconception, and toward model as an application	The pattern of a model (paradigm) is shown by the result of the procedure of modeling (an aspect–change as remodeling)
TLP 2.12, 4.01, 4.463	**PI 131, 141**	**RFM III 22, 41; IV 12, 31, 50**
"A Picture is a model of reality." / "A proposition is a picture of reality. A proposition is a model of reality as we imagine it." / "A proposition, a picture, or a model is, in the negative sense, like a solid body that restricts a freedom of movement of others, and, in the positive sense, like a space bounded by a solid substance in which there is room for a body."	"For we can avoid unfairness or vacuity in our assertions only by presenting the model (*das Vorbild*) as what it is, as an object of comparison — as a sort of yardstick; not as a preconception to which reality must correspond. (The dogmatism into which we fall easily in doing philosophy.)" / "Now evidently we accept two different kinds of criteria for this: on the one hand, the picture [a picture of a cube, 139–40] that he visualizes at some time or another; on the other, the application which — in the course of time — he makes of this image. (And isn't it obvious here that it is absolutely inessential that this picture be in his imagination, rather than in front of him as a drawing or a model (Modell); or again, as something that he himself constructs as a model?)"	"When I say "a proof is a picture" — it can be thought of as a cinematographic picture. We construct the proof once for all. A proof must of course have the character of a model. The proof (the pattern of the proof) shows us the result of a procedure (the construction)…" / "A proof is a new paradigm." / "We do not judge pictures, we judge by means of the pictures." / "In the course of the proof our way of seeing is changed — and it does not detract from this that it is connected with experience. Our way of seeing is remodeled." (*gemodelt, umgemodelt*) / "The reason why one really cannot say that one learns that formal proposition from experience is — that one only calls this experience when this process leads to this result. The experience meant consists as such of this process with this result. This is why it is more than experience: seeing a pattern."

Let us start with stating some evident similarities and dissimilarities. Obviously enough, in all three quotations there is a continuity in use of the quite frequently mentioned concept of "picture" and also the not so frequently mentioned concept of "model". However, there are differences.

(c3.1) Picture and model are taken as synonyms in the TLP quotation, regardless of whether there are two senses of picture and model, and in RFM "being pictured" and "being modeled" are taken as synonyms as well.

(c3.2) In the quotation from PI there is a novelty, namely a criticism of model as "a preconception to which reality must correspond", but it is not clear whether it is going toward the presupposed and criticized Augustinian concept of language and meaning, toward his own standpoint from TLP, or perhaps both? Perhaps the use of the concept of "imagination" in a positive sense in TLP, but in a negative sense in PI, suggests that the criticism is directed to the TLP concept of a "model". Contrary to model as "being presupposed", in PI there is model as "a yardstick", and contrary to the "visualization" (similar to "imagination") there is "an application" of model.

(c3.3) In the RFM quotation, "proof" is similar to "picture" and "model". However, here some characteristics of model as "application" seem to be explicitly stated. First of all, a proof is connected not just to a model but to a pattern and a paradigm as well (concepts which are essentially morphological). Second, a proof as a model, a pattern, and a paradigm is connected with "the course of the proof", i.e. "proving". One can say that by the action of "proving" our "way of seeing is changed", and the result of "proving" is that "our way of seeing is remodeled". "A change of our way of seeing" is perhaps connected to the concept of an aspect–change in the light of the idea that such experience, activity, or practice of proving leads to "seeing a pattern" (which again is a morphological concept).

The "application of a model" or modeling seems to be important in the following way. In PG Wittgenstein writes: "We also say: I understand the picture exactly, I could model it in clay" (PG I 7). It could be said that this means that if one understands a model, one can model it in some way; or that the ability to model something manifests that a person understands the model of it.

Later on, he draws a parallel between a proposition and a picture claiming that the possibility that a picture does something means that something is "capable of being painted", and "capable of being modeled" (PG I 82). These quotes from PG and the explications, if correct, seem to go along with RFM quotations (as quoted in Table 4.1) and with their explication in (c3.3).

Before offering some concluding remarks, let us turn to a section from his later works, namely from *Remarks on Color* (ROC I 32), which shows a continuous interest in these issues, and it reads as follows:

"Sentences (*Sätze*) are often used (*gebraucht*) on the borderline between logic and the empirical, so that their meaning changes back and forth and they count now as expressions of norms (*Norm*), now as expressions of experience (*Erfahrung*). (For it is certainly not an accompanying mental phenomenon — this is how we imagine ‚thoughts' — but the use (*Verwendung*), which distinguishes the logical proposition from the empirical one.)" (ROC I 32, the metaphor of rotation and change in ROC I 32 compared with OC 94–99 and OC 152 is showed in Table 4.2).

(What should be emphasized here, according to a reviewer's objection, is that propositions and remarks are not the same, meaning that propositions are mostly logical constructs, while remarks are either experiential or grammatical. The discussion on OC 95–9 was made by authors elsewhere).

Table 4.2:
Similarities and differences between the various conceptions of axis remarks.
Tablica 4.2:
Sličnosti i razlike između raznih koncepcija napomena osi vrtnje.

ROC I 32	OC 94-99	OC 152
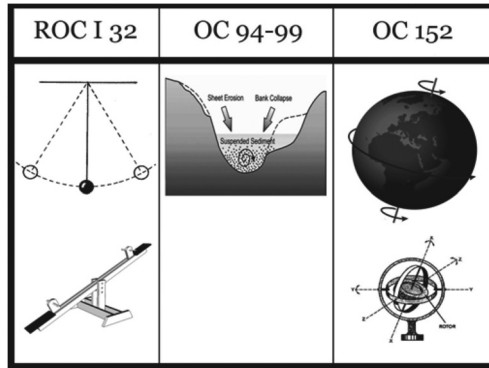
There is the logical and the empirical, and they are separate; they share a border. Sentences are not rarely but often used on this borderline. Moreover, the same sentence can be used on both sides of the border and change its position back and forth many times (changing an aspect perhaps). The same sentence can be considered at one point as an expression of experience, and at another as an expression of a norm. This is so because the use of a sentence determines whether it is an expression of a norm or an expression of an experience.

An example of axis remarks: Take the following sentence: (1) "Squirrels (don't) grow on trees." Sentence (1) can be a result of an observation of squirrels on trees compared with an observation of acorns on trees. They are both on trees. So, (1) can be similar to sentence (2) "Acorns grow on trees in Akron, Ohio." and (3) "Squirrels are on trees as well as acorns." In this respect, (1) swings to being an expression of experience; let us call it (1a). However, sentence (1) can be attested by comparison with various norms, such as (4) "Squirrels are animals, not plants.", and (5) "Acorns are parts of plants, not of animals." These sentences are expressions of norms, and compared with them (1) can be negated, therefore one gets the sentence "Squirrels don't grow on trees."; let us call it (1b). Here, (1b) is closer to an expression of a norm, than of an experience. This example is perhaps too obvious because there are obvious norms (4 and 5), and obvious experiences (2 and 3), and the sentence (1) can obviously be used as both (1a and 1b). However, what about sentences such as: "Caterpillars die on trees.", or "Imaginal cells are neither caterpillars nor butterflies."? These pose not just a scientific, but a conceptual issue as well. Here we have obvious norms such as

(6) "Caterpillars and butterflies are animals, not plants." or (7) "An imaginal cell is an animal, not a plant cell.", and obvious experiences such as (8) "Caterpillars die on trees.", or (9) "Butterflies grow on trees." Here the sentence (10) "Imaginal cells are modeling caterpillars into butterflies." is obviously not enough, neither it is a norm, nor an experience, but it can be used as both. Before the moment at which imaginal cells start to act together, they are acting individually and are attacked by a caterpillar immune system, but when they start to act together, they are starting to model butterfly organs from destroyed caterpillar organs. So, during the process of observation and reasoning, the sentence (10) can be used both as an expression of a norm and as a description of an experience.

Therefore, ROC I 32 can be seen as a more grammatical remark of which OC 94–99 is a nice metaphor that perhaps makes the idea even more precise: the "river sand" has a dual function, that is, it can be eroded and sedimented and thus it can change not only the river banks (norms), but also the river flow (experiences), or it can function as both. OC 152 offers something new, because such sentences of dual nature and of dual use function as an axis (not as hinges, which are grammatical sentences) on which our world–view rotates, and what is more, the immovability of the axis is not primarily the precondition of rotation, but the rotation is what creates or results in the axis being immovable.

Concluding remarks: Wittgenstein's modeling and contemporary philosophy of scientific models

If what was previously said, especially in (c3.3), is correct in terms of explication, then two following elements can be emphasized:

> (d1) The first is the continuity of importance of pictures and visual images in general, here obviously connected to models.

> (d2) The second is discontinuity, because it seems that the TLP concept of model, applied under the influences of Hertz and Boltzmann, is more static, and is taken as a prerequisite of isomorphism, while the PI and RFM concept of model, applied under Hertz and Boltzmann's influence, but also under Goethe and Spengler's, is more dynamic, practical, and taken not as a prerequisite, but rather as essentially manifested by the activity of modeling.
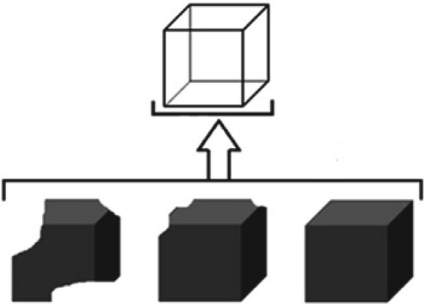
The second element in (d2) can be graphically represented as follows (as shown in Table 5).

Table 5:
A possible difference between understanding models and modeling in TLP
and in PI, RFM, and related Wittgenstein's works.
Tablica 5:
Moguća razlika između razumijevanja modela i modeliranja u TLP i u PI,
RFM i srodnim radovima.

If the interpretation (d2) based on (c3.3) is plausible, it should help us to understand Wittgenstein's reasons for the change in treatment of pictures and models. First of all, there is no clear reason to consider this change from TLP and PI/RFM standpoints as a radical one in terms of some kind of "model vs. modeling" dilemma.

(e) What seems to be more plausible is that the TLP concept of a model is being incorporated in the wider and more practical, perhaps pragmatic, concept of modeling (as the application of a model) in PI/RFM, since a model lives only in its application, as meaning lives only in its use.

Perhaps the TLP concept of a model is just a chapter in the PI/RFM concept of modeling as TLP logic is a chapter in PG/PI philosophical grammar.

(f) Second, if this change in continuity is a plausible interpretation, it seems to be consistent with the more general change in his philosophy from TLP to PI standpoints, i.e., the questions remained the same, however, the answers differed.

These sections as they are interpreted fit in neatly within a broader context of Wittgenstein's general shift from formal and logical to practical and grammatical aspects of his approach. More about this issue cannot be claimed based solely on quoted sections.

A more detailed analysis of these and similar sections would be helpful as well as a connection of these with analyses of other quotations and similar concepts, most notably pictures, patterns, and paradigms. Finally, it is difficult to say how this shift can help contemporary philosophy of science that discusses scientific models and modeling. For instance, contemporary philosophy of science of models discusses the question "How does a model represent?", and Wittgenstein would probably call this a metaphysical mumbo–jumbo, since a model "is" a representation, and modeling is the "activity" of representing.

However, perhaps Wittgenstein's emphasis on the practice and application of a model, i.e. modeling, which essentially "manifests models" or, we can say, "a modeling–action", and which results with "aspect change" by "seeing a pattern or a paradigm", is the direction in which future research should go. In terms of further research, the difference in understanding models and modeling as presented in Wittgenstein's works (as shown in Table 5) can perhaps help in contemporary debates concerning models and modeling as fictional or non–fictional entities in sciences, particularly concerning the following issue mentioned by R. Frigg and S. Hartmann in their text "Models in Science" (we could not go into the issue of the use of fiction in Wittgenstein's modeling, as suggested by an anonymous reviewer, but we recognize the importance of the issue).

> "The issue of how to understand fictions in science has been the subject matter of a recent debate in the philosophy of modeling. Barberousse and Ludwig (2009), Contessa (2010), Frigg (2010a, 2010b), Godfrey–Smith (2006, 2009), Leng (2010) and Toon (2010) develop views that see models as fictions of some kind. Giere (2009) denies that his earlier work should be understood in this way and argues against viewing models as fictions. Magnani (2012), Pincock (2012, Ch. 4) and Teller (2009) second Giere's anti–fictionalism and argue that models should not be regarded as fictions. Weisberg (2012) argues for a middle position which sees models as playing a heuristic role but denies that they form part of a scientific model." (Frigg and Hartmann 2012)

In Wittgenstein's terms models can be understood as fictional epiphenomena that supervene the actual or real activity of modeling, which manifests them simply by being performed.

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Sažetak

WITTGENSTEINOV OBRAT OD MODELA K MODELIRANJU

KRISTIJAN KRKAČ, JOSIP LUKIN

U tekstu izlažemo rješenje razlike između Wittgensteinove uporabe modela u TLP i modeliranja u PI, RFM i OC temeljem analize razlike u izboru samih riječi i opisa pojmova fenomena u njegovim napomenama. Metoda je pojmovna analiza Wittgensteinova tretiranja znanstvenih modela i modeliranja. Dodatno, tekstualna raščlamba i poredba korištena je pri istraživanju njegovih glavnih napomena na tu temu. Ako je otkrivena razlika između modela i modeliranja i kontinuitet ovisnosti oba pojma o pojmu slike, pri čemu je razlika slike i oslikavanja uzeta kao zadana, zaključujemo kako je Wittgenstein svoj stari pojam modela nastojao inkorporirati u novi pojam modeliranja. Eksplicirana razlika i inkorporiranje pridonosi pojašnjenju teme Wittgensteinovog posrednog i neposrednog utjecaja na filozofiju znanosti (napose Toulmina, Hansona, Kuhna i Feyerabenda).

Ključne riječi: model, modeliranje, filozofija znanosti, Filozofska istraživanja, znanost, TLP, Wittgenstein

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