GEOFIZIKA VOL. 24 NO. 1 2007

Original scientific paper
UDC 551.511.3

Analytical model of equatorial waves with CAPE and moisture closure

Željka Fuchs

Physics Department and Geophysical Research Center, New Mexico Institute of Mining and Technology, Socorro, New Mexico, USA

Received 23 February 2007, in final form 11 April 2007

The convective available potential energy (CAPE) closure and the moisture closure is implemented on an analytical linearized model for large-scale motions. The model includes cloud-radiation interaction (CRI), gross moist stability and wind-induced surface heat exchange (WISHE). The model is done in an equatorial non-rotating atmosphere and is vertically resolved.

As the gravity waves in non-rotating atmosphere map to Kelvin waves in rotating atmosphere, the modeled modes are fast Kelvin waves that resemble adiabatic modes, convectively coupled Kelvin modes that are damped and move with the observed phase speed of 17 ms^{-1} and the unstable slow moisture mode. The slow moisture mode owes its propagation speed to WISHE and instability to CRI and gross moist instability. It is thought that it can be related to the easterly waves and perhaps even the Madden-Julian oscillation (MJO).

Keywords: Kelvin waves, moisture mode

1. Introduction

A majority of the analytical models (Emanuel, 1987; Neelin et al., 1987; Neelin and Yu, 1994; Majda et al., 2004; Khouider and Majda, 2006; Fuchs and Marki, 2007 etc.) describing the interactions between large-scale motions and deep convection in the tropics use the convective available potential energy (CAPE) closure. The CAPE closure implies that the vertically integrated convective heating is proportional to CAPE.

Fuchs and Raymond (2002) developed a simple analytical model that in addition to CAPE closure had the moisture closure. The moisture closure implies that more moisture there is more precipitation is produced. As a consequence of the moisture closure an interesting unstable mode appeared that was called the moisture mode (Sobel and Hourinachi, 2000). The model was not vertically resolved and thus failed to produce convectively coupled Kelvin waves.

Using only the moisture closure, Fuchs and Raymond (2007) developed a vertically resolved that in addition to slow moisture mode modeled the convectively coupled Kelvin waves of the observed phase speed (Straub and Kiladis, 2002). However, the modeled convectively coupled Kelvin waves were not unstable what is in disagreement with the observations show.

Fuchs and Marki (2007) used the CAPE closure on a vertically resolved model of Fuchs and Raymond (2007). The modeled convectively coupled Kelvin waves had the observed phase speeds, but were also stable.

The purpose of this paper is to combine the convective closure that is mainly used today, i.e. the CAPE closure with the moisture closure on a vertically resolved model. The motivation for CAPE closure has been discussed in Fuchs and Marki (2007) while he motivation for the moisture closure is the following:

1. Using the daily-mean sounding data averaged over the five KWAJEX (Kwajalein Experiment) locations, Sobel et al. (2004) show that the correlation between CAPE and precipitation is weak and negative while the correlation between the relative humidity and precipitation is positive. Bretherton et al. (2004) show a strong correlation and almost no phase lag between the actual precipitation and the precipitation predicted from the relative humidity. They suggest that the schemes such as Betts-Miller (1986) should use a moisture adjustment time of 12 h rather than 1 – 2 h.

2. Sobel et al. (2001) explored the shallow-water equations under the weak temperature gradient (WTG) approximation where they assume that the vertical structure of the temperature is confined to a single profile associated with deep convection. The convective heating is controlled by a moisture variable advected by the flow. The system of equations is balanced by the temperature equation rather than the momentum equation (geostrophic balance) because of the WTG approximation. The consequence is that there are no gravity modes as they are assumed to propagate rapidly. The mode that the authors found of particular interest was the eastward propagating mode on f plane. It is propagating eastward for the moisture decreasing poleward in the background state. That mode is unstable for low wavenumbers and it arises from the model irrespective of WTG approximation. In their derivation, Sobel et al. (2001) use the quantity that controls the difference between the precipitation and the moisture convergence that is essentially Neelin and Held's (1987) gross moist stability (GMS). They do not consider the case when GMS is unstable, but from their dispersion relation it is apparent that if GMS is negative, the eastward propagating mode would be unstable for all wavenumbers. Sobel and Bretherton (2003) simulated numerically a similar mode to the one of Sobel et al. (2001) and called it the stationary moisture mode.

3. Using the data from Tropical Ocean Global Atmsophere Coupled Ocean-Atmosphere Response Experiment (TOGA COARE) Carrillo and Raymond (2005) calculate the reduced Bernoulli function that is essentially just the negative of the nominator in Neelin and Held's GMS (the minor differences

between the two are irrelevant for this paper). They show that the Bernoulli function is positive (GMS negative) when the spatially averaged infrared temperature is dominated by the shallower clouds and negative when it is dominated by deeper clouds, thus moistening and drying the atmosphere respectively.

The moisture closure (Fuchs and Raymond, 2002, 2007) implies that the precipitation rate increases linearly with the relative humidity and that the time needed to relax the moisture profile to equilibrium is one day. It reproduces the moisture mode of Sobel et al. (2001) and Sobel and Bretherton (2003). The CAPE closure (Fuchs and Marki, 2007) implies that the increased CAPE, represented by decreased midlevel potential temperature results in increased precipitation.

The model in this paper incorporates the assumptions of Fuchs and Raymond (2002, 2007) and Fuchs and Marki (2007). It thus incorporates both, the CAPE closure and the moist convection (moisture closure), as well as cloud-radiation interactions (CRI), the gross moist stability and wind-induced surface heat exchange (WISHE). It is restricted to a non-rotating atmosphere for simplicity, but it is vertically resolved. As the gravity modes in non-rotating atmosphere map into Kelvin modes in rotating atmosphere, the gravity modes are called the Kelvin modes in this paper. The model calculates the free Kelvin modes, convectively coupled Kelvin modes of the observed speed 17 ms^{-1} that are damped, and the moisture mode. It gives us a better understanding of Kelvin waves as well as the other tropical disturbances such as the Madden-Julian oscillation (MJO) or easterly waves as they might be connected to the moisture mode.

2. Model

The model consists of six linearized governing equations for large scale motions in the non-rotating atmosphere (the horizontal momentum equation, the hydrostatic equation, the continuity equation, the buoyancy equation, the moisture equation and the moist entropy equation):

$$\frac{\partial u}{\partial t} + \frac{\partial \Pi}{\partial x} = 0 \tag{1}$$

$$\frac{\partial \Pi}{\partial z} - b = 0 \tag{2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

$$\frac{\partial b}{\partial t} + \Gamma_B w = S_B \tag{4}$$

$$\frac{\partial q}{\partial t} + \Gamma_Q w = S_Q \tag{5}$$

$$\frac{\partial e}{\partial t} + \Gamma_E w = S_E \tag{6}$$

where u, Π, w, b, q, e, Γ_B, Γ_Q, Γ_E, S_B, S_Q, S_E are given in table 1. All the variables are perturbations.

As in Fuchs and Raymond (2002, 2007) I assume that the vertically integrated dry entropy source term, B, depends on scaled perturbation in precipitation rate, P, minus the vertically integrated radiative cooling rate, R:

$$B = \int_0^h S_B(z)dz = P - R \tag{7}$$

The integrated moisture source term, Q, depends on surface evaporation rate, E, minus the precipitation rate:

$$Q = \int_0^h S_Q(z)dz = E - P \tag{8}$$

The integrated moist entropy source term, Ξ, depends on evaporation rate minus the radiative cooling rate:

$$\Xi = \int_0^h S_E(z)dz = E - R \tag{9}$$

h is the depth of the troposphere. Precipitation rate P is parametrized as:

$$P = \alpha\int_0^h q(z)dz - \eta\int_0^h b dz \tag{10}$$

where α is moisture relaxation rate taken as 1 day^{-1}, and η is the buoyancy relaxation rate. This equation implements two very important physical mechanisms and combines the two closures from Fuchs and Raymond (2007) and Fuchs and Marki (2007). The first term tells us that with more moisture there is more precipitation; that is called the moisture closure. The second term represents CAPE and tells us that increased CAPE, represented by decreased midlevel potential temperature, results in increased precipitation; that is called the CAPE closure. Radiative cooling rate is:

$$R = -\alpha\varepsilon \int_0^h q(z)dz \tag{11}$$

where ε is the cloud-radiative feedback parameter taken as $\varepsilon \approx 0.2$ (see Fuchs and Raymond, 2002) and surface evaporation rate is:

$$E = C\mu u_s \Delta q \tag{12}$$

Table 1. Parameters and their values

Parameter	Expression	Value/dimensions
horizontal velocity	u	[ms^{-1}]
Exner function	$\tilde{\Pi}$	m^{2}s^{-2}K^{-1}
Exner function*mean potential temperature	$\Pi = \theta_0\tilde{\Pi}$	[m^{2}s^{-2}]
vertical velocity	w	[ms^{-1}]
scaled dry entropy	$b = gs_d / C_p$	[ms^{-2}]
scaled mixing ratio	$q = gLr / C_pT_R$	[ms^{-2}]
scaled moist entropy	$e = gs/C_p$	[ms^{-2}]
Brunt-Väisälä frequency	$\Gamma_B = \frac{g}{C_p}\frac{ds_{d0}}{dz}$	[s^{-2}]
	$\Gamma_Q = \frac{gL}{C_pT_R}\frac{dr_{t0}}{dz}$	[s^{-2}]
moist static stability	$\Gamma_E = \frac{g}{C_p}\frac{ds_0}{dz}$	[s^{-2}]
scaled dry entropy source	$S_B = \frac{g}{C_pT_R}Q$	[ms^{-3}]
scaled moisture source	$S_Q = -\frac{gL}{C_pT_R}P_r$	[ms^{-3}]
scaled moist entropy source	$S_E = \frac{g}{C_pT_R}Q_R - \frac{g}{C_p}\frac{\partial F_e}{\partial z}$	[ms^{-3}]
acceleration of gravity	g	[9.81 ms^{-2}]
specific heat of air at constant pressure	C_p	1005 Jkg^{-1}K^{-1}
vertical wavenumber	$m = k\Gamma_B^{1/2} / \omega$	[m^{-1}]
first baroclinic vertical wavenumber	$m_0 = \pi / h$	[m^{-1}]
depth of the troposphere	h	15 km
horizontal wavenumber	k	[m^{-1}]
frequency	ω	[s^{-1}]
dimensionless phase speed	$\Phi = \Omega / \kappa$	calculated
dimensionless frequency	$\Omega = \omega / \alpha$	calculated
dimensionless horizontal wavenumber	$\kappa = k\Gamma_B^{1/2} / \alpha m$	from 1.3 till 15
planetary zonal wavenumber	$l = 2\pi / 40000$	[km^{-1}]
moisture relaxation rate	α	1/day
buoyancy relaxation rate	η	1/day
nondimensional CAPE parameter	$\gamma = \eta / \alpha$	
cloud-radiative feedback parameter	ε	0.2
WISHE parameter	$\delta = mC\mu\Delta q / 2$	-5×10^{-8}s^{-2}
dimensionless WISHE parameter	$\Lambda = \delta(1 + \varepsilon) / \alpha\Gamma_B^{1/2}$	–0.4
scaled minimum moist entropy height	$H = d / h$	0.25, 0.5, 0.75
scaled moist entropy difference	$\Delta e = \Delta e_0 / (h\Gamma_B)$	0.13, 0.26, 0.39
dimensionless gross moist stability	$\Gamma_M = \frac{\int_0^h \Gamma_E(z)w(z)dz}{\Gamma_B\int_0^h w(z)dz}$	

where C is the transfer coefficient, $\mu = -u_0 / u_{eff}$ is the negative ratio between the velocities of the ocean relative to the ambient air and the effective wind. For strong ambient easterly winds that are chosen in this linearized parameterization of WISHE: $\mu = -1$. The parameter u_s is the surface wind velocity and Δq is the scaled difference between the saturation mixing ratio at the sea surface temperature and the subcloud mixing ratio. The surface sensible heat fluxes are ignored as they are small compared to the latent heat fluxes over the tropical oceans.

To solve the governing system of equations (1) – (6), the vertical velocity profile from Fuchs and Raymond (2007) is used:

$$w(z) = \frac{Bm_0}{2\Gamma_B(1-\Phi^2)}\left[\sin(m_0 z) + \Phi\exp(-i\frac{\pi}{\Phi})\sin(mz)\right] \tag{13}$$

where $m_0 = \pi / h$. $m = k\Gamma_B{}^{1/2} / \omega$ where k is the horizontal wavenumber and w the frequency. The nondimensionalized phase speed is then $\Phi = m_0 / m$ or $\Phi = \Omega / \kappa$ where the nondimensionalized horizontal wavenumber is $\kappa = k\Gamma_B{}^{1/2} / \alpha m_0$ and the nondimensionalized frequency is $\Omega = \omega / \alpha$. For more details on parameters see table 1. To obtain (13) Fuchs and Raymond (2007) assumed that the heating vertical profile has the structure of the first baroclinic mode:

$$S_B(z) = 0.5Bm_0 \sin(m_0 z) \tag{14}$$

where the vertically integrated heating: B is an unknown (note that the integral of (14) through the troposphere indeed results in B). The equation (14) will further be used in solving the governing system of equations and obtaining the dispersion relation.

3. Calculating the dispersion relation

By assuming that all the variables in the governing system of equations (1) – (6) are proportional to $\exp[i(kx - \omega t)]$, it is straightforward to write the polarization relations for the governing system of equations. To obtain the dispersion relation I will need to combine the polarization relations and the thermodynamic assumptions of the model as a function of the unknown B, which will ultimately cancel out of each term. I first combine the equations (7), (10) and (11):

$$B = \int_0^h S_B(z)dz = P - R = \alpha(1 + \varepsilon)\int_0^h q(z)dz - \eta\int_0^h b(z)dz \tag{15}$$

The scaled moist entropy perturbation, e, can be written as $e = b + q$ (Fuchs and Raymond, 2002). The vertically integrated heating can then be written as

$$B = \alpha(1 + \varepsilon)\int_0^h e(z)dz - \alpha(1+\varepsilon+\gamma)\int_0^h b(z)dz \tag{16}$$

where the nondimensional CAPE parameter γ is: $\gamma = \eta / \alpha$. It is straightforward to calculate the vertical integral of the scaled entropy perturbation, b, from the polarization relation for buoyancy (i.e. equation 4)

$$\int_0^h b(z)dz = \frac{i}{\omega}\left[\int_0^h S_B(z)dz - \Gamma_B \int_0^h w(z)dz\right] \tag{17}$$

The vertical velocity perturbation is given by (13). Its integral is then:

$$\int_0^h w(z)dz = \frac{B}{\Gamma_B(1-\Phi^2)}\left\{1 + \frac{\Phi^2}{2}\exp\left(-i\frac{\pi}{\Phi}\right)\left[1-\cos\left(\frac{\pi}{\Phi}\right)\right]\right\} \tag{18}$$

The vertical integral of the moist entropy perturbation is not that obvious. First the moist entropy equation (6) needs to be integrated where the right-hand-side comes from equation (9):

$$\frac{\partial}{\partial t}\int_0^h e(z)dz + \int \Gamma_E(z)w(z)dz = E - R \tag{19}$$

Noting the expressions (11) and (12) for radiative cooling rate and the surface evaporation rate:

$$\int_0^h e(z)dz = \frac{1}{\alpha\varepsilon + iw}\left[\int_0^h \Gamma_E(z)w(z)dz + \alpha\varepsilon\int_0^h b(z)dz - C\mu u_s \Delta q\right] \tag{20}$$

where:

$$u_s = \frac{i}{\kappa}\left(\frac{\partial w}{\partial z}\right)_{z=0} = i\frac{m_0^2 B}{2k\Gamma_B(1-\Phi^2)}\left[1+\exp\left(-i\frac{\pi}{\Phi}\right)\right]. \tag{21}$$

$C\mu u_s \Delta q$ is the WISHE term. The integral in height of vertical velocity perturbation multiplied by moist static stability, which is a function of height, is related to the gross moist stability of Neelin and Held (1987), and it comes from the moist entropy equation. Fuchs and Raymond (2007) took it as the unknown and varied the vertical profiles of $\Gamma_E(z)$. To understand the nondimensional parameters in dispersion relation, the following is a review of the gross moist stability parametrization of Fuchs and Raymond (2007).

The nondimensionalized gross moist stability Γ_M is defined as:

$$\Gamma_M = \frac{\int_0^h \Gamma_E(z)w(z)dz}{\Gamma_B \int_0^h w(z)dz} \tag{22}$$

where

$$\Gamma_E(z) = \frac{g}{C_p}\frac{ds_0(z)}{dz} = \frac{de_0(z)}{dz} \tag{23}$$

The quantity s_0 is the mean moist entropy profile and e_0 is the scaled moist entropy profile. To estimate the value of Γ_M it is assumed that the scaled mean moist entropy e_0, obeys the following relations:

$$\Gamma_{E1} = -\frac{\Delta e_0}{d} \qquad 0 < z < d \tag{24}$$

$$\Gamma_{E2} = \frac{\Delta e_0}{h-d} \qquad h > z > d \tag{25}$$

where the moist entropy takes the same value at the top of the troposphere as at the surface.

It is now possible to write the last unknown integral needed for dispersion relation:

$$\int_0^h \Gamma_E(z) w(z) dz = \frac{B\Delta e}{(1-\Phi^2)H(1-H)}\left\{H - \frac{1}{2}[1-\cos(\pi H)]\right\} +$$

$$+ \frac{B\Delta e\,\Phi^2}{2(1-\Phi^2)H(1-H)}\exp(-i\frac{\pi}{\Phi})\left\{H\left[1-\cos\left(\frac{\pi}{\Phi}\right)\right] - \left[1-\cos\left(\frac{\pi}{\Phi}H\right)\right]\right\} \tag{26}$$

where $\Delta e = \Delta e_0 / h\Gamma_B$ is the nondimensionalized scaled moist entropy difference and $H = d / h$ is the nondimensionalized minimum entropy height.

Plugging the equation (18) into (17) gives the integral of the scaled entropy perturbation b. Plugging the equations (26), (21) and (17) into (20) gives the integral of the scaled moist entropy perturbation e. Such obtained integrals for b and e are then plugged into (16) resulting in the dispersion relation for moist convection with gross moist stability, CRI, WISHE and CAPE:

$$\kappa\Phi^3 + i\Phi^2 - \kappa\Phi + i\varepsilon +$$

$$+ 0.5i(1+\varepsilon)\Phi^2 \exp\left(-i\frac{\pi}{\Phi}\right)\left[1-\cos\left(\frac{\pi}{\Phi}\right)\right] - \frac{\Lambda}{\kappa}\left[1+\exp\left(-i\frac{\pi}{\Phi}\right)\right] +$$

$$+ \frac{\gamma\Phi(\varepsilon + i\kappa\Phi)}{\kappa}\left\{1 + 0.5\exp\left(-i\frac{\pi}{\Phi}\right)\left[1-\cos\left(\frac{\pi}{\Phi}\right)\right]\right\} - i\varphi\left\{H - \frac{1}{2}[1-\cos(\pi H)]\right\} -$$

$$- i\varphi\frac{\Phi^2}{2}\exp\left(-i\frac{\pi}{\Phi}\right)\left\{H\left[1-\cos\left(\frac{\pi}{\Phi}\right)\right] - \left[1-\cos\left(\frac{\pi}{\Phi}H\right)\right]\right\} = 0 \tag{27}$$

where $\Delta e(1 + \varepsilon) / H(1 - H) = \varphi$, and ε is the cloud-radiative feedback parameter. $\Lambda = \delta(1 + \varepsilon) / (\alpha\Gamma_B{}^{1/2})$ is the WISHE parameter and $\delta = m_0 C\mu\Delta q / 2$. The parameters to vary are η^{-1} and Λ. The dispersion relation is solved numerically using Newton's method in Python.

4. Results

All the normal modes that come out of the dispersion relation (27) are now plotted in figure 1 for the case in which CRI and WISHE are included. The entropy profile parameters are $H = 0.5$ and $\Delta e = 0.26$ which corresponds to the mean moist entropy difference, Δs_0, of 40 Jkg^{-1}K^{-1} and a minimum moist entropy at half the height of the troposphere. The CAPE parameter $\gamma = 1$ which corresponds to $\eta^{-1} = 24$ hours. The nondimensional Φ, $\Phi = \Omega/\kappa$, that the dispersion relation is solved for, is a complex number where the real part of it corresponds to the nondimensional phase speed; if it is positive it shows the eastward propagation, negative westward. The imaginary part of Φ leads

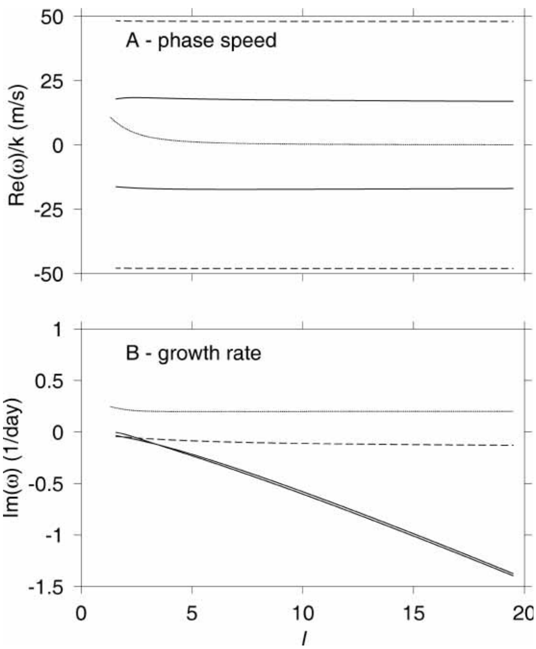

Figure 1. Dimensional dispersion curves as a function of the planetary wavenumber l when $\varepsilon = 0.2$, $\Lambda = -0.4$, $\Delta e = 0.26$, $H = 0.5$ and $\eta = 1$/day. Solid lines represent convectively coupled Kelvin modes, eastward and westward propagating; dashed lines represent fast Kelvin modes, eastward and westward propagating and dotted line represents the eastward propagating slow moisture mode. The upper panel shows the phase speeds of the modes, while the bottom panel shows the growth rates.

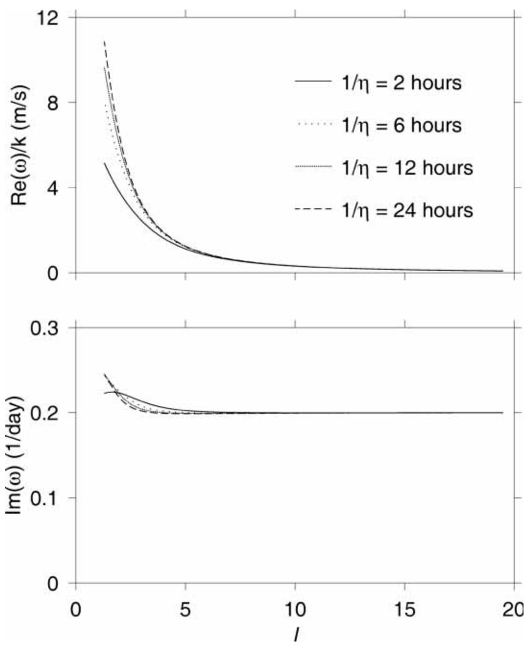

Figure 2. As in figure 1 but for the moisture mode as a function of planetary wavenumber l when the CAPE parameter $\gamma = \eta / \alpha$ is varied with four different values of buoyancy relaxation time η^{-1}.

to the growth rate of the mode as multiplied by κ it gives nondimensional frequency that can easily be converted to the real frequency ω. It is called the growth rate as the field variables' of the model have the exponential dependence on time: $\exp(-i\omega t)$; the positive imaginary part of the frequency gives the exponential growth of the mode and negative the decay. In figures 1, 2, 3 the upper panel shows the dispersion curves of the dimensional phase speeds and the bottom panel shows the growth rates. The phase speeds and the growth rates of the modes are plotted as a function of the planetary wavenumber l, where $l = 1$ corresponds to a wavelength equal to the circumference of the earth.

The modeled modes in figure 1 are the following:

1. The convectively coupled Kelvin modes, eastward and westward (note that there is no analogue to the westward Kelvin wave in the real atmosphere) that have the phase speeds of 17 ms^{-1}. Their phase speed varies slightly with the wavelength. The modes are damped regardless to the strong influence of the diabatic effects, moisture and CAPE closure.

2. The free Kelvin modes or the fast Kelvin modes, eastward and westward that have phase speeds of 48 ms^{-1} and decay at a very slow rate.

3. The slow moisture mode that moves eastward under the influence of WISHE and is the only unstable mode.

Figure 2 shows the moisture mode dependence on CAPE: as the buoyancy relaxation time η^{-1} shortens, the mode propagates slower. The influence on the growth rates is slight and only for the long wavelengths. The influence of CAPE on the convectively coupled Kelvin wave is small (Fuchs and Marki, 2007) and is not shown here.

Figure 3 shows interesting influence of WISHE on moisture mode: it makes it move eastward, while the growth rate is only slightly affected and only for the long wavelengths. The convectively coupled Kelvin mode is not affected much by WISHE.

The results compare well with Fuchs and Raymond (2007) and Fuchs and Marki (2007). The convectively coupled Kelvin wave is stable what shows that neither the CAPE closure nor the moisture closure are responsible for the observed instability. Furthermore the model shows that neither closure is responsible for the modeled phase speeds of the convectively coupled Kelvin waves, which agrees with the observations, but is a sole consequence of the system dynamics. The moisture mode is the addition to Fuchs and Marki model and it shows that the CAPE closure has little influence on it.

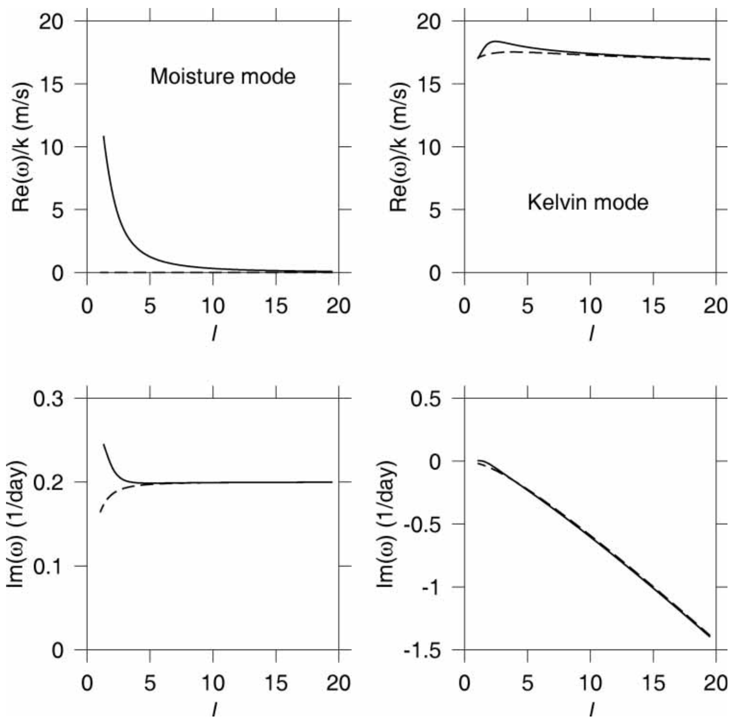

Figure 3. As in figure 1 but for the moisture mode and convectively coupled Kelvin mode with WISHE effect (solid lines) and without it (dashed lines) when η = 1/day.

5. Conclusions

A simple linearized model for large-scale disturbances in the tropical non-rotating atmosphere is presented. It includes cloud-radiation interactions (CRI), wind-induced surface heat exchange (WISHE), gross moist stability, sensitivity to CAPE and moist convection. The moist convection enters the model through a parameterization of precipitation rate that is assumed to be directly proportional to the precipitable water. This parameterization is called the moisture closure and it is implemented with a relaxation time chosen to be one day. The CAPE closure says that increased CAPE will indirectly cause increased precipitation via the decrease in midlevel tropospheric temperature.

The model is vertically resolved (Fuchs and Raymond, 2007) and assumes a first baroclinic mode structure for the vertical heating profile. The other variables' profiles are calculated with a radiation boundary condition. The calculated vertical velocity consists of two sinusoidal components with different vertical wavelengths. One corresponds to the imposed heating profile or deep convection component and the other to a shallow mode component that comes from satisfying the boundary conditions. The latter determines the phase speed of the convectively coupled Kelvin mode that agrees with observations (Straub and Kiladis, 2002). As the gravity modes in a non-rotating atmosphere map to Kelvin modes in the equatorial beta plane case, I identify the gravity mode of this model to be the Kelvin mode. The model shows that all the diabatic effects presented in the model have little or no influence on the phase speed of the convectively coupled Kelvin waves nor are they responsible for the observed instability of the Kelvin wave, perhaps due to lack of the convective inhibition.

The slow moisture mode is the same mode as that of Fuchs and Raymond (2002, 2007) and it comes out of the model as a consequence of the implemented moisture closure. It propagates eastward under the influence of WISHE as a direct result of the linear parameterization of WISHE in the mean ambient easterly flow, and is stationary without it. It is unstable because of the CRI effect and the gross moist instability. WISHE makes the moisture mode unstable only for a very long wavelengths. The disadvantage of the linear parameterization of WISHE is that the observations show an MJO-like prominent in the regions of mean surface westerlies. When combined with nonlinearity it can be speculated that the moisture mode could lead to the MJO as well as other slow-moving disturbances such as easterly waves. The temperature perturbations for the moisture mode are weak (Fuchs and Raymond, 2007) which suggests a relationship to the WTG mode (Sobel et al., 2001; Sobel and Bretherton, 2003). Neelin and Yu's (1994) propagating deep convective mode, that they compare to the moist Kelvin mode and the MJO, could perhaps also be compared to the moisture mode as WISHE makes it unstable in a rotating atmosphere without CRI and gross moist instability for planetary wavenumber one (Fuchs and Raymond, 2005). The main difference between Neelin and

Yu and this model is that they used the quasi-equilibrium assumption and obtained a different dispersion relation.

The presented model simultaneously captures the convectively coupled Kelvin waves of the observed phase speeds along with the slow moisture mode. The moisture mode did not exist in the vertically resolved model which only had the CAPE closure (Fuchs and Marki, 2007) and thus the moisture closure enables us to capture more of the physics in a still simple analytical model. The model further shows that no matter what diabatic closure is used, the phase speed of the convectively coupled Kelvin wave is intact as it is the sole consequence of the dynamics of the system. The damping of the convectively coupled Kelvin wave in a model that implements the moisture closure, the CAPE closure, CRI, WISHE and gross moist stability indicates that the full physics of the interaction between Kelvin waves and convection is not captured, but it does show which mechanisms are not responsible for it.

Acknowledgments – I thank the reviewers.

References

Betts, A. K., and Miller, M. J. (1986): A new convective adjustment scheme. Part II: Single column tests using GATE wave, BOMEX, ATEX and arctic air-mass data sets. *Q. J. Roy. Meteor. Soc.*, **112**, 693–709.

Bretherton, C. S., Peters, M. E. and Back, L. E. (2004): Relationships between water vapor path and precipitation over the tropical oceans. *J. Climate*, **17**, 1517–1528.

Carrillo, C. L. and Raymond, D. J. (2005): Moisture tendency equations in a tropical atmosphere. *J. Atmos. Sci.*, **62**, 1601–1613.

Emanuel, K. A. (1987): An air-sea interaction model of Intraseasonal Oscillations in the Tropics. *J. Atmos. Sci.*, **44**, 2324–2340.

Fuchs, Z. and Marki, A. (2007): Large-scale modes of the tropical atmosphere. Part II: Analytical modeling of Kelvin waves using the CAPE closure, *Geofizika*, (published in this issue).

Fuchs, Z. and Raymond, D. J. (2002): Large-scale modes of a nonrotating atmosphere with water vapor and cloud-radiation feedbacks. *J. Atmos. Sci.*, **59**, 1669–1679.

Fuchs, Z. and Raymond, D. J. (2005): Large-scale modes in a rotating atmosphere with radiative-convective instability and WISHE. *J. Atmos. Sci*., **62**, 4084–4094.

Fuchs, Z. and Raymond, D. J. (2007): A simple, vertically resolved model of tropical disturbances with a humidity closure. *Tellus*. (in press)

Khouider, B. and Majda, A. J. (2006): A simple multicloud parameterization for convectively coupled tropical waves. Past I: Linear Analysis. *J. Atmos. Sci.*, **63**, 1308–1323.

Majda, A. J., Khouider, B., Kiladis, G. N., Straub, K. H. and Shefter, M. G. (2004): A model for convectively coupled tropical waves: Nonlinearity, rotation, and comparison with observations. *J. Atmos. Sci.*, **61**, 2188–2205.

Neelin, J. D., Held, I. M. and Cook, K. H. (1987): Evaporation-wind feedback and low-frequency variability in the tropical atmosphere. *J. Atmos. Sci.,* **44**, 2341–2348.

Neelin, J. D. and Held, I. M. (1987): Modeling tropical convergence based on the moist static energy budget. *Mon. Weather. Rev.*, **115**, 3–12.

Neelin, J. D. and Yu, J.-Y. (1994): Modes of tropical variability under convective adjustment and the Madden-Julian oscillation. Part I: Analytical theory. *J. Atmos. Sci.*, **51**, 1876–1894.

Sobel, A. H. and Bretherton, C. S. (2003): Large-scale waves interacting with deep convection in idealized mesoscale model simulations. *Tellus*, **55A**, 45–60.

Sobel, A. H. and Horinouchi, T. (2000): On the dynamics of easterly waves, monsoon depressions, and tropical depression type disturbances. *J. Meteorol. Soc. Jpn.*, **78**, 167–173.

Sobel, A. H., Nilsson, J. and Polvani, L. M. (2001): The weak temperature gradient approximation and balanced tropical moisture waves. *J. Atmos. Sci*., **58**, 3650–3665.

Sobel, A. H., Yuter, S. E., Bretherton, C. S. and Kiladis, G. N. (2004): Large-scale meteorology and deep convection during TRMM KWAJEX. *Mon. Weather. Rev.*, **132**, 422–444.

Straub, K. H., and Kiladis, G. N. (2002): Observations of a convectively coupled Kelvin wave in the Eastern Pacific ITCZ. *J. Atmos. Sci.*, **59**, 30–53.

SAŽETAK

Analitički model ekvatorijalnih valova uz konvekcijsku raspoloživu potencijalnu energiju i vlagu

Željka Fuchs

Na analitički linearni model za dugoperiodičke valove implementirana je ovisnost oborina o konvekcijskoj raspoloživoj potencijalnoj energiji i vlazi kojom se promatra povezanost konvekcije i valova. Model također uključuje međudjelovanje oblaka i zračenja, osnovnu vlažnu nestabilnost i površinsku izmjenu topline uzrokovanu vjetrom. Model je primijenjen na ekvatorijalnoj ne-rotirajućoj atmosferi i uključuje ovisnost modeliranih polja po vertikali.

Težinski valovi u ne-rotirajućoj atmosferi su ekvivalentni Kelvinovim valovima u rotirajućoj atmosferi. Modelirani valovi su brzi stabilni adijabatski Kelvinovi valovi, stabilni Kelvinovi valovi povezani s konvekcijom koji propagiraju faznom brzinom od 17 ms^{-1} i nestabilni vlažni val male fazne brzine. Vlažni val propagira radi površinske izmjene topline uzrokovane vjetrom, a nestabilan je radi međudjelovanja oblaka i zračenja i osnovne vlažne nestabilnosti. Moguće je da vlažni val predstavlja jedan od mehanizama za tropske istočne valove i Madden-Julian oscilaciju.

Ključne riječi: Kelvinovi valovi, vlažni val

Author's address: Željka Fuchs, Physics Department and Geophysical Research Center, New Mexico Institute of Mining and Technology, Socorro, New Mexico, USA, Tel: + 1 505 83 55 407. Fax: + 1 505 83 55 707, e-mail: zeljka@kestrel.nmt.edu