GEOFIZIKA VOL. 26 No. 1 2009

Short communication
UDC 551.509.25

Assessment of Monin-Obukhov scaling over small slopes

Branko Grisogono and Dragana Zovko Rajak

Department of Geophysics, Faculty of Science, University of Zagreb, Zagreb, Croatia

Received 2 March 2009, in final form 5 May 2009

In this note we combine two recent findings related to sloped stably-stratified boundary layers (SBL). One of them contrasts Monin-Obukhov height, L, and the low-level jet (LLJ) height, z_j, induced by simple katabatic flows. Another result connects turbulent Prandtl number, Pr, to gradient Richardson number, Ri. In this way, one finds the ratio of the two heights (L/z_j) as a single value function of the underlying slope and Ri, thus giving a criterion where L may not be the most relevant near-surface layer scaling parameter for turbulent processes. For $(L/z_j) > 1$ it is the LLJ which determines most of near-surface turbulent properties. This has significant consequences on properties of various near-surface fluxes as treated in NWP, air-chemistry and wind-energy exploring models because most of them deploy L in one or another way for describing the lower boundary condition. This note shows that for ever finer horizontal resolution in our models, L should accommodate slope effects on the near-surface turbulent fluxes.

Keywords: diffusion, parameterization, Prandtl number, gradient Richardson number, stably-stratified turbulence, very stable boundary layers.

1. Introduction

Apparently, turbulent features of the (very) stable boundary layers (SBL) are still not well understood or treated adequately in various numerical models (Kim and Mahrt, 1992; Mahrt, 1998, 2007, 2008; King et al., 2001; Mauritsen et al., 2007; Steeneveld et al., 2007; Grisogono and Belušić, 2008). In the SBL there are many different mechanisms and processes modifying and altering turbulent fluxes under relatively weak mixing conditions (e.g. Baklanov and Grisogono, 2007). These fluxes are often governed by flows and effects above the surface such as the low-level jet (LLJ), near-surface inversion, background stratification, etc. Therefore, the related turbulence can be of different nature and properties than that generated over nearly-horizontal surfaces, say, over slopes ≤ 1° (e.g. Van der Avoird and Duynkerke, 1999; Grisogono and Oerlemans, 2001; Cuxart and Jiménez, 2007; Banta, 2008). At the same time, most of NWP, air-chemistry, climate, wind-energy, etc. models still use Monin-Obukhov length, L, for parameterizing near-surface turbulent fluxes (e.g.

Stull, 1988; Jeričević and Grisogono, 2006; Söderberg and Parmhed, 2006; Baklanov and Grisogono, 2007; Steeneveld et al., 2007). Since most of numerical models deploy L, in one way or another, to formulate the lower boundary condition, it is of considerable importance to be aware of the (in)applicability of the underlying assumptions pertaining to L.

Typical consequences of (over)use of L are manifolds; the latter range from the SBL over-diffusion to its frictional decoupling, often followed by runaway cooling and the SBL artificial collapse (e.g. King et al., 2001; Jeričević and Grisogono, 2006; Söderberg and Parmhed, 2006; Zilitinkevich et al., 2008; Grisogono, 2009). Therefore, various generalizations and extensions of the formulation of L are being conducted as in e.g. Zilitinkevich and Esau (2007), Argain et al. (2009). These few bold statements should suffice as a background and motivation for this scientific note; a more detailed overview is provided in the cited references. Here, we combine two recent findings about the SBL. The focus is on the lower part of the SBL, conveniently but also arguably called the stable surface layer. One of the recent results contrasts L and the height of LLJ, i.e. z_j, induced by simple katabatic flows, thus implying the conditions where L is an inappropriate scaling for the near-surface turbulent fluxes (Grisogono et al., 2007). Another result relates turbulent Prandtl number, Pr, to gradient Richardson number, Ri, under stable conditions (Kim and Mahrt, 1992; Zilitinkevich et al., 2008). In this way, for a given slope and Ri, we shall estimate the applicability of Monin-Obukhov scaling for the near-surface fluxes. The measure of this reliability of L is the goal of this communication.

2. Low-level jet-height versus Monin-Obukhov height

Recall the definition of L (e.g. Stull, 1988; Zilitinkevich et al., 2008):

$$L_{MO} = -\frac{\bar{\theta}}{gk} \cdot \frac{u_*^3}{\overline{w'\theta'}}, \qquad (1)$$

where all symbols have their usual meaning, i.e. u_* is friction velocity, $\overline{w'\theta'}$ is surface heat flux (already divided by density and specific heat at constant pressure), g is acceleration due to gravity, $\bar{\theta}$ is a relevant potential temperature and k is von Karman constant. This is the relevant length for scaling the fluxes, provided that the turbulent flow is horizontally homogeneous, i.e. without dynamically relevant underlying slopes influencing the flow.

Over sloped flows, L often becomes a less relevant scale; there the wind receives a direct contribution from the buoyancy force. Since we are concerned with the SABL here, negative buoyancy, generating katabatic flow, may produce wind speeds of e.g. ~ 5 m s^{-1} in an otherwise quiescent atmosphere. A simple model of Prandtl is invoked to estimate the height of the LLJ, i.e.

$$z_j = \frac{\pi}{4}\left(\frac{4K^2\,\mathrm{Pr}}{N^2 \sin^2(\alpha)}\right)^{1/4}, \tag{2}$$

where K is eddy heat conductivity (yielding momentum eddy diffusivity if multiplied by Pr), N is buoyancy frequency and α is slope angle (e.g. Grisogono et al., 2007). Although Parmhed et al. (2004) show that (2) usually somewhat overestimates actual LLJ positions, for the sake of the argument here (2) suffices. Invoking the usual formulation for the relevant parameters in (1) and (2) based on K–theory, i.e. $u_*^2 = K\mathrm{Pr}\frac{\partial U}{\partial z}$, $-\overline{w'\theta'} = K\frac{\partial \Theta}{\partial z}$, and $Ri = \frac{N^2}{(\partial U/\partial z)^2}$, where U is the mean wind speed, the squared ratio of (1) and (2) can be expressed:

$$(L_{MO}/z_j)^2 = br = \frac{8}{(k\pi)^2}\left|\sin(\alpha)\right|\left(\frac{\mathrm{Pr}^5}{Ri^3}\right)^{1/2}, \tag{3}$$

as in Grisogono et al. (2007). Obviously, if $br < 1$ (e.g. $\alpha << 0.1$ rad, and for $Pr \sim Ri \sim 1$), then the classical Monin-Obukhov theory and its scaling applies as usual, and one proceeds as before in modeling the traditional SBL flows. A typical example where $L_{MO}/z_j < 1$ is a (weakly) stratified Ekman layer; this flow type has been studied to appreciable length and depth elsewhere (e.g. Zilitinkevich et al., 2002). Nonetheless, (3) may also give $br > 1$ (e.g. for sufficiently stratified flows over moderate slopes $\alpha \le 0.1$ rad); then L is too large for estimating the near-surface fluxes determined now by the LLJ. In other words, L cannot sense any LLJ positioned so that $z_j < L$.

For a given slope, which is an external parameter, (3) still depends on two flow (i.e. internal) parameters, Pr and Ri, whose mutual relation has been largely ignored over the past years even though there has been evidence that Pr increases with Ri (Kondo et al., 1978; Kim and Mahrt, 1992; Monti et al., 2002). Zilitinkevich et al. (2008) find out that

$$Pr \approx 0.8 + 5\,Ri, \tag{4}$$

based on field and laboratory experiments and LES results; this simplifies (3) considerably. It is important to note, thanks to one of the reviewers, that Kim and Mahrt (1992) established a similar relation as (4), based on two field experiments, they proposed $Pr \approx 1 + 3.8\,Ri$; this note proceeds with (4) which is a newer result also including LES data. Notwithstanding now, only a single dimensionless dynamic parameter, Ri, is needed in order to determine a particular value of br over a given slope and see if L is the reliable scaling for the fluxes. Hence,

$$(L_{MO}/z_j)^2 = br = \frac{8}{(k\pi)^2}\left|\sin(\alpha)\right|\left(\frac{(0.8+5Ri)^5}{Ri^3}\right)^{1/2}. \tag{5}$$

Obviously, $br \to \infty$, for $Ri \to \infty$; moreover, br is also very large for very small Ri. A closer analysis shows that for a given slope (5) possesses a minimum at $Ri = Ri_{min} = 0.24$. Accidentally, this value is very close to a critical Ri value related to Kelvin-Helmholtz instability. Around $Ri = Ri_{min}$, (5) (or (3)) gives the smallest br over a given α, thus yielding most use of L in numerical models. At the same time, as a side note, there is no typical critical Ri for (non)turbulent atmospheric flows (Mauritsen et al., 2007; Zilitinkevich and Esau, 2007; Grisogono and Belušić, 2008). Figure 1 displays (5) as the main result of this note.

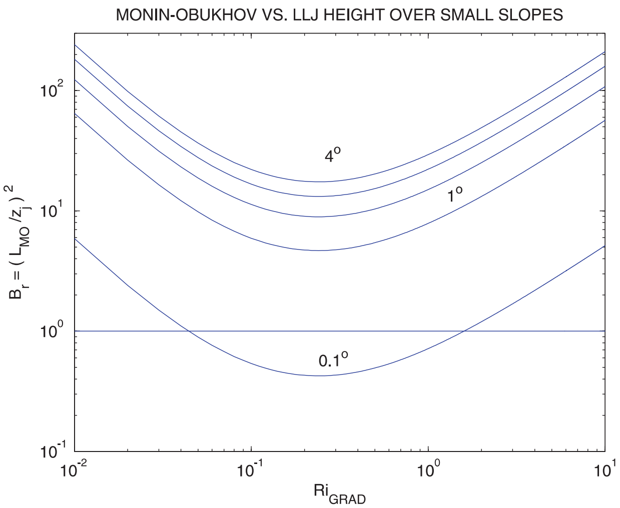

Figure 1. Summarizing the main finding of this note: eqn. (5) is plotted with $k = 0.4$, $Ri = \mathrm{Ri}_{\mathrm{GRAD}}$, $L = \mathrm{L}_{\mathrm{MO}}$. Several slopes α are indicated; for most of α, $L > z_j$, and thus L is not representative for the near-surface turbulent fluxes over most of the slopes.

Having provided Ri_{min}, one may ask for the maximum terrain slope over which L may still provide a reasonable scaling for the fluxes without the need for involving the LLJ consideration. The inquiry translates to equating (5), at $Ri = Ri_{min}$, to one, and finding the slope $\alpha = \alpha_{max}$. Such α_{max} is a surprisingly small value of 0.24° (i.e. only 0.0041 rad) which reduces L's credibility and is obtained because of two reasons. First reason is the neglecting of the roughness length, finiteness, two-dimensionality and variability of actual slopes as these would all lift z_j, etc. but are not accommodated by Prandtl's model (e.g. Grisogono and Oerlemans, 2001; Kavčič and Grisogono, 2007). Second reason relates to (4), because it actually represents an interpolation among different data sets. Adequately resolving the first reason will require a dedicated data analysis and numerical simulation which is beyond the scope of this note. Fortunately, the second reason, i.e. uncertainty in (4), thus affecting (5), can be easily tackled by various means. One can find the relative errors in (5) when (4) is written as

$$Pr \approx a + b\, Ri, \tag{6}$$

with a being quite certainly $0.7 < a \leq 1$, and $1 < b \leq 5$, where the emphasis is put toward lowering Pr with respect to that in (4) in order to allow for relatively larger α while keeping $br \leq 1$ in (5). This also agrees with Kim and Mahrt (1992), and generalizes $Ri_{min} = 3a/(2b)$, embracing the former $Ri_{min} = 0.24$ as a special case; moreover, $Ri_{min} = 3a/(2b)$ implies that e.g. relatively smaller b increases Ri_{min}, and vice versa. Plugging $Ri_{min} = 3a/(2b)$ in (5) yields the critical slope, where L is still the useful scaling in the limit i.e. at $br = 1$

$$\left|\sin(\alpha_{\max})\right| \approx \alpha_{\max} = \frac{(k\pi)^2}{4}\sqrt{\frac{3^3}{5^5}}(ab^{3/2})^{-1}. \tag{7}$$

For particular dependencies in (6), i.e. $Pr(Ri)$, one finds from (7) up to which maximum slope α_{max} one may freely deploy L. For instance, if $(a, b) = (0.8, 2)$, then L is applicable up to $\alpha_{max} \approx 1°$ slope; if $(a, b) = (1, 1)$, which is a hardly justified choice (Kim and Mahrt, 1992; Zilitinkevich et al., 2008), then L would be applicable up a maximum slope of $\alpha_{max} \approx 2.1°$. We leave additional tests for the interested reader and further investigation. The use of (7) is in determining the largest terrain slope for which Monin-Obukhov scaling is still applicable, provided a linear relationship (6).

As a final remark, we hypothesize what could be done next using our result, pertaining to high-resolution simulations and data analysis. A pragmatic but plausible way to include our findings in numerical models is to implement the estimation of z_j side by side with L. A modified L, L_{MOD}, could be obtained as e.g.

$$L_{MOD} = min(L, C_0\, z_j), \tag{8a}$$

or interpolating, like suggested in Grisogono et al. (2007),

$$\frac{1}{\mathrm{L_{MOD}}} = \frac{c}{L} + \frac{d}{z_j}, \tag{8b}$$

where c and d are to be obtained from measurements while the coefficient C_0 in (8a) is loosely guessed to be $0.7 < C_0 \leq 0.9$. For a particular data set from Pasterze glacier, Austria, based on Greuell et al. (1997), we find that $c = 1$ and $d = 0.7$ in (8b) representing L_{MOD} as a smooth function. Since these coefficients might be site and/or flow dependent, we believe that (8a) should be preferred in practice; there, $C_0\, z_j$ is chosen simply as a large fraction of zj (at z_j the mechanical production of turbulence ceases to exist). Of course, much more testing and model comparisons using very high vertical resolution against various data sets is needed before (8) would be more firmly recommended to modelers. However, such work is beyond the scope of this short communication.

3. Conclusion

Two results pertaining to the lower part of the SBL are inter-related in this note. One of them, by Grisogono et al., (2007) compares Monin-Obukhov height, L, with the position of LLJ, i.e. z_j, generated by pure katabatic flow. The other result is by Zilitinkevich et al. (2008), continuing on Kim and Mahrt (1992) and relating Prandtl number, Pr, to gradient Richardson number, Ri. The ratio of the two heights (L/z_j) now becomes a single value function of the underlying terrain slope and Ri, an external and one internal parameter, respectively. This gives the criterion where L may ($L < z_j$), or may not be the most relevant scaling parameter for the near-surface turbulent fluxes. Namely, for $(L/z_j) > 1$ it is usually the LLJ which determines most of near-surface turbulent properties (e.g. Van der Avoird and Duynkerke, 1999; Grisogono and Oerlemans, 2001; Cuxart and Jiménez, 2007; Banta, 2008; Grisogono and Belušić, 2008).

Although it seems that the particular $Pr(Ri)$ deployed is too strong, i.e. (4), discrediting L even for small slopes $\alpha < 1°$, which may appear as too strict in (5), we have derived a simple and robust test for the applicability of L. Further examinations and improvements for L are indicated. These may have strong impacts on modeled properties of various near-surface fluxes in NWP, air-chemistry and wind-energy related models because most of them still use L straightforwardly, in one or another way, in formulating the lower boundary condition as a perfectly horizontal source of turbulence. This note indicates that for refining horizontal resolution in our numerical models, L should accommodate slope effects on the near-surface turbulent fluxes. The latter ought to deploy firstly LLJ as a modulator of L. Hopefully, this study will be of some use for further simulations of the SBL flow regimes with various numerical models, such as WRF, HIRLAM, EMEP, etc. dealing with NWP, air-pollution, wind energy issues, etc.

Acknowledgements – The authors are thankful to the two unknown reviewers for their constructive recommendations, and to Michiel van den Broeke for providing us with the data from Pasterze. This study is supported by the Croatian Ministry of Science, Education and Sports, projects BORA No. 119–1193086–1311 and by EMEP4HR project number 175183/S30 provided by the Research Council of Norway.

References

Argain, J.L., Miranda, P.M.A. and Teixeira, M.A.C. (2009): Estimation of the friction velocity in stably stratified boundary-layer flows over hills, *Bound. Lay. Meteorol.*, **130**, 15–28.

Baklanov, A. and Grisogono, B. (Eds.), (2007): *Atmospheric Boundary Layers: Nature, Theory and Applications to Environmental Modelling and Security*. Springer, New York, 241 pp.

Banta, R.M. (2008): Stable-boundary-layer regimes from the perspective of the low-level jet, *Acta Geophysica*, **56**, 58–87.

Cuxart, J. and Jiménez, M.A. (2007): Mixing processes in a nocturnal low-level jet, an LES study, *J. Atmos. Sci.* **64**, 1666–1679.

Grisogono, B. (2009): Generalizing »z-less« mixing length for stable boundary layers, *Q. J. Roy. Meteor. Soc.* Tentatively accepted.

Grisogono, B. and Belušić, D. (2008): Improving mixing length-scale for stable boundary layers, *Q. J. Roy. Meteor. Soc.*, **134**, 2185–2192.

Grisogono, B. and Oerlemans, J. (2001): Katabatic flow: analytic solution for gradually varying eddy diffusivities, *J. Atmos. Sci.*, **58**, 3349–3354.

Grisogono, B., Jeričević, A. and Kraljević, L. (2007): The low-level katabatic jet height versus Monin-Obukhov height, *Q. J. Roy. Meteor. Soc.*, **133**, 2133–2136.

Greuell, W., Knap, W. and Smeets, P. (1997): Elevational changes in meteorological variables along a midlatitude glacier during summer, *J. Geophys. Res.*, **102**(D22), 25941–25954.

Jeričević, A. and Grisogono, B. (2006): The critical bulk Richardson number in urban areas: Verification and application in a numerical weather prediction model, *Tellus*, **58A**, 19–27.

Kavčič, I. and Grisogono, B. (2007): Katabatic flow with Coriolis effect and gradually varying eddy diffusivity, *Bound. Lay. Meteorol.*, **125**, 377–387.

Kim, J. and Mahrt, L. (1992): Simple formulation of turbulent mixing in the stable free atmosphere and nocturnal boundary layer, *Tellus*, **44A**, 381–394.

King, J.C., Conneley, W.M. and Derbyshire, S.H. (2001): Sensitivity of modelled Antarctic climate to surface and boundary-layer flux parameterizations, *Q. J. Roy. Meteor. Soc.*, **127**, 779–794.

Kondo, J., Kanechika, O. and Yasuda, N. (1978): Heat and momentum transfers under strong stability in the atmospheric surface layer, *J. Atmos. Sci.*, **35**, 1012–1021.

Mahrt, L. (1998): Stratified atmospheric boundary layers and breakdown of models, *Theor. Comp. Fluid Dyn.*, **11**, 263–279.

Mahrt, L. (2007): Bulk formulation of the surface fluxes extended to weak-wind stable conditions, *Q. J. Roy. Meteor. Soc.,* **134**, 1–10.

Mahrt, L. (2008): Mesoscale wind direction shifts in the stable boundary-layer, *Tellus*, **60A**, 700–705.

Mauritsen, T., Svensson, G., Zilitinkevich, S., Esau, I., Enger, L. and Grisogono, B. (2007): A total turbulent energy closure model for neutral and stably stratified atmospheric boundary layers, *J. Atmos. Sci.*, **64**, 4113–4126.

Monti, P., Fernando, H.J.S., Princevac, M., Chan, W.C., Kowalewski, T.A. and Pardyjak, E.R. (2002): Observations of flow and turbulence in the nocturnal boundary layer over a slope, *J. Atmos. Sci.*, **59**, 2513–2534.

Parmhed, O., Oerlemans J. and Grisogono, B. (2004): Describing surface-fluxes in katabatic flow on Breidamerkurjökull, Iceland, *Q. J. Roy. Meteor. Soc.,* **130**, 1137–1151.

Söderberg S. and O. Parmhed (2006): Numerical modelling of katabatic flow over melting outflow glacier, *Bound. Lay. Meteorol.*, **120**, 509–534.

Steeneveld, G.J., Van de Wiel, B.J.H. and Holtslag, A.A.M. (2007): Diagnostic equations for the stable boundary layer height: evaluation and dimensional analysis, *J. Appl. Meteorol. Clim.,* **46**, 212–225.

Stull, R.B. (1988): *An Introduction to Boundary Layer Meteorology*. Kluwer Academic Publishers, Dordrecht, the Netherlands, 666 pp.

Van der Avoird, E. and Duynkerke, P.G. (1999): Turbulence in a katabatic flow. Does it resemble turbulence in stable boundary layers over flat surfaces?, *Bound. Lay. Meteorol.*, **92**, 39–66.

Zilitinkevich, S.S. and Esau, I. (2007): Similarity theory and calculation of turbulent fluxes at the surface for the stably stratified atmospheric boundary layers, *Bound. Lay. Meteorol.*, **125**, 193–205.

Zilitinkevich, S.S., Baklanov, A., Rost, J., Smedman, A.-S., Lykosov, V. and Calanca, P. (2002): Diagnostic and prognostic equations for the depth of the stably stratified Ekman boundary layer, *Q. J. Roy. Meteor. Soc.*, **128**, 25–46.

Zilitinkevich, S.S., Elperin, T., Kleeorin, N., Rogachevskii, I., Esau, I., Mauritsen, T. and Miles, M.W. (2008): Turbulence energetics in stably stratified geophysical flows: strong and weak mixing regimes, *Q. J. Roy. Meteor. Soc.,* **134**, 793–799.

SAŽETAK

Procjena Monin-Obukovog skaliranja za blago nagnute padine

Branko Grisogono i Dragana Zovko Rajak

U ovom priopćenju povezujemo dva nedavna rezultata o nagnutim stabilnim graničnim slojevima. Jedan rezultat uspoređuje Monin-Obukhovu duljinu, L, i visinu niske mlazne struje z_j, uzrokovane katabatičkim strujanjem. Drugi rezultat povezuje Prandtlov broj, Pr, s gradijentnim Richardsonovim brojem, Ri. Tako se dolazi do omjera spomenutih duljina, (L/z_j) kao jednoznačne funkcije nagiba terena i Ri, što daje kriterij adekvatnosti L u skaliranju prizemnih turbulentnih procesa. Naime, u slučaju $(L/z_j) > 1$ niska mlazna struja određuje prizemne karakteristike turbulencije, a ne skaliranje pomoću L. To ima značajne posljedice na modelirane osobine prizemnih turbulentnih tokova u numeričkim prognostičkim modelima, kemijskim modelima, modelima za procjenu energije vjetra, itd. jer većina tih modela koristi L za definiranje donjeg rubnog uvjeta. Ova nota pokazuje da u modelima sve detaljnije i finije horizontalne rezolucije, L treba uvažavati efekte nagnutosti podloge radi adekvatnije procjene prizemnih turbulentnih tokova.

Ključne riječi: difuzija, parametrizacija, Prandtlov broj, gradijentni Richardsonov broj, stabilno-stratificirana turbulencija, vrlo stabilni granični slojevi.

Corresponding author's address: Branko Grisogono, AMGI, Department of Geophysics, Faculty of Science, University of Zagreb, Zagreb, Croatia, e-mail: bgrisog@gfz.hr, tel.:+385 1 460 5927