GEOFIZIKA VOL. 27 2010

*Original scientific paper*
*UDC 551.515.5*

# Convectively coupled Kelvin waves and convective inhibition

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*Received 9 April 2009, in final form 25 September 2009*

We use a thermodynamic assumption that the vertical heating profile has the shape of the first baroclinic mode, and that the analytical expression for vertical velocity has two modes, representing shallow and deep convection. The thermal assumption of the model is given through the convective inhibition closure, i.e. negative convective inhibition results in increased precipitation. These modeled modes are the free Kelvin waves and the convectively coupled Kelvin waves. We find the latter mode to be unstable, with maximum growth rate at wavelengths of 6 000 kilometers. The model successfully captures the observed nature of the Kelvin waves and shows that convective inhibition closure is sufficient to trigger the observed destabilization of the convectively coupled Kelvin mode.

*Keywords:* Large-scale modes, tropics, destabilization

## 1. Introduction

Equatorial waves are disturbances that are trapped about the equator. They can propagate eastward or westward and they decay as we move away from the equator. The biggest disturbance among the equatorial waves is the Madden-Julian oscillation (MJO). The MJO is a wave envelope of planetary wavenumber $l$ = 1, 2, 3 and a period of 30–60 days. It brings bad weather and lots of precipitation. Another equatorially trapped wave is the Kelvin wave. Its wavelength is smaller than the MJO's, but it can also be unstable and thus bring bad weather. The Rossby waves and inertia-gravity waves are also equatorially trapped waves, but will not be discussed in this work. Figure 1 shows the space – time spectrum of outgoing longwave radiation (OLR) symmetric about the equator, the equatorially trapped waves, their periods and zonal wavenumbers.

Due to the specific characteristics of the tropical atmosphere, we need to consider the influence of diabatic source terms on the dynamics of the waves.

In this model we research the equatorial convectively coupled Kelvin waves. The observations show (Straub and Kiladis, 2002) that the convectively coupled Kelvin waves slow down compared to their free modes. The way that the local convection interferes with the dynamics of the Kelvin wave is still not well known.

Matsuno (1966) is the father of the analytical theory for the equatorially trapped waves. In his work Matsuno assumes a shallow-water model where Coriolis parameter linearly varies with the longitude: $f = \beta \cdot y$. When zonally propagating waves are assumed as the solution, the result is a dispersion relation with several modes as a solution: inertia-gravity waves, Rossby wave and Kelvin wave (when meridional velocity is zero). In the Kelvin wave solution the phase speed of the wave is nondispersive and equal to $(gh)^{1/2}$, i.e. it is equal to the phase speed of shallow water gravity waves. Matsuno's model does not include thermal effects, i.e. it is an adiabatic model and thus the modeled modes are free. Matsuno's modes are shown in solid line in Figure 1.

As local convection in the tropics is not well understood, it is difficult to model the unstable Kelvin waves coupled with convection. However, recent work by Raymond and Fuchs, 2007 (RF2007) and Khouider and Majda, 2006 (KM2006) has shown that such modeling is possible.

The main difference between KM2006 and RF2007 is the assumed heating profile. KM2006's vertical heating profile is more complex as they assume two different profiles; one corresponds to deep convection and the other to stra-

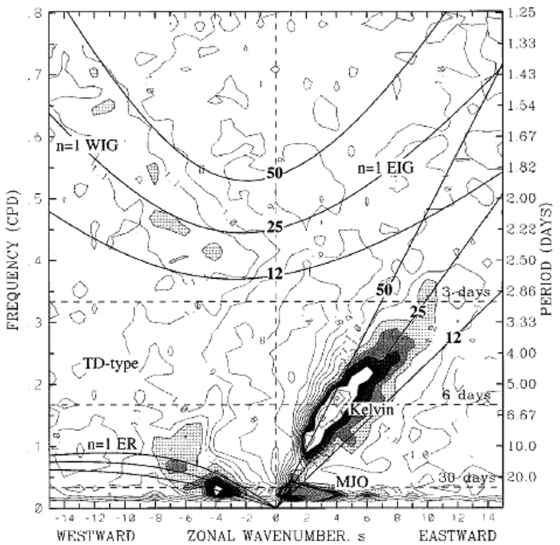


**Figure 1.** Space-time spectrum of OLR symmetric about the equator and equatorial waves: Kelvin waves (Kelvin), Madden-Julian oscillation (MJO), Rossby waves (ER) and eastward (EIG) and westward (WIG) inertio-gravity waves (Wheeler and Kiladis, 1999).

tiform convection. Therefore, this model has two equations for the vertical velocity and consequently for all other fields. As a result the model can only be solved numerically. KM2006's Kelvin wave phase speed agrees with the observations.

Raymond and Fuchs solve their model analytically using a simple sinusoidal heating profile with a wavelength equal to twice the depth of the troposphere. Despite this assumption, RF2007's model obtains an analytical expression for the vertical velocity that has both types of the heating profiles. When the model is solved with the expression for vertical velocity, the result is an unstable convectively coupled Kelvin wave with a phase speed and instability properties that agree with the observations. Though RF2007 agrees with the observational data, it is still rather complex. The goal of this work is to create a model that is simpler than RF2007, but that still produces modes which agree with the observations.

The model in this paper is a simple non-rotating model of the tropical atmosphere. It is based on work by RF2007 as the assumed vertical heating profile is of the shape of the first baroclinic mode and as the vertical velocity expression is taken from their work. The assumption of the non-rotating atmosphere is justified as the Kelvin waves in a rotating atmosphere correspond to gravity waves in non-rotating atmosphere. For that reason we call those waves Kelvin waves.

In this paper we assume that the only condition for the strong precipitation connected to the Kelvin waves is suppressed convective inhibition. It is an idealization, but the purpose of this work is to reduce the pool of diabatic effects and get a clearer picture of which mechanism is responsible for the instability of the wave.

We find the model with the above hypothesis successfully generates a convectively coupled Kelvin wave with the same phase speed and instability as the observed one. This points to suppressed convective inhibition as the primary mechanism responsible for the convectively coupled Kelvin waves in the tropics.

Section two shows the basic theory behind the model, section 3 gives the results, while conclusions are given in section 4.

## 2. Model

### *2.1. Basic equations*

We begin the derivation of Kelvin waves with the basic governing equations:

Momentum equation:

$$\frac{\mathrm{d}\vec{V}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla p - 2\vec{\Omega}\times\vec{V} + \vec{g} + \frac{1}{\rho}\vec{F} \tag{1}$$

Continuity equation:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t}+\rho\nabla\cdot\vec{V}=0 \tag{2}$$

Thermodynamic equations:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t}=S \tag{3}$$

$$\frac{\mathrm{d}\theta_e}{\mathrm{d}t}=S_e \tag{4}$$

$$\frac{\mathrm{d}\theta_q}{\mathrm{d}t}=S_q \tag{5}$$

where $\theta$ is the potential temperature, $\theta_e$ is the equivalent potential temperature and $q$ is the mixing ratio. As we are considering the diabatic case, the right side of the thermodynamic equations is not zero. We further neglect the processes that are not directly responsible for the Kelvin waves:

1. We assume a non-rotating atmosphere, i.e. $2\vec{\Omega}\times\vec{V}=0$. The Kelvin waves in rotating atmosphere map into gravity waves in non-rotating atmosphere which justifies this assumption and is a reason why we call them Kelvin waves.
2. We neglect the friction: $F = 0$.
3. We assume the fluid is incompressible ($\frac{\mathrm{d}\rho}{\mathrm{d}t}=0$), which according to equation (2) implies that $\nabla\cdot\vec{V}=0$.
4. We assume the fluid is in hydrostatic equilibrium, $\frac{\mathrm{d}w}{\mathrm{d}t}=0$. In the vertical momentum equation the gravitational term is assumed to be balanced by the pressure gradient.
5. We consider the $(x, z)$ plane only.

After the above approximations we are left with:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0 \tag{6}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z}+g=0 \tag{7}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{8}$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+w\frac{\partial \theta}{\partial z}=S \tag{9}$$

$$\frac{\partial \theta_e}{\partial t}+u\frac{\partial \theta_e}{\partial x}+w\frac{\partial \theta_e}{\partial z}=S_e \tag{10}$$

$$\frac{\partial q}{\partial t}+u\frac{\partial q}{\partial x}+w\frac{\partial q}{\partial z}=S_q \tag{11}$$

The equations (6) and (7) are the $x$ and $z$ components of the momentum equation. Equation (8) follows from the continuity equation. Equations (9), (10) and (11) come from the thermodynamic equations.

From the Poisson equation $\theta = T\left(\frac{p_0}{p}\right)^{\frac{R}{c_p}}$ and the equation of state $p = \rho RT$:

$$\frac{\mathrm{d}p}{\rho}=RT\frac{\mathrm{d}p}{p}=R\theta\left(\frac{p}{p_0}\right)^{\frac{R}{c_p}}\frac{\mathrm{d}p}{p}=\theta c_p \mathrm{d}\left(\frac{p}{p_0}\right)^{\frac{R}{c_p}}$$

and therefore

$$\frac{\mathrm{d}p}{\rho}=\theta \mathrm{d}\tilde{\Pi} \tag{12}$$

where $\tilde{\Pi}=c_p\left(\frac{p}{p_0}\right)^{\frac{R}{c_p}}$ is the Exner function.

Equations (6) and (7) can now be rewritten as:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}+\theta\frac{\partial \tilde{\Pi}}{\partial x}=0 \tag{13}$$

$$\theta\frac{\partial \tilde{\Pi}}{\partial z}+g=0 \tag{14}$$

### *2.2. Linearization*

We linearize the model using the perturbation method and expressing every variable as a superposition of the equilibrium state and its perturbation. The basic state of every variable satisfies the governing system of equations, and the perturbation values are small enough for their multiplication to be neglected.

We start with:

$$\begin{aligned}
&u(x,z,t)=u_0(z)+u'(x,z,t)\\
&w(x,z,t)=w_0(z)+w'(x,z,t)\\
&\tilde{\Pi}(x,z,t)=\tilde{\Pi}_0(z)+\tilde{\Pi}'(x,z,t)\\
&\theta(x,z,t)=\theta_0(z)+\theta'(x,z,t)\\
&\theta_\mathrm{e}(x,z,t)=\theta_{\mathrm{e}0}(z)+\theta_\mathrm{e}'(x,z,t)\\
&q(x,z,t)=q_0(z)+q'(x,z,t)
\end{aligned} \tag{16}$$

We next assume that the mean horizontal and vertical velocities are zero, and therefore:

$$\begin{aligned}
&u(x,z,t)=u'(x,z,t)\\
&w(x,z,t)=w'(x,z,t)
\end{aligned} \tag{17}$$

After linearization the system of equations (6)–(11) simplifies to:

$$\frac{\partial u}{\partial t}+\theta_0\frac{\partial \tilde{\Pi}}{\partial x}=0 \tag{18}$$

$$\theta_0\frac{\partial \tilde{\Pi}}{\partial z}=g\frac{\theta}{\theta_0} \tag{19}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{20}$$

$$\frac{\partial \theta}{\partial t}+w\frac{\mathrm{d}\theta_0}{\mathrm{d}z}=S \tag{21}$$

$$\frac{\partial \theta_e}{\partial t}+w\frac{\mathrm{d}\theta_{e0}}{\mathrm{d}z}=S_e \tag{22}$$

$$\frac{\partial q}{\partial t}+w\frac{\mathrm{d}q_0}{\mathrm{d}z}=S_q \tag{23}$$

where for simplicity we write $u' \to u$, $w' \to w$, etc.

We recognize the term in equation (19) as buoyancy:

$$g\frac{\theta}{\theta_0}=b \tag{24}$$

We further introduce $S_B = \frac{g}{\theta_0} S$ and to simplify the notation we redefine the Exner function as $\Pi = \theta_0 \tilde{\Pi}$. The system of equations (18)–(23) can now be written as:

$$\frac{\partial u}{\partial t} + \frac{\partial \Pi}{\partial x} = 0 \tag{25}$$

$$\frac{\partial \Pi}{\partial z} = b \tag{26}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{27}$$

$$\frac{\partial b}{\partial t} + \Gamma_B w = S_B \tag{28}$$

$$\frac{\partial q}{\partial t} + \Gamma_Q w = S_Q \tag{29}$$

$\Gamma_B$ is the square of the Brunt-Väisälä frequency: $\Gamma_B = N^2 = \frac{g}{\theta_0} \frac{\partial \theta_0}{\partial z}$, while $\Gamma_Q = \frac{gL}{c_p T_r} \frac{\mathrm{d} q_0}{\mathrm{d} z}$.

We use this system of equations to derive the convectively coupled Kelvin waves in moist atmosphere.

### *2.3. Thermodynamics of moist atmosphere*

In the tropical atmosphere precipitation is an important heating source as the latent heat is released by condensation of water vapor. Another heating mechanism is latent or sensible heat transport from the surface by evaporation or induced by wind (wind induced surface heat exchange).

The heat source comes into the equations through the source terms. As we consider a non adiabatic case the heat is exchanged with the environment. We integrate the buoyancy source term $S_B$ through the entire troposphere:

$$B = \int_0^h S_B dz \tag{30}$$

Fuchs and Raymond (2007) assumed a simple sinusoidal heating profile (the first baroclinic mode), with the heating maximum in the middle, and falling to zero at the bottom (surface) and the top of the troposphere:

$$S_B = \left(\frac{m_0 B}{2}\right)\sin(m_0 z) \tag{31}$$

$m_0 = \pi/h$ is the vertical wavenumber where half of its wavelengths is equal to the depth of the troposphere. Troposphere depth is taken as $h$ = 15 000 m. This type of heating profile corresponds to deep convection and we will use it in this paper.

Convective inhibition energy (CIN) is the amount of energy that will prevent an air parcel from rising from the surface to the level of free convection, $z_{LFC}$:

$$CIN = \int_{z}^{z_{LFC}} g\left(\frac{T_{V_{parcel}} - T_{V_{environment}}}{T_{V_{environment}}}\right)\mathrm{d}z \tag{32}$$

Convective inhibition disables the updrafts, thus preventing the development of convective clouds and precipitation. Convective inhibition is a consequence of stable stratification. In that situation the parcels that are being lifted come to the environment that is warmer than themselves and thus the convection ceases. In-situ measurements (Raymond et al., 2003) show that a stable layer just above the boundary layer is sufficient to inhibit the development of deep convection and the associated cloudiness. This disables the precipitation from convective clouds. Convective inhibition is parameterized through the buoyancy perturbation.

The crucial hypothesis of this model is that the precipitation is controlled by changes in convective inhibition. The modulation of convection can be explained by wave-related adiabatic lifting of the capping layer just above the boundary layer.

Convective inhibition is parameterized through the precipitation term ($P$). Raymond and Fuchs (2007) analyzed the data collected by the ship *Ron Brown*, and particularly the situations when the precipitation was connected to small or negative values of convective inhibition. They suggested the following parameterization:

$$P = -\mu_{CIN}\lambda_t b(D) \tag{33}$$

$D$ is the dimensionless height of the stable layer, normalized by the depth of the troposphere $h$. $\mu_{CIN}$ is a parameter that defines sensitivity of the precipitation to convective inhibition (see Raymond and Fuchs, 2007 for more details), while $\lambda_t$ is defined as: $\lambda_t \equiv \left[1 + \frac{L}{c_p}\left(\frac{\partial r_s}{\partial T}\right)_p\right]$. $L$ is the specific latent heat, $c_p$ is the specific heat capacity at constant pressure and $r_s$ is the saturation mixing ratio at level $D{\cdot}h$. In tropical atmosphere $\lambda_t \approx 3.5$ at level $D{\cdot}h$.

To be able to isolate the most important mechanism for the convectively coupled Kelvin waves we neglect all other effects and thus assume that total heating is caused by latent heat release from precipitation. Compared to

RF2007 this simplifies the parameterization of precipitation, as it is only done by changes in CIN:

$$B = \int_0^h S_B(z)dz = P = -\mu_{CIN}\lambda_t b(D) \tag{34}$$

To solve (25)–(29), we assume the following form of the variables:

$$u(x,\ z,\ t)\ =\ u(z)\exp[i(kx-\omega t)] \tag{35}$$

$$w(x,\ z,\ t)\ =\ w(z)\exp[i(kx-\omega t)] \tag{36}$$

$$\Pi(x,\ z,\ t)\ =\ \Pi(z)\exp[i(kx-\omega t)] \tag{37}$$

$$b(x,\ z,\ t)\ =\ b(z)\exp[i(kx-\omega t)] \tag{38}$$

$$q(x,\ z,\ t)\ =\ q(z)\exp[i(kx-\omega t)] \tag{39}$$

thus obtaining the polarization relations:

$$u(z) = \frac{i}{k}\frac{\partial w(z)}{\partial z} \tag{40}$$

$$\Pi(z) = \frac{i\omega}{k^2}\frac{\partial w(z)}{\partial z} \tag{41}$$

$$b(z) = \frac{i}{\omega}[S_B(z) - \Gamma_B w(z)] \tag{42}$$

$$q(z) = \frac{i}{\omega}[S_Q(z) - \Gamma_Q(z)w(z)] \tag{43}$$

Vertical velocity perturbation is taken from RF2007 where the model was solved on the $x$–$z$ plane:

$$w(z) = \frac{m_0 B}{2\Gamma_B(1-\Phi^2)}\left[\sin(m_0 z) + \Phi\exp\left(-\frac{i\pi}{\Phi}\right)\sin(mz)\right] \tag{44}$$

$\Phi$ is a nondimensional phase speed: $\Phi = \dfrac{\omega}{\alpha\kappa} = \dfrac{m_0}{m}$. $\kappa$ is a nondimensional horizontal wavenumber: $\kappa = \dfrac{h\Gamma_B^{1/2}}{\alpha\pi}k$. $\alpha$ is a moisture relaxation rate (Fuchs and Raymond, 2001) in units of day$^{-1}$. $m_0$ is the vertical wavenumber of the first baroclinic mode while $m$ is the vertical wavenumber of the calculated mode. Vertical velocity is a result of updrafts due to deep and stratiform convection and thus both components appear in the vertical velocity expression. The

$\sin(m_0 z)$ term corresponds to the deep convection, while the $\exp(-i\pi/\Phi)\sin(mz)$ term corresponds to stratiform convection.

Given we already know the vertical velocity profile $w$, and the vertical heating source profile $S_B$, it is straightforward to obtain the dispersion relation. From equations (38), (31) and (44) we get the buoyancy perturbation as a function of height:

$$b(z) = \frac{-iBm_0}{2\alpha\kappa(1-\Phi^2)}\left[\Phi\sin(m_0 z) + \exp\left(-i\pi\frac{i\pi}{\Phi}\right)\sin(mz)\right] \tag{45}$$

Using equations (44), (45), (31) and (28) we find the dispersion relation to be:

$$2\alpha\kappa\Phi^2 + i\Phi\mu_{CIN}\lambda_t m_0 \sin(m_0 D) - 2\alpha\kappa + i\mu_{CIN}\lambda_t m_0 \exp\left(-i\frac{\pi}{\Phi}\right)\sin(mD) = 0 \tag{46}$$

Finally, we define $\chi_t = \dfrac{\mu_{CIN}\lambda_t m_0}{2\alpha}$ and obtain the nondimensional dispersion relation:

$$\kappa\Phi^2 + i\Phi\chi_t \sin(D\pi) - \kappa + i\chi_t \exp\left(-i\frac{\pi}{\Phi}\right)\sin\left(\frac{D\pi}{\Phi}\right) = 0 \tag{47}$$

Parameter $\chi_t$ defines the sensibility to stable layers. It is taken to be equal to 12, as in the control case from RF2007 who based it on the extended thermodynamic parameter analysis. Nondimensional height $D$ is the ratio of the height of the stable layer and the height of the troposphere $h$. As the stable layer is usually at $z = 2$ km, the parameter $D$ is 0.17.

The dispersion relation is solved numerically using the Newton's method in Mathematica.

## 3. Results and discussion

Dispersion relation (46) is solved numerically for $\Phi$ with $D = 0.17$ and $\chi_t = 12$ (Raymond and Fuchs, 2007). $\Phi = \omega/\alpha\kappa$ is a complex number whose real part is non-dimensional phase speed. If positive, the wave propagates eastward; if negative, it propagates westward. By multiplying $\Phi$ by wavenumber $\kappa$, we get $\Omega \equiv \Phi\cdot\kappa = \omega/\alpha$, where $\omega$ is the wave frequency in units of day$^{-1}$. Imaginary part of the wave frequency is growth or decay rate of the mode as all the fields have wave dependence in time that is proportional to $\exp(-i\omega t)$. If the imaginary part is positive, the result is exponentially growing mode, while if it is negative, the result is decaying mode.

To better visualize the results we define the planetary wavenumber $l=k/k_1$, where $k_1 = 2\pi/40\,000$ km$^{-1}$. It is equal to 1 for waves with wavelength equal to

*Table 1. Parameters and their values.*

| Symbol | Parameter | Value/dimension |
|---|---|---|
| $u$ | Horizontal speed | $[\mathrm{m \cdot s^{-1}}]$ |
| $\tilde{\Pi}$ | Exner function | $[\mathrm{m^2 \cdot s^{-2} \cdot K^{-1}}]$ |
| $\Pi$ | Exner function · mean potential temperature | $[\mathrm{m^2 \cdot s^{-2}}]$ |
| $w$ | Vertical speed | $[\mathrm{m \cdot s^{-1}}]$ |
| $B$ | Buoyancy | $[\mathrm{m \cdot s^{-2}}]$ |
| $Q$ | Scaled mixing ratio | $[\mathrm{m \cdot s^{-2}}]$ |
| $\Gamma_B = \dfrac{g}{\theta_0}\dfrac{\mathrm{d}\theta}{\mathrm{d}z}$ | Squared Brunt-Väisälä frequency | $[\mathrm{s^{-2}}]$ |
| $G$ | Gravitational acceleration | $[\mathrm{m \cdot s^{-2}}]$ |
| $c_p$ | Specific heat capacity at constant pressure | $1\,005\ \mathrm{J\ kg^{-1}\ K^{-1}}$ |
| $m = k\Gamma_B^{1/2}/\omega$ | Vertical wavenumber | $[\mathrm{m^{-1}}]$ |
| $m_0 = \pi/h$ | First baroclinic vertical wavenumber | $2.09\times10^{-4}\ \mathrm{m^{-1}}$ |
| $h$ | Depth of the troposphere | 15 km |
| $k$ | Horizontal wavenumber | $[\mathrm{m^{-1}}]$ |
| $\omega$ | Frequency | $[\mathrm{s^{-1}}]$ |
| $\Phi$ | Nondimensional phase speed | Calculated |
| $\Omega = \omega/\alpha$ | Nondimensional frequency | Calculated |
| $\kappa = k\Gamma_B^{1/2}/\omega m$ | Nondimensional horizontal wavenumber | 0.7 to 20 |
| $l = 2\pi/40\,000$ | Planetary zonal wavenumber | $[\mathrm{km^{-1}}]$ |
| $\alpha$ | Moisture relaxation rate | $[\mathrm{day^{-1}}]$ |
| $\chi_t$ | Constant | 12 |
| $\lambda_t$ | Constant | 3.5 |
| $D = z{\cdot}h$ | Nondimensional height of the stable layer | 0.17 |
| $L$ | Specific latent heat | $[\mathrm{J \cdot kg^{-1}}]$ |

circumference of the earth. Figures 2–7 show the phase speed and growth rate as a function of $l$ for all the obtained modes – the free Kelvin waves and the convectively coupled Kelvin waves.

Figures 2 and 3 show the free Kelvin waves that propagate eastward and westward with a phase speed of ±48 $\mathrm{m{\cdot}s^{-1}}$. Figure 4 shows the growth rate of the free Kelvin wave. Imaginary part of the frequency is zero which means that they are neither growing nor decaying, they are neutral.

Figure 5 shows the convectively coupled Kelvin wave that propagates eastward and Figure 7 the one that propagates westward. The phase speeds are

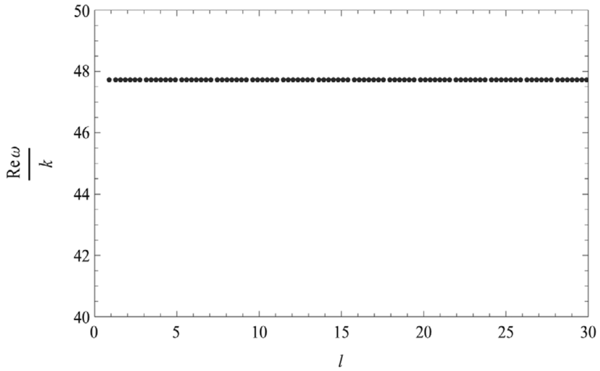


**Figure 2.** Phase speed of the free Kelvin waves as a function of planetary wavenumber. The propagation is eastward.

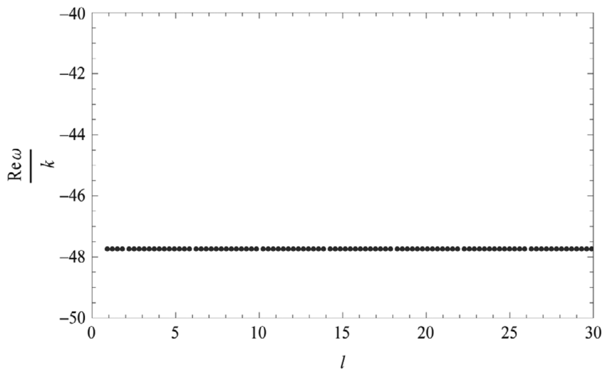


**Figure 3.** The same as figure 2, but the propagation is westward.

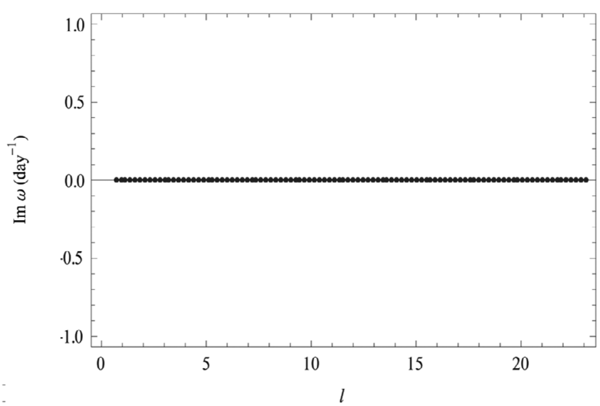


**Figure 4.** Growth rate for the free Kelvin waves as a function of planetary wavenumber.

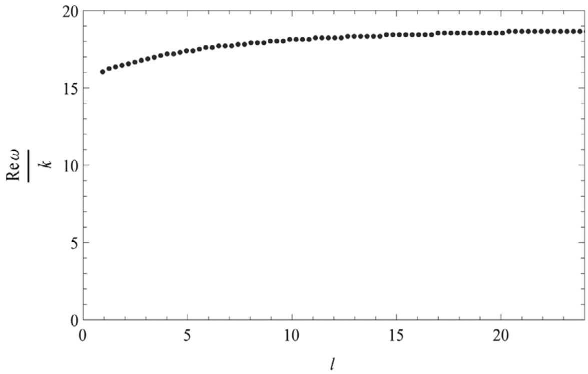


**Figure 5.** Phase speed of the eastward convectively coupled Kelvin wave as a function of planetary wavenumber.

±18 m·s$^{-1}$ and vary slightly with the wavenumber. The westward propagating convectively coupled Kelvin wave does not have its analog in reality so we neglect it. The imaginary part of the frequency is positive (Figure 6) for most wavelengths which means that the wave is unstable or growing in time.

The modeled phase speed of the convectively coupled Kelvin wave (16–19 m·s$^{-1}$) is in good agreement with observations (Straub & Kiladis, 2002). The imaginary part of the frequency reaches its maximum for planetary wavenumber $l$ = 7. This indicates that the convectively coupled Kelvin waves show the biggest growth rate for the wavelengths around 6 000 km. This result is in good agreement with observations as the highest spectral energy for this mode was found around that wavelength (Wheeler and Kiladis, 1999). The growth rate decreases as we go towards higher wavenumbers; after $l$ = 15 the modes decay.

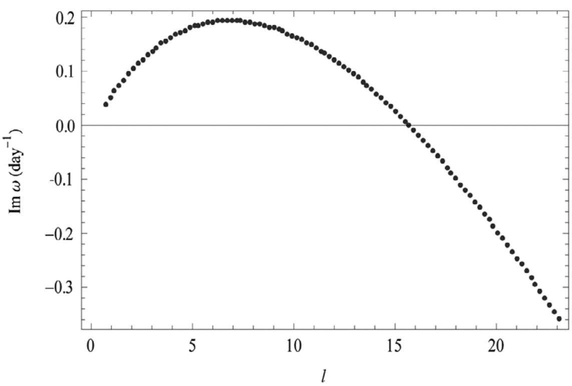


**Figure 6.** Growth rate of the eastward and westward propagating convectively coupled Kelvin waves as a function of planetary wavenumber.

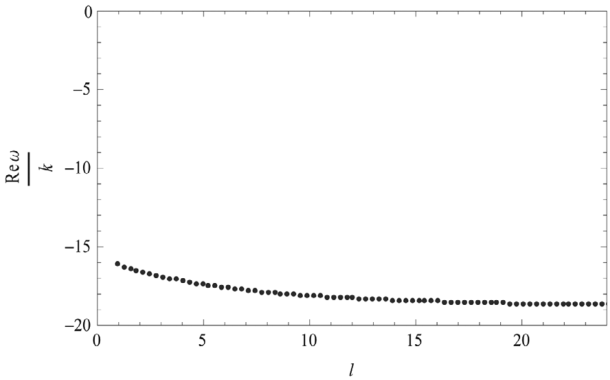


**Figure 7.** Phase speed of the westward convectively coupled Kelvin waves as a function of planetary wavenumber.

## 4. Conclusions

We present an idealized, vertically resolved model of the tropical atmosphere. We assume that the heating profile is sinusoidal with half of the wavelength equal to the depth of the troposphere (the first baroclinic mode). The vertical velocity profile is bimodal and taken from Raymond and Fuchs (2007).

Heat release through precipitation is the only diabatic mechanism of importing heat in the system. Precipitation rate is parameterized by variations in convective inhibition (CIN), motivated by wave-related adiabatic lifting of the capping layer just above the boundary layer. The reason for such a simplified treatment is our intent to investigate the feasibility of CIN variations as the basic mechanism for generating convectively coupled Kelvin waves.

Two modes are modeled: the free Kelvin waves and the convectively coupled Kelvin waves. We find that the free Kelvin waves have a phase speed of approximately 48 $m\cdot s^{-1}$, close to that seen in observations (Andrews et al., 1987). They propagate eastward and westward, and are found to be neutral. The modeled free Kelvin waves follow the theory of free gravity waves. We find convectively coupled Kelvin wave that propagates with a phase speed of 16–19 $m\cdot s^{-1}$, varying only slightly with wavelength. It is unstable with the biggest growth rate for planetary wavenumber $l = 7$, corresponding to $\lambda = 6\,000$ km. Phase speed and growth rate agree with the observations (Wheeler and Kiladis, 1999; Straub and Kiladis, 2002). The phase speed is likely to be a consequence of vertical velocity profile, i.e. of the wave dynamics while the instability is a result of the CIN modulation.

The presented model is an idealized analytical model for the Kelvin waves. Our results follow from only two assumptions: i) the first baroclinic mode heating profile (leading to bimodal vertical velocity profile; RF2007), and ii) the parameterization of precipitation by variations of CIN. More complex models such as Raymond and Fuchs (2007) also include cloud-radiation interactions, precipitation dependence on humidity and wind induced surface heat exchange (WISHE). The dynamics in this model is the same as in RF2007, and the agreement between our results and RF2007 is very strong especially when compared to their case without WISHE. This shows that the basic mechanism for the instability of the convectively coupled Kelvin wave is the variation in convective inhibition.

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SAŽETAK

## Konvektivno udruženi Kelvinovi valovi i konvektivna inhibicija

*Dea Doklestić, Željka Fuchs i Antun Marki*

Termodinamičke pretpostavke modela uključuju vertikalni profil grijanja koji ima oblik prvog baroklinog moda. Pretpostavljeni oblik vertikalne brzine sastoji se od dva dijela koji odgovaraju plitkoj i dubokoj konvekciji. Za parametrizaciju oborine uzeta je negativna konvektivna inhibicija (CIN).

Dobiveni modovi su slobodni Kelvinovi valovi i Kelvinov val povezan s konvekcijom. Kelvinov val povezan s konvekcijom je nestabilan, a najveću nestabilnost pokazuje pri valnoj duljini od 6 000 km. Ovim modelom uspješno je reproducirana opažena priroda Kelvinovih valova, a iz modela se vidi da je konvektivna inhibicija dovoljna za modeliranje opažene nestabilnosti Kelvinovih valova povezanih s konvekcijom.

*Ključne riječi:* Modeli velikih razmjera, tropi, destabilizacija

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