GEOFIZIKA VOL. 34 2017

DOI: 10.15233/gfz.2017.34.3
Professional paper
UDC 550.347.62

Robust network adjustment of vertical movements with GNSS data

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Received 25 October 2016, in final form 17 February 2017

Long operation periods of GNSS stations give a possibility to use the data in analyzing vertical crustal movements with the accuracy better than ± 0.5 mm/y. During the analysis, the reliability must be considered. This involves the choice of the vertical crustal movements network adjustment method. In most cases, the vertical crustal movements network models are designed as absolute and related to the ellipsoid, where the movement is calculated on the basis of estimated station coordinates. The other option is choosing differential relative models, where GNSS vector coordinates are used. In this case, GNSS stations are connected and vertical movements between them are calculated. In the next stage, the network of vertical crustal movements is adjusted and the accuracy is assessed. The aim of this article is to calculate and adjust the unadjusted trend based on GNSS time series in an area located in Central Europe. The article presents the robust adjustment method with a weighting scheme. The obtained results show that the accuracy of vertical crustal movements model of 0.5 mm/y can be obtained from the GNSS observations processing. Also the benefits coming from the application of robust adjustment method are emphasized.

Keywords: vertical crustal movements in GNSS data, robust adjustment, weight in adjustment

1. Introduction

The knowledge about vertical crustal movements is widely applied in climate change monitoring (glaciers melting) and tectonics (Fjeldskaar et al., 2000; Trần et al., 2013), seismology (Romaniuk, 2014), determining places for oil and gas extraction (Kenselaar and Quadvlieg, 2001), location of strategic engineering buildings, the analysis of changes in water table (Bednarczyk et al., 2015), the analysis of geological changes (Badura, 2007), the verification of vertical crustal movements models determined with the use of different techniques (Sowa et al., 2013) and retaining current state of global and regional frames and references systems (Ågren and Svensson, 2007).

The creation of Global Positioning Systems (GPS) (Hofmann-Wellenhof et al., 2012), Synthetic Aperture Radar (SAR) (Bürgmann et al., 2000), and Satellite Laser Ranging (SLR) (Pearlman et al., 2002) as well as the Very-long-baseline interferometry (VLBI) (Campbell, 2000) have enabled the possibility to monitor the Earth on a global scale. Afterward, the use of all mentioned systems has permitted to change previously used 3D to 4D frames, which have made it possible to monitor changes in the location of tectonic plates in time (Altamimi et al., 2002; Matsumura et al., 2004; Sacher et al., 2008).

This involves choosing the vertical crustal movements network adjustment method. In most cases, the vertical crustal movements network models are designed as absolute and related to the ellipsoid, where the movement is calculated on the basis of estimated station coordinates. The other options are differential relative models, where GNSS vector coordinates are used.

The first models of vertical crustal movements in Europe were made with geodetic observations in the 1960s and the 1970s on the basis of precise levelling data (Gopwani and Scheidegger, 1971; Randjärv, 1968; Vaníček and Christodulidis, 1974; Wyrzykowski, 1971). The frequency of the models' creation depended on the frequency of the levelling data updates. The models were more precise than the models of horizontal velocities. Since the operation of GPS and the long time series availability, the models of vertical crustal movements have been created with the use of GNSS data, mainly as the absolute velocities of permanent stations (Vaniček and Krakiwsky, 1986; Kontny and Bogusz, 2012).

As it was presented in Wöppelmann et al. (2009), there is a theoretical possibility to determine the vertical crustal movements with the precision of ±0.1mm/y to ±0.2 mm/y. However, in practice, achieving such precision is complex. The main factor that influences the precision in estimating the absolute velocity of a station is the operation time (Kowalczyk, 2015; Ihde and Augath, 2001). The combination of levelling data and GNSS data is difficult to make due to various systems and working methods (Kenyeres et al., 2013) and the necessity to consider the geoid models, and the knowledge of the changes in the geoid in time (Torge, 1989). An additional impediment is the application of various solutions in the process of creating vertical crustal movement models, from mathematical models to geophysical models (Ågren and Svensson, 2007).

The knowledge of the velocity of changes is obtained on the basis of various types of observations and measurements. Very precise measurements are made with the use of geodetic techniques, namely precise levelling. The measuring time and the equipment used in observations vary in relation to the applied measuring method. Three main measuring methods can be distinguished:

mareographic measurements, precise levelling measurements and GNSS measurements.

Precise levelling as the technology is laborious and thus, the data gained from a precise levelling campaign have been updated on average every 20 years (or in shorter periods as complementary measurements or control measurements) (Sacher et al, 2008; Kowalczyk, 2008; Kowalczyk and Rapinski, 2013). The range of the networks and various measuring epochs in many European countries hinder the creation of a reliable and current model of vertical crustal movements with the use of levelling data. Such diversified time ranges to a large extent influence the validity of regional and global models of vertical crustal movements.

The use of GNSS data would permit to create models of vertical crustal movements in almost real time (daily, weekly or monthly updated, depending on the geological and climate needs for the evaluation of GNSS networks condition). The estimation of factors that negatively influence the quality of the determined movements and their elimination would permit to create reliable models of vertical crustal movements. Currently, this is the key issue for the Regional Reference Frame Sub-Commission for Europe (EUREF) (*symposium EUREF Budapest 2013 http://euref2013.fomi.hu/*, *symposium EUREF Vilnius 2014 http://www.nzt.lt/euref2014/*). For the realization of the project, a special working group has been created. Its aim is to obtain velocity models and significantly improve the prediction of the coordinates' time evolution, and overcome the limitations in the use of the ETRS89 (Lidberg et al., 2014). The long-term goal is to establish a velocity model of crustal deformations (Lidberg et al., 2014) based on the "known" 3D crustal velocities of GNSS reference stations (Kenyeres et al., 2013; Caporali et al., 2013).

In the literature, vertical crustal movements (from GNSS stations) are drawn up as **absolute models related to the ellipsoid** (Kontny and Bogusz, 2012; Kenyeres et al., 2013). Consequently, the so-called absolute movements are calculated. In order to connect the GPS, levelling, mareographic data and geophysics studies, relative models or the observed models, *i.e.* related to the sea level, are required. That is why in this article a method of estimating the relative motion and considering the mean Baltic Sea level as a reference in the levelling is proposed. The first tests indicated the correctness of the assumption (Kowalczyk, 2015). The innovative approach forced by the main assumption in creating the model is the use of height differences between the stations instead of using absolute heights, which would eliminate the influence of the ellipsoid change in time and also the necessity to reduce ellipsoid heights by introducing a quasigeoid model.

The additional innovation also uses the algorithm that analyzes, verifies and determines a linear trend in time series, and evaluates its precision for time series decomposition (Rapiński and Kowalczyk, 2016). Vertical movements will be adjusted, not as a network of heights, but as a network of vertical movements, similarly as it was presented in the article by Kowalczyk and Rapiński (2013).

The vectors between the GNSS stations make a network of triangles (Fig. 1) which permits to use the loops misclosure criterion. Such solution is used for levelling and double-levelling networks (Kakkuri and Vermeer, 1985). To evaluate the loops closure criterion, the original formula presented in (Kowalczyk, 2015) can be used.

To depict the map of vertical crustal movements, the Kriging interpolation was used (Kowalczyk et al., 2010).

The aim of this article is to calculate the unadjusted trend based on GNSS time series and their adjustment in an area located in Central Europe. The article presents the robust adjustment method with weighting scheme taking into account differences in epochs and error a' posteriori.

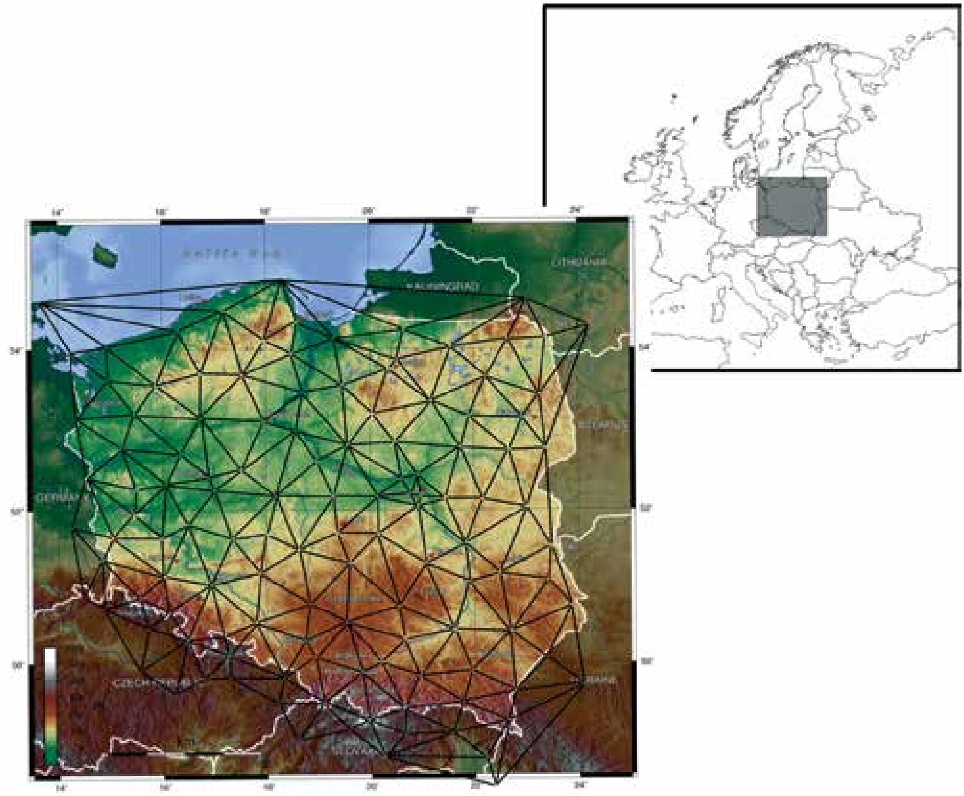

Figure 1. The baselines between the GNSS stations selected for the analysis (Central Europe) (topographic map © worldmap.pl by Webrange).

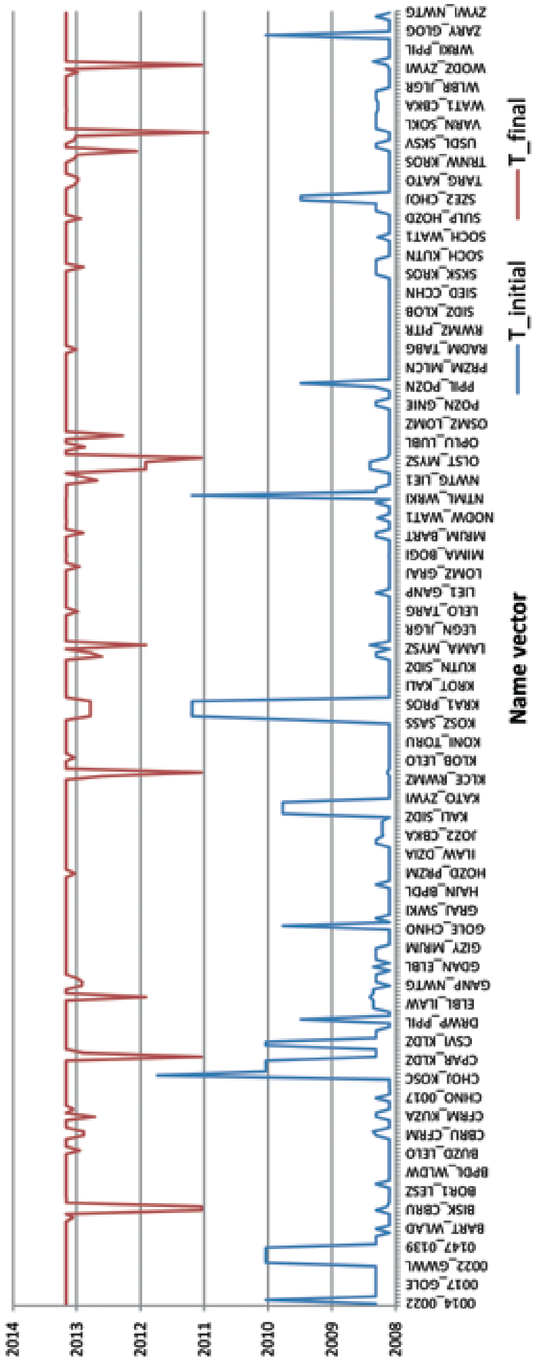

Figure 2. Initial and final epochs for particular time series on vectors.

2. Description and characteristics of data

The testing area included a part of Central Europe (Fig. 1) located on the West-European Platform, in the Sudetes, the Carpathians and on the East-European Platform. Three hundred forty-six height differences were estimated by the Military University of Technology EPN Local Analysis Centre (MUT LAC) based on the daily ASG EUPOS Polish permanent stations data and several stations from the neighboring countries: Lithuania (LITPOS), Germany (SAPOS), Czech Republic (CZEPOS) and Slovakia (SKPOS) (Kowalczyk et al., 2014).

The time series is presented in Fig. 2: seven vectors from 1 to 2 years, thirty-six vectors from 3 to 4 years, and the other vectors from 4 to 5 years. Figure 3 presents a theoretical number of measurement epochs in time series on vectors - from 1200 to 1850 daily epochs; the lower amounts are scarce. Epoch inconsistencies appear in all time series, *i.e.* there are single or several dozens of breaks of epochs. It makes from several to between ten and twenty percent of the theoretical number of epochs.

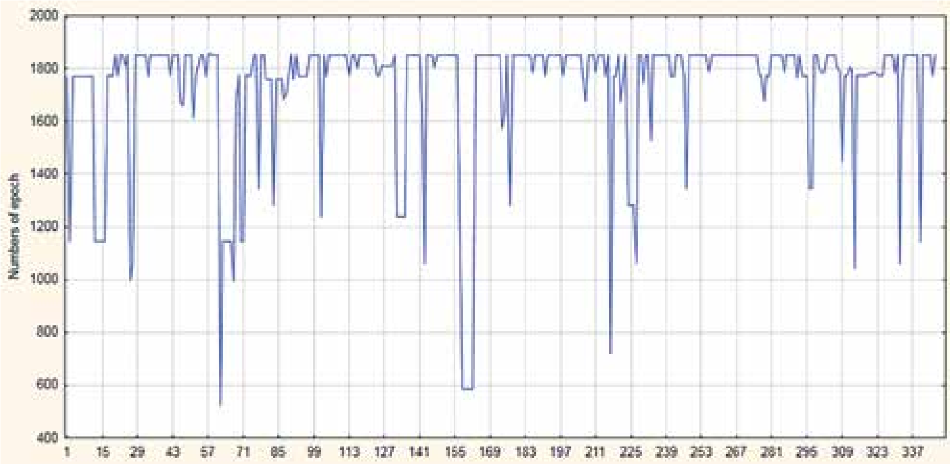

Figure 3. The characteristics of the number of measurement epochs for particular time series on vectors.

3. The estimation of unadjusted velocities of height change.

To estimate the velocity of height changes between GNSS stations (the linear trend), the algorithm presented in the work by Rapiński and Kowalczyk 2016, with later supplementation, was used. The algorithm *Vertical Switching Edge Detection (VSED)* is used to detect the discontinuities in time series and to calculate the values of "shifts" at the same time based on the least squares method.

To detect places in which "steps" occur, the switching edge detector algorithm was used (Smith, 1998). In the first step, moving averages H_{i+} and the variances were constructed using the n-point window (*e.g.* 1).

$$H_{i\pm} = \frac{\sum_{k=1}^{n} h_{i\pm k}}{n} \tag{1}$$

The next step was to construct $S_{i\pm}$ from these averages over two windows of n points (2) based on the moving averages (1).

$$S_{i\pm} = \frac{\sum_{k=1}^{n} \left(h_{i\pm k} - H_{i\pm}\right)^2}{n} \tag{2}$$

If t H_{out} for a certain epoch exceeds a threshold value μ then a "shifts" is detected. The value of μ is calculated on the basis of a priori error $m_{t\Delta v_{GNSS}^{N}}$. This error is calculated according to the formula (Kowalczyk, 2015):

$$\sigma_{\Delta v_{GNSS}^{N}} = 2.908\ \Delta T^{-1.349} \tag{3}$$

where ΔT – epochs differences ($T_i - T_e$).

The output function of the switching edge detector is defined as:

$$H_{out}(t_i) = g_{i+}H_{i+} + g_{i-}H_{i-} \tag{4}$$

where g_{i-} and g_{i+} are the switching factors defined as:

$$g_{i+} = \frac{S_{i-}^{2r}}{S_{i+}^{2r}+S_{i-}^{2r}} \qquad g_{i-} = \frac{S_{i+}^{2r}}{S_{i+}^{2r}+S_{i-}^{2r}} \tag{5}$$

The switching edge detector results in a **C** matrix containing zeroes and ones.

The mathematical model used in this article (Rapiński and Kowalczyk, 2016) is a straight line with "steps" in particular epochs:

$$h(t) = vt + h_0 + c_1 s_1 + c_2 s_2 + \cdots + c_m s_m \tag{6}$$

where:

t - epoch,

$h(t)$ - height difference in epoch t,

v - velocity between stations,

h_0 - height difference at epoch 0,

c_1, c_2, ... c_m - elements from matrix **C**,

s_1, s_2, ... s_m - magnitude of "shifts"

The final result of the calculations is the unadjusted linear trend on Δv_{GNSS}^N vector, the number of identified jumps and the mean error a posteriori of the determined $m_{p\Delta v_{GNSS}^N}$ trend. Figure 4 presents the estimated values and the calculated mean a posteriori error.

The trend values fluctuate from –3 mm/y to +3 mm/y, occasionally over the range, and to –6.5 mm/y at most.

The mean a posteriori errors of a trend fluctuate from ±0.0 mm/y to ±2.6 mm/y. It is highly correlated with the number of epochs (correlation coefficient 0.71) which results from the assumptions presented in Kowalczyk (2015), where the formula for (3) was derived. The correlation between the a posteriori error and the estimated linear trend is 0.15. In 21 cases (6% of the trial), the error exceeds ±0.6 mm/y and it is over ±1.0 mm/y in 7 cases (2% of the trial). In 70% of the trial, the error does not exceed ±0.1 mm/y.

The mean a priori error of $m_{t\Delta v_{GNSS}^N}$ trend was calculated on the basis of the assumptions presented in (Ihde and Augath 2002) (eq. 7) and Kowalczyk (2015) (eq. 3).

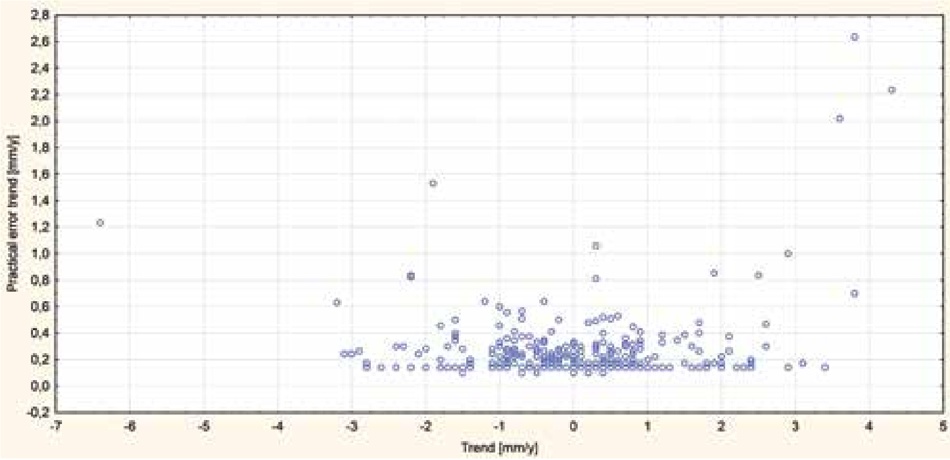

Figure 4. Unadjusted values of the linear trend and its mean a posteriori error.

$$m_{V_h} = m_h \sqrt{2}/\sqrt{365}/\text{year} \tag{7}$$

where: m_h - daily repeatability in the height component.

The a posteriori errors and a priori errors are presented in Fig. 5. The vast majority of the a posteriori errors is equal or below the a priori errors (80%). About 15% is slightly higher (from ±0.1 mm/y to ±0.2 mm/y), and in 6% of cases, the difference is higher than ±0.6 mm/y.

The correlation between the number of jumps and a posteriori error (Fig. 6) is 0.3 mm/y which indicates a lack of visible relation. A large number of jumps may show the influence of outer factors on the estimated differences in heights between GNSS stations.

Table 1 presents the list of vectors with the worst parameters.

4. The preparation of data for adjustment.

In the data adjustment process, the data should be reliable. To evaluate the loops closure criterion, the formula (*e.g.* 8) presented in (Kowalczyk, 2015) and (Kowalczyk and Rapiński, 2016) was used:

$$\varphi_L^{GNSS} = 1.5 \cdot \sum_{i=1}^{n} (\sigma_{\Delta v_{GNSS_i}^N}) \tag{8}$$

where: n - the number of vectors in the loop, considering the fact that the a priori error of the trend is taken as the $\sigma_{\Delta v_{GNSS}^N}$ standard deviation. Additionally, the loops misclosure was calculated with the use of the mean a posteriori error of the trend. The estimated values of particular loops misclosure are presented in Fig. 7.

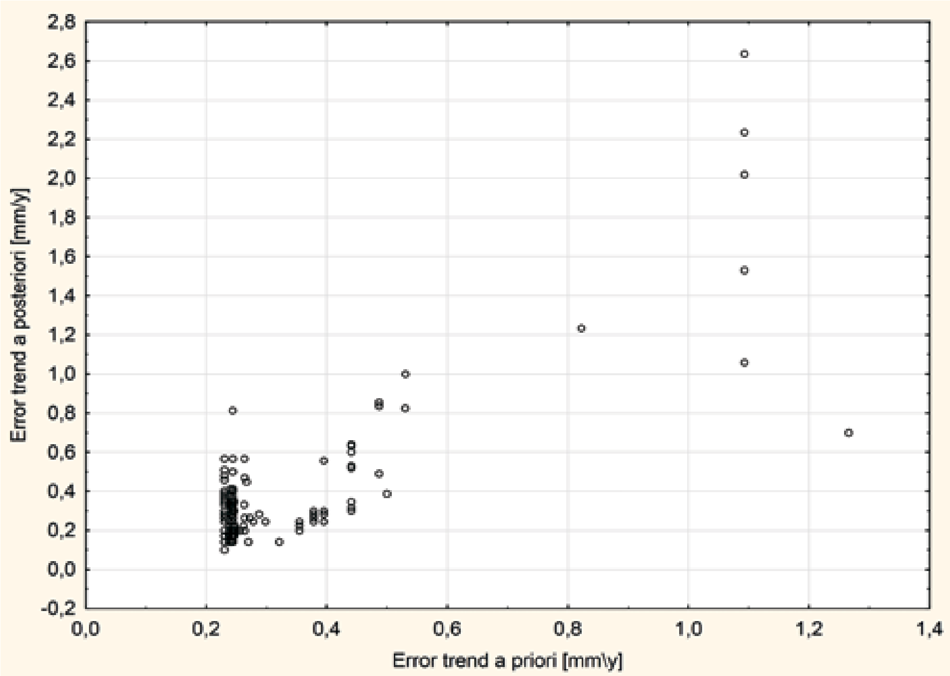

Figure 5. The comparison graph – a priori and a posteriori trend errors.

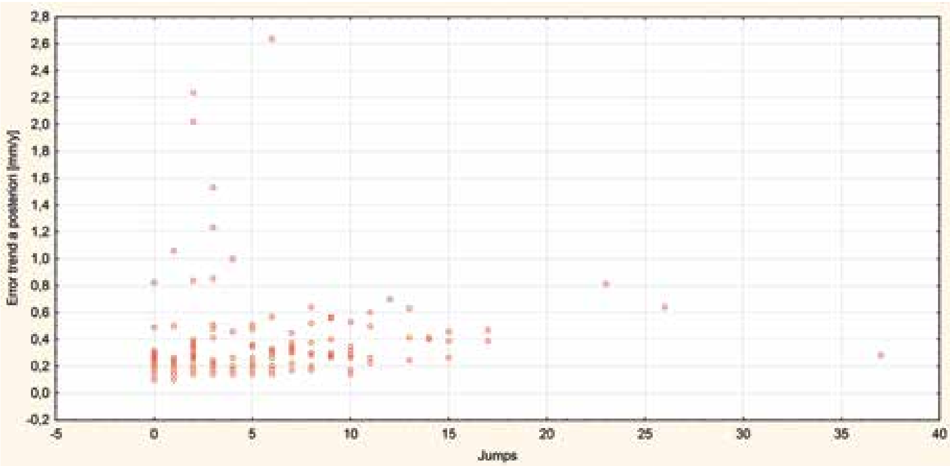

Figure 6. The list of the number of jumps and the a posteriori errors.

According to the conducted analyses, the material is diversified in the accuracy in estimating the trends. In 10% of cases, the maximal loops misclosure is exceeded including 4% cases, where the misclosure exceeds over half value of the accepted misclosure. The part of the data may negatively influence the reliability of the adjusted network. The solutions to the problems can be as follows:

Table 1. The characteristics of data of the estimated worst parameters.

Name	Jumps	Trend [mm/y]	Error trend a priori (*T*) [mm/y]	Error trend a posteriori (*P*) [mm/y]	Diferents errors trend (*T-P*) [mm/y]	Time interval [y]
0139_CLIB	13	–3,2	0,4	0,6	–0,2	3,1
0139_JLGR	26	–1,2	0,4	0,6	–0,2	3,1
0139_LEGN	8	–0,4	0,4	0,6	–0,2	3,1
0147_0014	11	–1	0,4	0,6	–0,2	3,1
BISK_CBRU	0	–2,2	0,5	0,8	–0,3	2,7
BISK_NYSA	2	–2,2	0,5	0,8	–0,3	2,9
CLIB_0147	12	3,8	1,3	0,7	0,6	1,4
CSUM_BISK	4	2,9	0,5	1,0	–0,5	2,7
KAM1_0781	9	–0,9	0,4	0,6	–0,2	3,4
KLDZ_**BISK**	2	2,5	0,5	0,8	–0,3	2,9
KRA1_KATO	6	3,8	1,1	2,6	–1,5	1,6
KRA1_LELO	1	0,3	1,1	1,1	0,0	1,6
KRA1_NWTG	2	3,6	1,1	2,0	–0,9	1,6
KRA1_PROS	2	4,3	1,1	2,2	–1,1	1,6
KRA1_ZYWI	3	–1,9	1,1	1,5	–0,4	1,6
NWSC_**KRA1**	3	–6,4	0,8	1,2	–0,4	2,0
NWSC_SKSL	9	–0,7	0,2	0,6	–0,3	4,9
PITR_KLOB	6	–0,7	0,2	0,6	–0,3	5,1
SKSL_NWTG	9	–0,7	0,3	0,6	–0,3	4,6
SKSV_UZHL	23	0,3	0,2	0,8	–0,6	4,9
WODZ_**BISK**	3	1,9	0,5	0,9	–0,4	2,9

1. Determining the weight of the trend and applying a robust method in network adjustment,

2. Identifying and removing the time series for which maximal loop misclosures exceeds a certain criterion,

3. Identifying and analyzing in details time series that cause the exceeding of maximal loop misclosures and a new determination of the trend,

4. Identifying and analyzing in details time series for which the a posteriori error is higher than the a priori error,

5. Moderating the loops misclosure criterion,

6. Removing all time series in which the a posteriori error is said to be greater than the determined criterion.

Considering the automatization of the calculation process, the realization of point 1 with determining the weight of the trend for all time series, identification and minimization of the influence of the series that cause the exceeding of maximal loops misclosures by the weight close to zero, or even removing the series

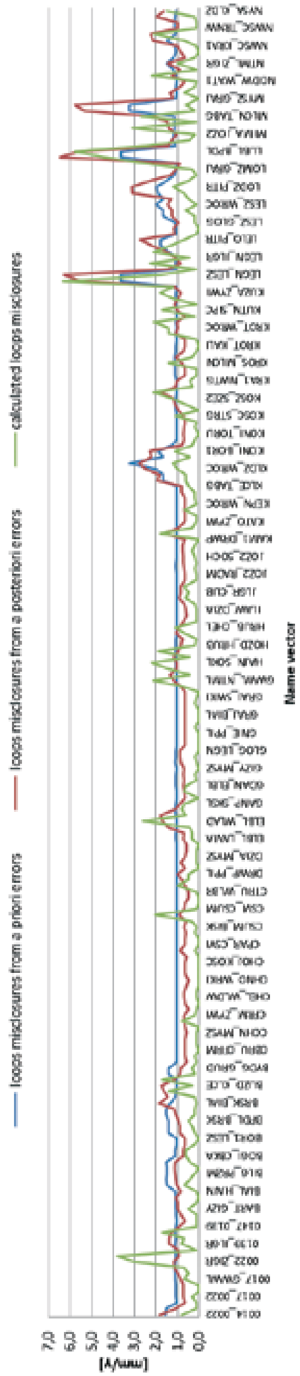

Figure 7. Maximal and estimated loops misclosure.

seem to be the most proper. In the first case, the network adjustment should be conducted with the robust method.

Taking into account the assumed accuracy of vertical crustal movements, for example, 0.1 mm/y or ±0.2 mm/y, the realization of point 6 would be the best solution. However, in the second case, there may be a necessity to reject numerous vectors (the loss of observation and results) which in the end may disturb its cohesion.

In both situations, one should identify the time series that do not fulfil the criteria or the accuracy. Based on the loops misclosure criterion, the identification of the "suspicious" time series should relate to the visual analysis:

- identifying the time series that appear in two neighboring loops and which do not fulfil the loop misclosure criterion,

- identifying the time series for which an unadjusted trend differs much from the neighboring values,

- identifying time series for which the a posteriori error of the trend is significantly bigger than the rest of the loop.

In the process of identification based on the loops misclosure criterion, there were 35 time series that fulfilled the above-mentioned criteria. The results of the identification are presented in Fig. 8.

In Fig. 8, the largest a posteriori errors of the estimated trends are marked with blue circles; white circles show the exceeding of the accepted loops misclosure; orange lines indicate the proportionally estimated values of the unadjusted trend; the time series that have an influence on unfulfilling the loops misclosure criterion are marked with dark blue. The biggest a posteriori error, trend values and loops misclosured appear around the KRA1 (station in Cracow). The station was excluded from the ASG EUPOS network in 2014. The distinct

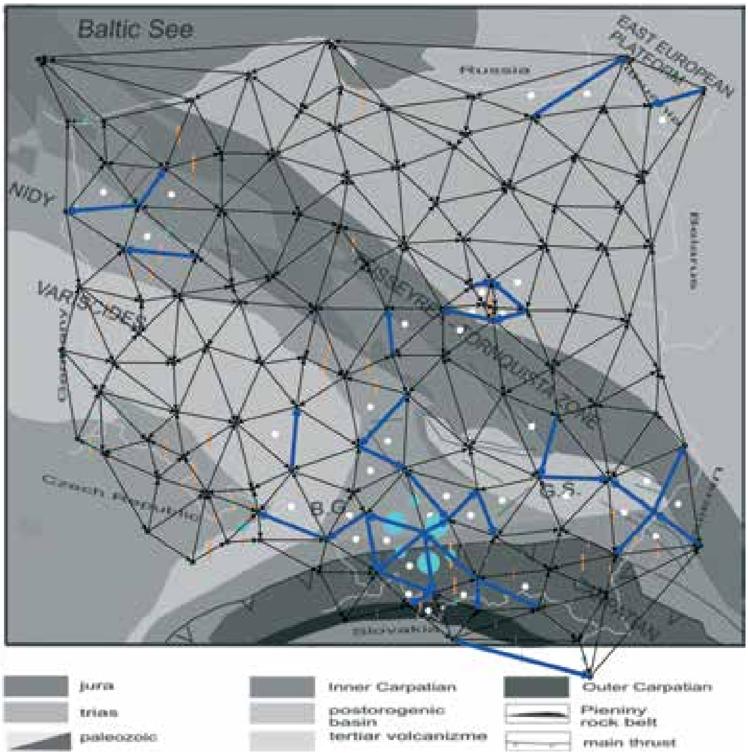

Figure 8. Results of the identification of time series that influence the unfulfillment of loops misclosure criterion.

relation between the appearance of the "suspicious" time series and the geological structure of the area were not stated.

5. Network adjustment

Before adjusting the network, the authors made the analysis of the occurrence of the a posteriori errors over ±0.2 mm/y in determining trends in the time series. The results of the analysis are presented in Fig. 9. Selecting vectors for which a posteriori error is smaller than ±0.2 mm/y causes the elimination of numerous observations and consequently, the velocities estimated at given points of the network cannot be determined precisely.

In the further adjustment process, the authors decided not to use the observation method in network adjustment.

Adjustment of the vertical crustal movements network is based on the following observation equation:

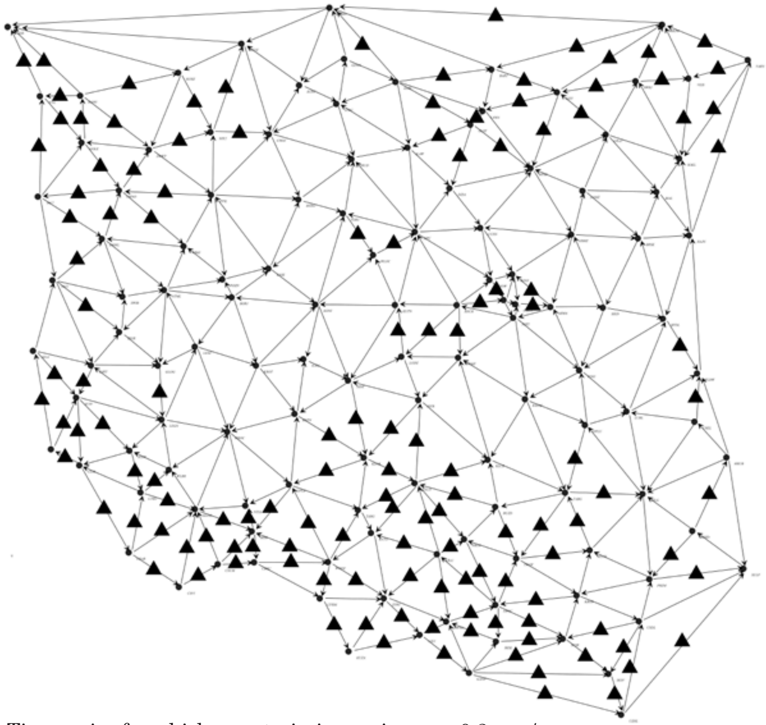

Figure 9. Time series for which a posteriori error is over ±0.2 mm/y.

$$v_{ij} = V_i - V_j \tag{9}$$

where v_{ij} is a relative velocity between two stations calculated on the basis of GNSS observations, V_i and V_j are station velocities for i and j stations respectively.

Hence, the equation of residuals takes the form:

$$\varepsilon_{ij} = V_i - V_j - v_{ij} + \xi_{ij} \tag{10}$$

where ε is a residuum and ξ is an unmodelled observation noise. The adjustment is based on the minimization of the following objective function:

$$\sum \varepsilon_{ij} p_{ij} \varepsilon_{ij} = min \tag{11}$$

where p_{ij} is a weight of an observation. In the standard approach, weights are calculated as:

$$p_{ij} = \frac{1}{\sigma_{ij}^2} \tag{12}$$

with σ_{ij}^2 being a mean a posteriori error of a particular observation. The objective function can be minimized using a standard least squares method. This approach is not immune to outliers in the observations. In order to minimize the impact of observations with large residuals, the iterative approach was introduced. In the first iteration, weights were calculated according to the equation (*e.g.* 12). In the second and further iterations, the weight of each observation was modified according to the attenuation function described by the equation (*e.g.* 13).

$$p_{ij}^{new} = \frac{p_{ij}}{2exp(-p_{ij}{}^{\varepsilon_{ij}^2/2})} \tag{13}$$

This reduces the weights of observations with large residuals in each iteration.

The iterative process is stopped when the increase in estimated parameters is smaller than 10^{-9} mm/y.

The adjustment was conducted in 4 variants:

1. Variant 1 – the adjustment in 1 iteration, the weights were

$$p_{\Delta v_{GNSS}^N} = \frac{1}{\left[m_{p\Delta v_{GNSS}^N}\right]^2} \tag{14}$$

2. Variant 2 – the adjustment in 5 iterations with the use of robust method; weights as in Variant 1.

3. Variant 3 - the adjustment in 1 iteration, weights were:

$$p_{\Delta v_{GNSS}^N} = \frac{\Delta T}{\left[m_{p\Delta v_{GNSS}^N}\right]^2} \tag{15}$$

4. Variant 4 – the adjustment in 5 iterations with the use of robust method; weights as in Variant 3.

WLAD station, located in the northern part of the network in the vicinity of a mareograph in Wladysławowo, was chosen as the main point of the network.

The results of the adjustment are shown in Fig. 10.

In Variant1 and Variant 3, the values of trends in network points were similar to the mean errors. Variant 1 visibly indicated centers that caused local changes in trends. In Variant 3, the influence was limited by considering the time of a station operation in the weighting process. In Variant 2 and Variant 4, the velocities were similar to those in Variant 1 and Variant 3, the mean errors were twice smaller than in Variant 1 and Variant 3, and they were evenly distributed. In Variant 2, like in Variant 1 and Variant 3, local centers with higher values of mean errors were indicated. In Variant 4, the influence of the

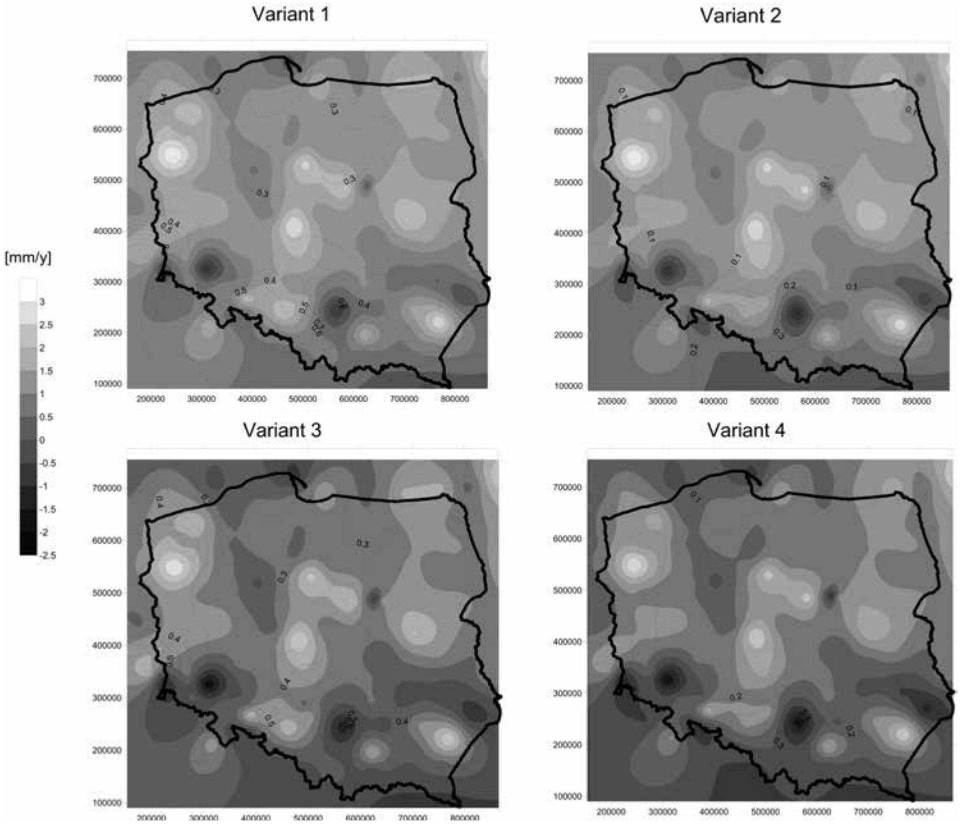

Figure 10. Variants of vertical crustal movements adjustments with the use of GNSS data; isolines mark the distribution of mean errors of the adjusted linear trend in points of the network.

"suspicious" time series was limited, mean errors were evenly distributed, local centers occurred occasionally. In all four variants, there was a local center with diverge values of mean errors in a point. They were places around KRA1 station (Cracow) which was excluded from the ASG EUPOS network.

6. The adjustment evaluation

The authors observed and compared the smallest weights (Variant 4) with the "suspicious" time series derived from the used analysis. The results are presented in Fig. 11.

Most of the indicated places were compatible. Single differences appeared scarcely. The adjustment with the use of the robust method permitted a more reliable identification of the "suspicious" time series and minimizing their influence in the final vertical crustal movement network.

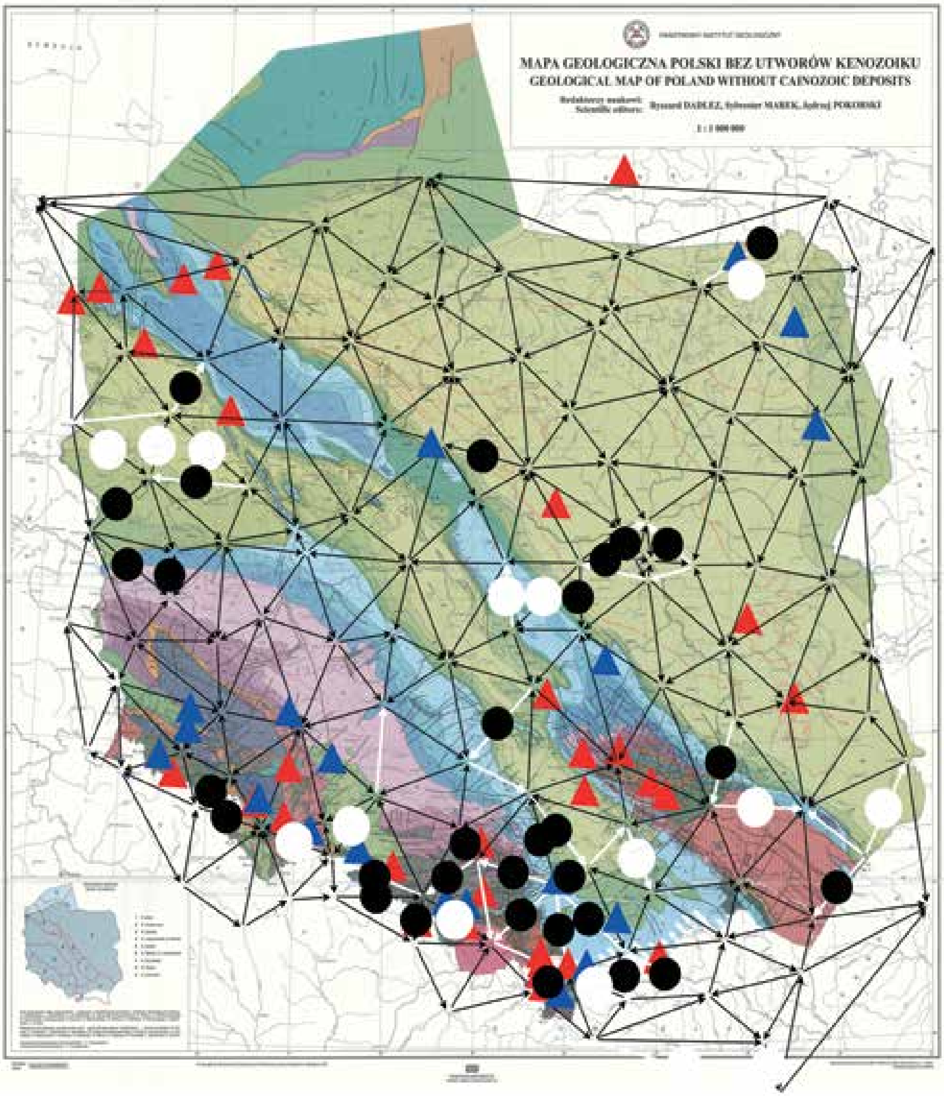

Figure 11. The smallest observation weights for Variant 4 (black and white circles), vectors identified as "suspicious". The localization of natural seismic activities epicenters (triangles) on the geological map of Poland is used as a background image (Lewandowska-Maciniak and Guterch, 2002).

Figure 12 presents the trends values and mean errors in particular variants, and their histograms in comparison to their normal distribution.

The smallest mean errors were received in Variant 4, between ±0.08 to ±0.32 mm/y (singly ±0.7 mm/y). In all the variants, the distribution of the collected point velocities was close to the normal distribution.

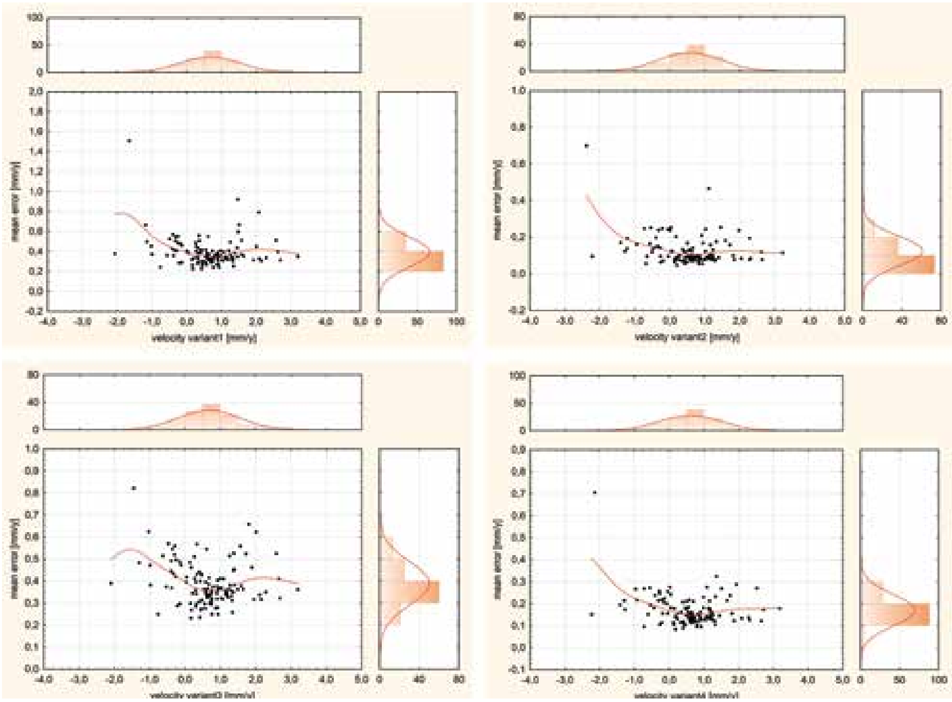

Figure 12. Trend values and mean errors after adjustment in particular variants.

7. Summary

The differential method (the relative method) permits the evaluation, control and improvement of results in particular stages of creating models of vertical crustal movements. The conducted analyses show that it is possible to create a model of vertical crustal movements from GNSS data diversified in time with the accuracy of ±0.5 mm/y and even below ±0.3 mm/y with the use of the method. However, it depends on five main factors: the GNSS station operation time, the reliability of the determined trend from time series (a posteriori error), proper selection of weights, and the choice of the main point in the networks and the selection of the adjustment method. The use of the relative method permits to include new GNSS stations as well as to evaluate the reliability of data gained by the stations.

Establishing the reliability criterion of the observation (the unadjusted linear trend) in adjustment process is an essential element. The elimination of ostensibly the worst connections may make the creation of a reliable model impossible. As it was shown in the conducted research, it does not always happen that the a posteriori error is lower than the a priori error. It is essential to consider the working time of the stations in the weighting process.

The robust method that regards the above-mentioned weights deals well with the "suspicious" time series as it shows them more precisely than it can be done visually, owing to which one can evade the element in the process of creating the model. The place that shows more mean errors of nodes of the network should be analyzed, and the connections with them should possibly be eliminated (KRA1 station). Permanent stations that belong to both national and private networks have longer and longer operation times (over 5 years), which enables the use of the relative methods in creating the model of vertical crustal movements from GNSS data with the accuracy close to ±0.1 mm/yin nodal points of the network.

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SAŽETAK

Robusna prilagodba vertikalnih pomaka na osnovi podata GNSS mreže

Kamil Kowalczyk i Jacek Rapiński

Na osnovi dugogodišnjeg niza podataka GNSS postaja omogućena je analiza vertikalnog gibanja kore s točnošću većom od –0,5 mm/god. Tijekom same analize potrebno je razmatrati pouzdanost, a što uključuje odabir metode DETINIRANJA MREŽE vertikalnih pomaka kore. U većini slučajeva te metode su dizajnirane apsolutne te povezane s elipsoidom, gdje se pomak računa za postaje procijenjenih koordinata. Druga mogućnost je odabir diferencijalnih relativnih modela, gdje se koriste GNSS vektorske koordinate. U tom slučaju, su postaje međusobno GNSS mreže povezane te se računa vertikalni pomaci između njih. U idućem koraku se vrijednosti pomaka kore podešavaju te se procjenjuje se pouzdanost. Cilj ovog rada je izračuna i prilagodba vrijednosti vertikalnih pomaka kore na osnovi vremenskih nizova zapisa GNSS mreže postaja na području središnje Europe.

Članak prikazuje robusnu metodu prilagođavanja s težinskom shemom. Dobiveni rezultati pokazuju da je na osnovi korištenih podataka GNSS mreže moguće postići pouzdanost od 0.5 mm / god. Također, naglašena je prednost primjene navedene metode.

Ključne riječi: vertikalni pomaci kore na osnovi GNSS podatak, robusna metoda prilagođavanja, težine pri prilagodbi

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