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# Quality evaluation of height movement kinematic model of the Earth's crust on the Croatian territory

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The paper presents the empirical use of height movement kinematic model of Earth's crust created for territory of the Croatia, Slovenia and Bosnia and Herzegovina, in order to determine the relative height displacements of crust discrete points between different epochs. Also, presents the use of this model for the purpose of direct levelling measurement reductions determination, from surveying epoch to another unambiguously selected epoch, *i.e.* in purpose of height movement effects elimination from levelling network node benchmarks. For Croatian territory the quality of the kinematic model is indirectly tested, founded on the comparison of levelling measurement accuracy criteria related to the state II order levelling networks constituted with original and reduced levelling measurements. Levelling lines of two levelling networks of the II order, on the area of two deliberately selected and representative levelling loops of the Croatian state levelling network of I order, were analyzed. An accuracy criterion, using the original measurements and comparatively using the reduced measurements from their survey epoch to the I order network surveying epoch, is determined. Comparative comparison of the original and reduced measurement accuracy criteria is not clearly and unequivocally confirmed, nor completely rejected, the adequacy of the kinematic model quality for measurement reductions determination. However, comparison points to the fact that the quality of the kinematic model enables reliable determination of the relative height displacements at the centimeter level.

*Keywords*: kinematic model, height displacements, height movement, quality, measurement reduction, Croatia

## 1. Introduction

For the territory of Croatia, Slovenia, and Bosnia and Herzegovina the kinematic model of recent movements of the Earth's crust is created, which is presented and explained in the Rožić (2015). This is a mathematical model that defines the law of uniformly accelerated or decelerated relative height movement of topographic surface, implying that as a contact surface between the solid (crust) and fluid Earth (atmosphere & hydrosphere) represent movements of the crust as a whole. According to Rožić and Razumović (2014) and Rožić (2015) kinematic model is created from the models of relative height displacements of topographic surface, which are the result of data processing and data modeling of geometric levelling networks of the highest accuracy order, *i.e.* so-called I order networks, Rožić and Razumović (2010) and Rožić et al. (2011). Namely, the survey data of three I order geometric levelling networks, carried out in the period from 1874 to 1973 and related to the three explicit mean epoch's 1892.8, 1949.0 and 1971.1, enabled the relative height displacement models creation, Rožić (2001). Since the levelling networks are established on the territory of Croatia, Slovenia, and Bosnia and Herzegovina, the coverage area of relative height displacement models, and consequently of the kinematic model, refers to the area defined by the boundaries of these neighbouring states. This is a quite inhomogeneous area due to the different relief, geological, tectonic, geophysical, geodynamic and other relevant properties. Furthermore, in accordance with the epoch's to which levelling networks surveying data are related, the use of the relative height displacement models and the kinematic model is acceptable only within the time interval from 1874 to 1973.

The kinematic model of the relative height motion of the Earth's crust, presented in Rožić (2015), consists of two components. The first model component consists of the fundamental equations of uniformly accelerated or decelerated relative height movement of topographic surface discrete material points. These are the equations to determine the points height distance traveled

$$\Delta H_i = F(\Delta H_o, v_o, a, t_o, t_i), \tag{1}$$

*i.e.* the points relative height displacements that was occurring between the epoch $t_i$ and the model initial epoch $t_o$, and the equations to determine the height movement present speed of these points in epoch's $t_i$,

$$v_i = G(v_o, a, t_o, t_i). \tag{2}$$

The second model component makes up of the grid models of kinematic parameters, *i.e.* the grid model of points relative height displacements $\Delta H_o$ in the initial epoch $t_o$, the grid model of points speed $v_o$ in the initial epoch $t_o$ and the grid model of points constant acceleration $a$. Grid models of kinematic parameters are created as a homogeneous rectangular grids, with grid cells roughly of square shape, having the same dimensions, resolutions and positions as referenced in relation to the body of the Earth. All grids nodes are unambiguously related to the unique values of the appropriate kinematic parameter that have dependent on modeling. Also, each grid node is unambiguously defined by an ellipsoidal position on the Bessel reference ellipsoid, using ellipsoidal longitude $\lambda$, referred to the zero meridian in Greenwich and ellipsoidal latitude $\varphi$. Grid models of kinematic parameters can be defined by 3D rectangular reference coordinate system, as ordered sets of nodes $(\lambda, \varphi, \Delta H_o)$, $(\lambda, \varphi, v_o)$ and $(\lambda, \varphi, a)$, which are the discrete points of belonging continuous spatial model surfaces. Grid models allow, using bilinear interpolation, unambiguous determination of the kinematic parameter values for any point of known ellipsoidal position ($\lambda, \varphi$). It is assumed that the kinematic parameters between grid nodes, or within a particular grid cell, linearly and continuously change their amount in both explicit directions, *i.e.* in the direction of the ellipsoidal longitude and latitude. That fact is essentially the result of the theoretical hypotheses introduced during the creation of the relative height displacement models, Rožić et al. (2011). Namely, the hypothesis that changes in the Earth's crust geometry and topographic surface height movements are continuous, so no landforms such as faults exists, which would influence on the discontinuities of the model surfaces.

Described kinematic model, which is exposed and explained in Rožić (2015), is available for download from the Internet, Rožić (2016). It should be noted that the model can be used only with the prerequisite of understanding its characteristics and quality. Model quality is basically determined by the quality of the used levelling source data, by the methods and techniques used in creation of the relative height displacement models and by the methods and techniques used in the creation of a kinematic model. Indicators of consistency, reliability and accuracy of source levelling data and the levelling networks are shown in Rožić (2001), inner accuracy of the relative height displacement models in Rožić et al. (2011) and Rožić (2015) and inner accuracy of the kinematic model in Rožić (2015). However, thorough analysis of the kinematic model external accuracy has not been yet systematically carried out. The fundamental problem is the lack of a proper set of control data, *i.e.* data that would be completely independent of the data used during the model creation, that would be of appropriate accuracy and that would be in number and distribution along the model coverage area homogeneously distributed. Such a control data set, if would be possible to set it up, could be formed using explicit empirical data of levelling network benchmarks relative height displacements $\Delta H_e$, taking into account their known ellipsoidal position ($\lambda, \varphi$), known explicit displacement epoch's, finally making a control data set consisting of triplets ($\lambda, \varphi, \Delta H_e$). In this case, benchmark empirical displacements $\Delta H_e$ and displacements $\Delta H$ generated from the kinematic model could be directly comparable. On the basis of differences, or discrepancies $\varepsilon = \Delta H - \Delta H_e$, the appropriate criteria of model external accuracy could be quickly and easily determined. But, by a combination of objective circumstances, creation of such control data set is not possible because all the available benchmarks contained in levelling networks of I order on the territory of Croatia, Slovenia, and Bosnia and Herzegovina have already been put to use, *i.e.* were used in the displacement models and kinematic model creation. The described method of evaluating the external quality of the kinematic model could be considered as a direct method, because relevant criteria of model external accuracy, such as the mean square error or standard deviation, can be determined directly from empirical differences $\varepsilon$.

According to the impossibility of the direct method application for evaluating the external quality of the kinematic model, there is a need of finding a suitable alternative procedure, *i.e.* the method that would indirectly evaluate the level of model quality. In general, such indirect method can be derived from the context of the realization of state height reference systems. As part of their realization is necessary, in accordance with the hierarchical principle, to integrate all the geometric levelling networks of the lower accuracy orders, *i.e.* the network of II order, III order, etc., that have been established on the state territory, with the state levelling network of I order. Since these networks, or levelling loops and lines that make them, are surveyed in different epoch's as a rule, and considering that according to the networks geometric configurations always several levelling lines are connected to the same node benchmarks, it is obvious that the height differences of levelling lines undoubtedly depend on benchmarks height movement or better to say vertical movement of the Earth's crust. It is quite realistic to expect that in the longer periods between field surveys of different levelling lines, but connected to the same benchmarks, the Earth's crust height displacements would achieve higher amounts, especially in case when Earth's crust motion is of significant amounts. In the realization of national height reference system it's certainly prudent, before lower order networks adjustment, to do the reduction of directly measured height differences of levelling lines from their survey epoch to the epoch of realization of state I order levelling network. In this context, the effects of height movements of the Earth's crust, reflected on the nodal benchmarks height position, can be qualified as systematic effects that lead to corresponding systematic height "errors" which in significant level may affect on the relative height measurement accuracy of levelling lines and the absolute accuracy of the benchmark height positions included in the height reference system, calculated on the basis of networks adjustment. These errors are indirectly contained in the height difference survey measurements data, since the levelling measurements are made in the exact but mutually different epochs in which the benchmark nodes had very particular height position. The systematic nature of these errors should not be questionable since the movements of the Earth's crust has a clear and significant systematic trend, at least in case of the Croatia, Slovenia and Bosnia and Herzegovina.

As for Croatian territory surveying data of II order levelling networks are available, it's possible to analyze the impact of the Earth's crust height movements on the accuracy of levelling measurements of these networks, and it's possible to compare the measurement accuracy criteria belonging to the originally surveyed and to the reduced levelling measurements. Reduction of measurements is allowed by the use of the kinematic model and it's clear that the reductions quality, or the quality of elimination of Earth's crust height movement errors from the surveyed height differences, on the indirect way can evaluate the kinematic model quality.

Should also be noted two facts, which in this particular case one should not forget. The fact that on the total Croatian territory surveying epoch's of II order levelling networks belong to longer time interval, from 1945 to 1963, and at the same time they are definitely different from the survey epoch of the Croatian I order levelling network which served as the basis for the realization of the official Croatian vertical reference system, *i.e.* the period from 1971 to 1973. Also, that "a priori" introduced accuracy measurement standard of II order levelling networks, which corresponds to the class of so-called precise levelling, is sufficiently demanding to allow recognition of errors caused by the height movement of the Earth's crust. These systematic errors certainly must have an impact on the height difference measurement accuracy, depended on the level of intensity and size of benchmarks height movements.

In accordance with the above, in the first part of this paper explicit application of the kinematic model for the purpose of the reduction calculation of levelling lines height differences, to eliminate systematic errors of Earth's crust height movements, is presented. In the second part of the paper the quality of the kinematic model on Croatian territory using indirect method is tested, based on the comparison of levelling measurement accuracy criteria determined from originally surveyed and reduced levelling measurements data contained in II order levelling networks realized in the areas of two purposely selected levelling loops of the Croatian I order levelling network.

## 2. Height differences reduction calculation

Height difference $\Delta h$ of any levelling line contained in levelling network is measured between the two specific benchmarks of that line, *i.e.* benchmarks that define the start and the end of the line. Generally, these benchmarks are so-called nodal benchmarks of levelling network, depending on the network geometric configuration. For any network node benchmark $R$, with known ellipsoidal position ($\lambda$, $\varphi$), the relative height displacement between two different epoch's $t_A$ and $t_B$ can be determined using the kinematic model. According to Rožić (2015), that displacement is determined by the expression

$$\Delta H_{AB} = \tfrac{1}{2}\Delta t_{AB}\left(2v_o + a\,(t_A + t_B - 3748.0)\right), \tag{1}$$

where symbols have the following meaning:

$\Delta t_{AB} = t_B - t_A$ - time interval between older $t_A$ and younger epoch $t_B$,

3748.0 - double amount of kinematic model initial epoch ($t_o$ =1874.0),

$v_o$- benchmark speed parameter at kinematic model initial epoch $t_o$,

$a$ - benchmark acceleration parameter.

The expression (1) is essentially resultant of two equations of benchmark $R$ height motion, which determine its distance traveled, or relative height displacement between the epoch's $t_A$ and $t_B$, but in relation to the initial epoch $t_o$ of kinematic model:

$$\Delta H_A = \Delta H_o + v_o\,(t_A - 1874.0) + \tfrac{1}{2}a\,(t_A - 1874.0)^2, \tag{2}$$

$$\Delta H_B = \Delta H_o + v_o\,(t_B - 1874.0) + \tfrac{1}{2}a\,(t_B - 1874.0)^2. \tag{3}$$

In fact, in the expressions (2) and (3), in relation to the expression (1), the initial position of the benchmark $\Delta H_o$ in the epoch $t_o$ appear as a parameter of the kinematic model. Difference of equations (2) and (3)

$$\Delta H_{AB} = \Delta H_B - \Delta H_A, \tag{4}$$

after elementary mathematical arrangement, gives the expression (1). Therefore, empirical determination of benchmark displacements is certainly possible directly by using the expression (1) or alternatively by using expressions (2), (3) and (4).

Mutual relationship of the relative height displacements $\Delta H_o$, $\Delta H_A$, $\Delta H_B$ and $\Delta H_{AB}$ is presented in Fig. 1. It shows the trajectory of uniformly accelerated or decelerated motion of benchmark $R$ (parabola) depending on the course of time, *i.e.* depending on the epoch's $t_o$, $t_A$ and $t_B$. It should be noted that the sign of the relative height displacements, determined by expression (1) or (4), defines the direction of benchmark height movement starting from the older to the younger epoch. The "+" sign means the elevation of benchmark and the "–" sign benchmark lowering. Benchmark kinematic parameters $\Delta H_o$, $v_o$ and $a$ are contained in the grid models of kinematic parameters visualized in Figs. 2–4, Rožić (2015). These parameters are uniquely determined depending on the benchmark $R$ ellipsoidal position $R\,(\lambda, \varphi)$.

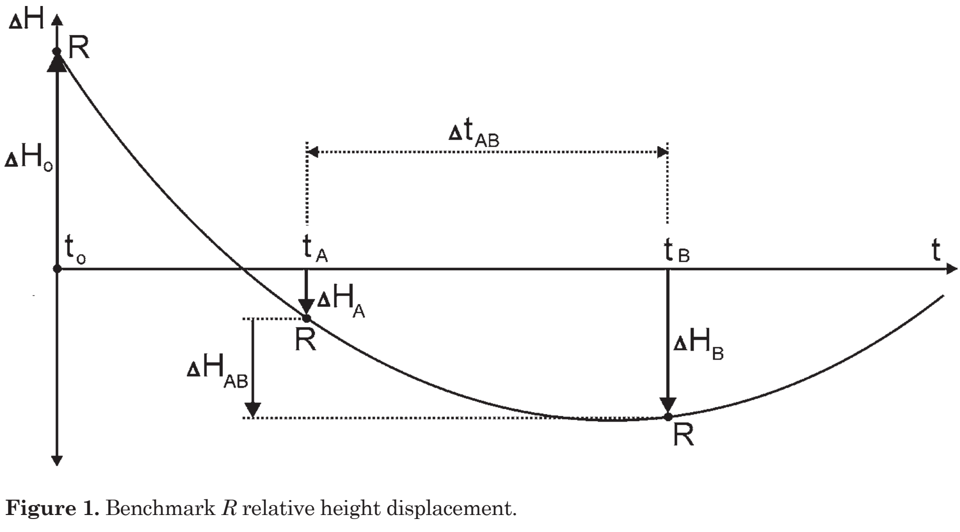


**Figure 1.** Benchmark $R$ relative height displacement.

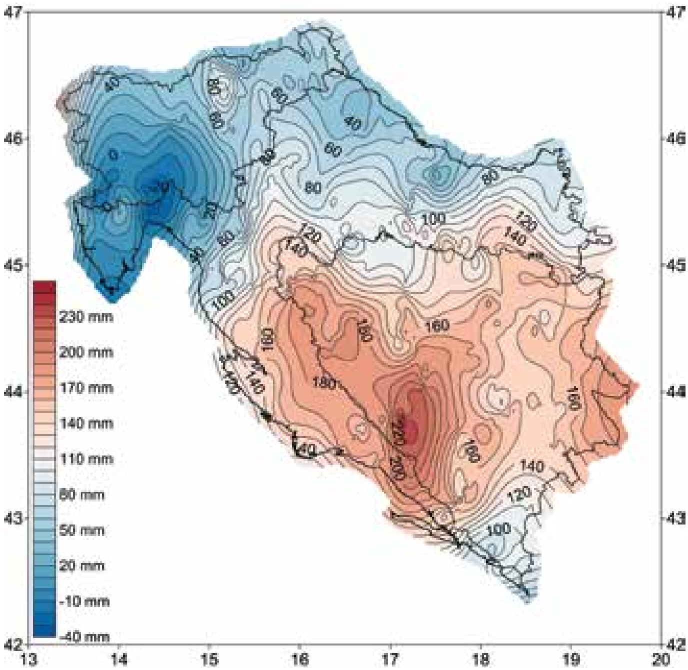


**Figure 2.** Grid model - parameter $\Delta H_o$.

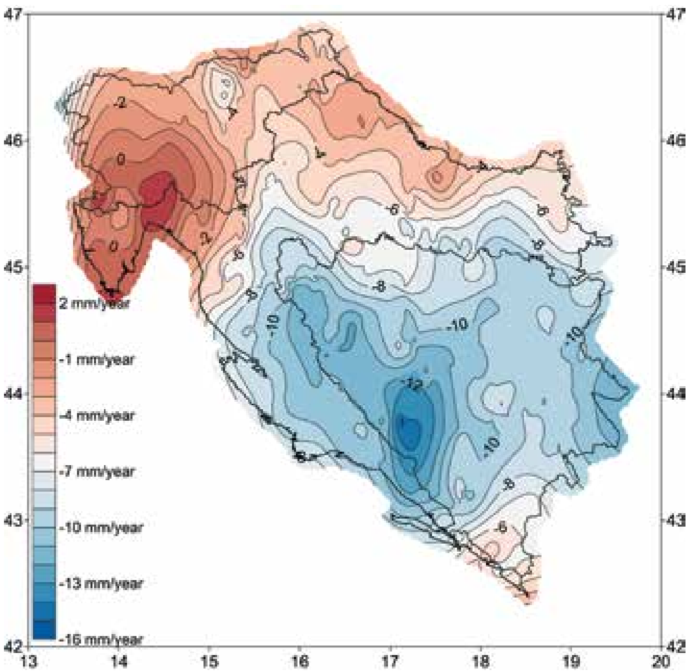


**Figure 3.** Grid model - parameter $v_o$.

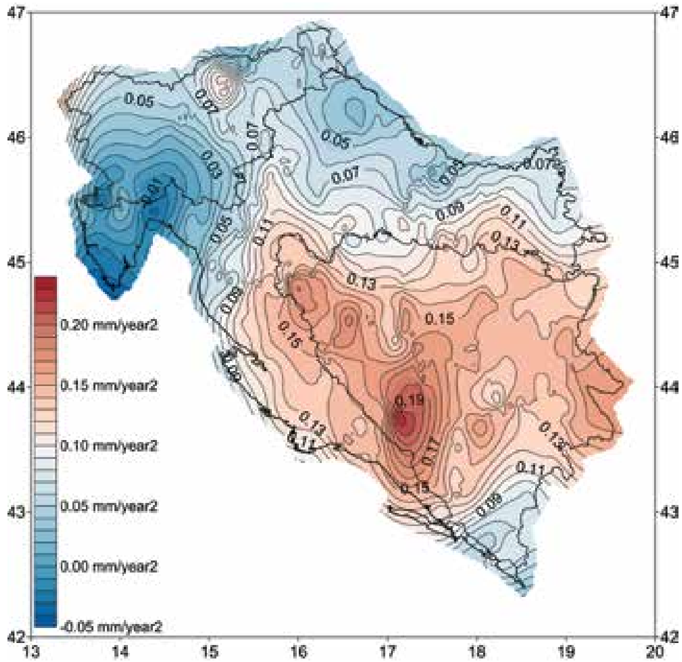


**Figure 4.** Grid model - parameter $a$.

Example of calculating the relative height displacements for two node benchmarks of II order levelling network in northern Croatia, which are defining levelling line no. 525 (Ivanić Grad - Bjelovar) of the total length of 56.39 km, clearly demonstrates the ease of the kinematic model application. These are benchmarks no. 2097 ($\lambda$ = 16° 24' 11", $\varphi$ = 45° 42' 26") and no. CMLXVIII ($\lambda$ = 16° 50' 58", $\varphi$ = 45° 53' 39"), which are connected using the precise levelling during 1949, by measuring the height difference in the amount of 21.2719 m. It's interesting to find the relative height displacements of these benchmarks in the timeframe given by levelling line survey epoch ($t_A$ = 1949.0) and by mean survey epoch of the Croatian I order levelling network which served as the basis for the realization of the Croatian height reference system ($t_B$ = 1971.1).

Based on benchmarks ellipsoidal position, and using the grid models of kinematic parameters $v_o$ and a presented in Figs. 2 and 3, kinematic parameters of benchmark motion:

- No. 2097, $v_o$=−4.9 mm/year, $a$ = 0.068 mm/year$^2$,

- No. CMLXVIII, $v_o$=−4.2 mm/year, $a$ = 0.057 mm/year$^2$,

enable, by the expression (1), determination of their relative height displacements:

- No. 2097, $\Delta H_{AB}^{2097} = 21.7\,\text{mm}$,

- No. CMLXVIII, $\Delta H_{AB}^{CMLXVIII} = 16.7\,\text{mm}$,

taking into account the time interval between epoch's $\Delta t_{AB} = t_B - t_A = 22.1$ year.

Alternatively, based on the expression (2) and (3) and the additional use of a grid model of kinematic parameters $\Delta H_o$, Fig. 2, follows:

- No. 2097, $\Delta H_o^{2097} = 79.2\,\text{mm}$,

- No. CMLXVIII, $\Delta H_o^{CMLXVIII} = 68.4\,\text{mm}$,

so relative benchmark height displacements regarding kinematic model initial epoch $t_o = 1874.0$ are determined:

- No. 2097, $\Delta H_A^{2097} = -94.1\,\text{mm}$, $\Delta H_B^{2097} = -72.4\,\text{mm}$,

- No. CMLXVIII, $\Delta H_A^{CMLXVIII} = -83.9\,\text{mm}$, $\Delta H_B^{CMLXVIII} = -67.2\,\text{mm}$,

and by means of expression (4) benchmark displacements between epoch's $t_A$ i $t_B$:

- No. 2097, $\Delta H_{AB}^{2097} = \Delta H_B^{2097} - \Delta H_A^{2097} = 21.7\,\text{mm}$,

- No. CMLXVIII, $\Delta H_{AB}^{CMLXVIII} = \Delta H_B^{CMLXVIII} - \Delta H_A^{CMLXVIII} = 16.7\,\text{mm}$.

Based on this example, it is clear that the use of a kinematic model to determine the benchmarks relative height displacements between different epoch's is really very simple. As the only demanding operation during calculation occurs the identification of the right cells in the grid models of kinematic parameters in accordance with the benchmarks position and application of bilinear interpolation.

Based on nodal benchmarks relative height displacements, reduction determination of directly measured levelling lines height differences from survey epoch's to any other epoch's is quite simple. According to Rožić (2015), height difference correction or reduction $r_{\Delta h}$ of directly measured levelling line height difference $\Delta h$, from an epoch $t_A$ to epoch $t_B$, is determined by the height displacements difference of levelling line starting benchmark $R_1$ and ending benchmark $R_2$ (network nodal benchmarks), ie.

$$r_{\Delta h} = \Delta H_{AB}^{R_2} - \Delta H_{AB}^{R_1}, \tag{5}$$

so the reduced height difference is

$$\Delta h_r = \Delta h + r_{\Delta h}\,. \tag{6}$$

The amount of the height difference reduction $r_{\Delta h}$, compared to the relative height displacements of both benchmarks is clearly illustrated in Fig. 5.

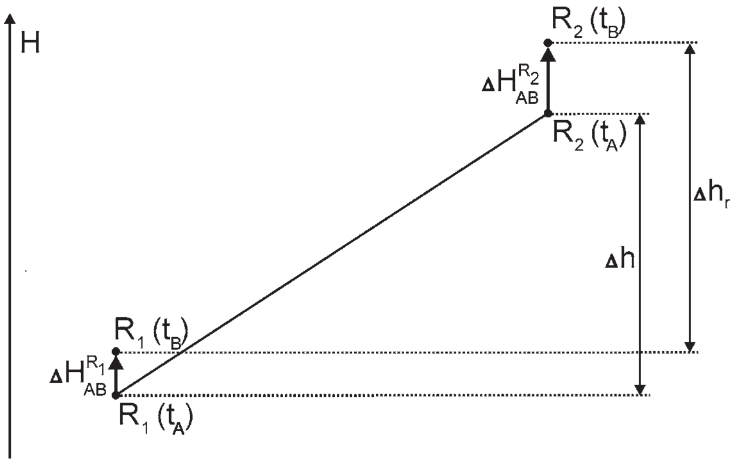


**Figure 5.** Height difference reduction regarding epoch's $t_A$ i $t_B$.

Following the previously presented example of benchmarks no. 2097 and no. CMLXIII forming the levelling line no. 525, reduction of originally measured height difference, in the amount of 21.2719 m, from the survey epoch 1949.0 to epoch 1971.1 is

$$r_{\Delta h} = \Delta H_{AB}^{CMLXVIII} - \Delta H_{AB}^{2097} = 16.7 - 21.7 = -5.0\,\mathrm{mm}$$

and finally the reduced height difference is

$$\Delta h_r = \Delta h + r_{\Delta h} = 21.2719 - 0.0050 = 21.2669\,\mathrm{m}.$$

It should be noted that in presented example the data to calculate the benchmarks relative height displacements and the amount of reduction of the levelling line measured height difference is consistent and in conformity with the intensity of the topographic surface kinematics, which is contained in the kinematic model in the area covering position of the levelling line no. 525. Benchmarks no. 2097 and no. CMLXVIII, which are located at a mutual distance of about 56 km, shows the same trend of height movement with respect to the direction and amount, although the amount of height displacement is not entirely coincidental. The amount of directly measured height difference reduction is – 5.0 mm, and as compared to the time interval of 22.1 years at first sight is not excessively large. However, it is significant in relation to “a priori” adopted Croatian standard of precise levelling measurement accuracy (II order levelling network). It is defined in Croatia with maximum allowed reference probable error for the impact of random errors in the amount of ± 2.0 mm/km and reference probable error for the impact of systematic errors in the amount of ± 0.4 mm/km.

Based on the reduction of the measured height difference it can be stated that height difference of the levelling line no. 525 in the epoch 1971.1 should be 5.0 mm smaller than it was in the epoch of its survey.

It is clear that at the state territory, by kinematic model use, reductions of the measured levelling line height differences belonging to the lower accuracy orders easily can be calculated, from their respective survey epoch's to the epoch of realization of the I order state network, originally used to realize the state height reference system. It can be set up a logical and realistic hypothesis that elimination of the systematic impact of benchmarks height movement should contribute to the realization of a more accurate height system, and should be visible in comparatively calculated measurement accuracy criteria using the original and reduced measurements.

## 3. Levelling data for the kinematic model quality testing

The height reference system of the Republic of Croatia was initially realized in 1992 by the I order levelling network which belongs to the class of so-called levelling of high accuracy. The network is surveyed from 1970 to 1973 and established covering the western part of the former Yugoslavia, Rožić (2009). Mean epoch of network realization, *i.e.* mean epoch of network levelling lines field survey, is 1971.1. The network is in the Croatian scientific and technical literature known as Second levelling of high accuracy or abbreviated IINVT. So far it's the youngest and highest quality levelling network which is realized on the Croatian territory, Tir et al. (2013). In a narrow sense the height system is realized by the IINVT network adjustment, using the indirect measurements model and the method of least squares. On 1 January 2010 the system was introduced into service as an official height reference system of the Croatia or abbreviated HVRS71. The system is normal-orthometric, because during the IINVT network survey gravimetric measurements were not done. System is oriented relative to the body of the Earth, by using tide gauge data from tide gauges in Koper, Rovinj, Bakar, Split and Dubrovnik. Like reference height surface geoid was adopted. Geoid surface height position at the locations of all tide gauges is fixed. Absolute heights of tide gauge benchmarks over the mean level of the Adriatic Sea are determined from the continuous tide gauge measurements during the interval of 18.6 years. Heights of tide gauges benchmarks have been introduced as fixed parameters of the Croatian height datum or abbreviated HVD71, which is related to the epoch 1971.5. It is noticeable that height datum epoch is not completely identical with the IINVT network mean survey epoch.

In accordance with the IINVT levelling network geometric configuration, *i.e.* the realization of the HVRS71 system, influence of the Earth's crust height kinematics on the benchmark height position changes can be analyzed using the II order state levelling networks. In doing so, the II order networks can be analysed separately at the level of individual levelling loops of the IINVT network, Fig. 6. The fitting of II order networks to the IINVT network frame it's possible to do within each levelling loop of the IINVT separately. As a relevant choice for testing the kinematic model quality the IINVT network levelling loop no. III and no. V are considered, since the loops are of relatively regular shape, proper size, the whole or predominantly on the Croatian territory and within them the II order networks are of coherent configuration.

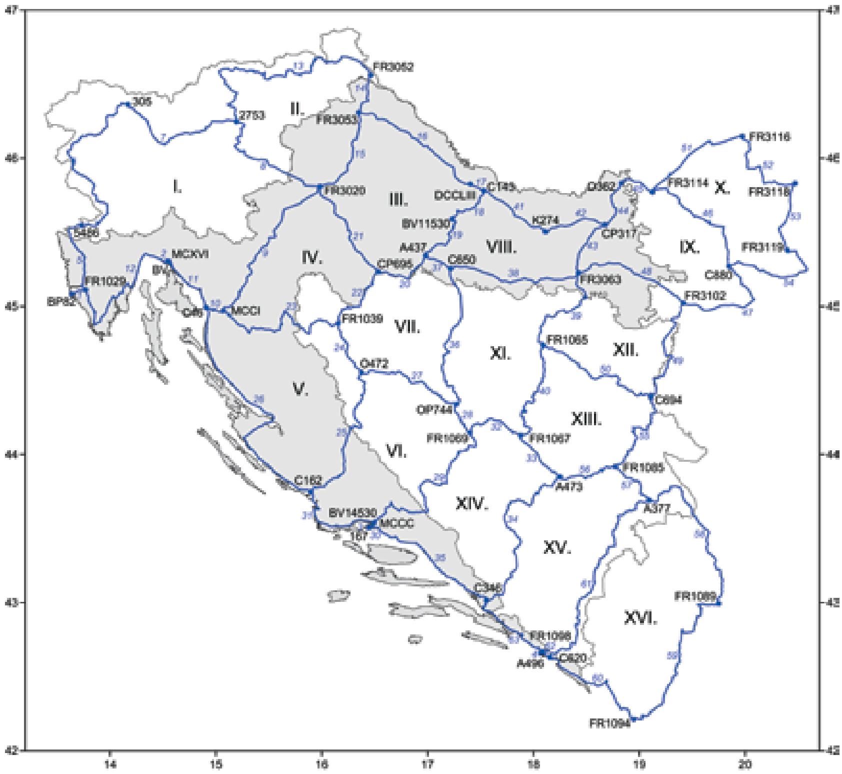


**Figure 6.** IINVT levelling network at territory of the Republic of Croatia.

Also, loops no. III and no. V are stretched along the Croatian territory having apparently different natural properties, where is especially interesting basically different relief height configuration on the one hand and different levels of height movements intensity of Earth's crust on the other side. Levelling lines in these loops are surveyed quite before the survey and realization of the IINVT levelling network, *i.e.* in the period 1945–1953. This fact points to the viability of the assumption that the network nodal benchmark relative height displacements could have a significant systematic influence on the benchmark height position accuracy. At the same time the IINVT network as a frame of reference system HVRS71, truly is realized in a very short period and it's certain that in such a short period changes in benchmark height positions are practically negligible.

In the area of levelling loop no. III a total of 25 levelling lines are surveyed, which form a II order network consisting of 13 levelling loops. Some lines are continuously linked to several network nodal benchmarks and are divided into several sections. Scheme of network geometrical configuration is shown in Fig. 7, including nodal benchmark labels and levelling loops numeration. Relevant data for the IINVT network benchmarks, which are already fixed by previous initial realization of the HVRS71 system, to which II order levelling lines are bound are known and at disposal. Table 1 lists the ellipsoidal positions of IINVT network fixed benchmarks and II order network nodal benchmark positions (Bessel ellipsoid, Greenwich). Table 2 provides the adjusted height differences between the fixed IINVT network benchmarks, which are forming levelling loop no. III, and directly measured height differences data of the II order levelling lines. Length of levelling lines $L$ is expressed in kilometers, the height differences $\Delta h$ in meters and survey epoch's are expressed in years.

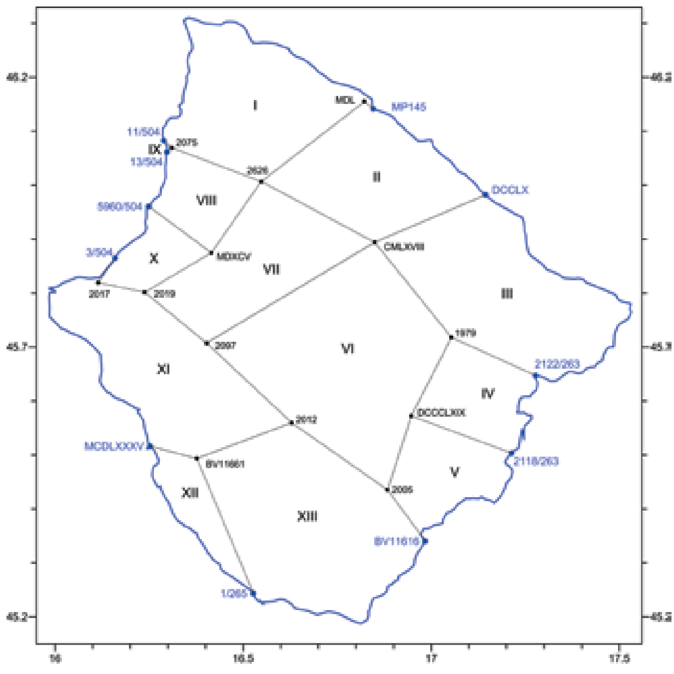


**Figure 7.** Scheme of II order levelling network geometrical configuration – loop no. III.

Levelling data contained in Tab. 2 shows that in terms of relief height configuration the area of loop no. III is very moderately formed. The biggest levelling line height difference is about fifty meters, and all the network node benchmarks are situated at heights ranging between about 102 and 204 meters above the HVRS71 reference surface. All levelling lines are surveyed in a relatively short period, between 1946.0 and 1951.0, *i.e.* about twenty years before the epoch of IINVT network survey.

*Table 1. Benchmarks position in the IINVT network levelling loop no. III.*

| Fixed IINVT benchmarks | | | | | | |
|---|---|---|---|---|---|---|
| Benchmark | $\varphi$ | | | $\lambda$ | | |
| 1/265 | 45º | 14' | 36" | 16º | 31' | 36" |
| 11/504 | 46º | 4' | 57" | 16º | 17' | 20" |
| 13/504 | 46º | 3' | 41" | 16º | 17' | 50" |
| 2118/263 | 45º | 30' | 11" | 17º | 12' | 49" |
| 2122/263 | 45º | 38' | 48" | 17º | 16' | 40" |
| 3/504 | 45º | 51' | 53" | 16º | 9' | 34" |
| 5960/504 | 45º | 57' | 38" | 16º | 14' | 54" |
| BV11616 | 45º | 20' | 24" | 16º | 59' | 2" |
| DCCLX | 45º | 58' | 55" | 17º | 8' | 40" |
| MCDLXXXV | 45º | 30' | 58" | 16º | 15' | 7" |
| MP145 | 46º | 8' | 29" | 16º | 50' | 45" |

| II order network nodal benchmarks | | | | | | |
|---|---|---|---|---|---|---|
| Benchmark | $\varphi$ | | | $\lambda$ | | |
| 1979 | 45º | 43' | 4" | 17 | 3' | 12" |
| 2005 | 45º | 26' | 7" | 16 | 53' | 1" |
| 2012 | 45º | 33' | 34" | 16 | 37' | 43" |
| 2017 | 45º | 49' | 31" | 16 | 6' | 53" |
| 2019 | 45º | 48' | 6" | 16 | 14' | 17" |
| 2075 | 46º | 4' | 9" | 16 | 17' | 58" |
| 2097 | 45º | 42' | 26" | 16 | 24' | 11" |
| 2626 | 46º | 0' | 24" | 16 | 32' | 53" |
| BV11661 | 45º | 29' | 36" | 16 | 22' | 35" |
| CMLXVIII | 45º | 53' | 39" | 16 | 50' | 58" |
| DCCCLXIX | 45º | 34' | 20" | 16 | 56' | 47" |
| MDL | 46º | 9' | 55" | 16 | 49' | 19" |
| MDXCV | 45º | 52' | 27" | 16 | 24' | 55" |

*Table 2. Levelling lines data in the IINVT network levelling loop no. III.*

| IINVT network adjusted levelling data | | | | | |
|---|---|---|---|---|---|
| Line | From bench-mark | To bench-mark | $L$ [km] | $\Delta h$ [m] | Epoch [year] |
| | MP145 | 11/504 | 81.15 | 11.3777 | 1971.1 |
| | DCCLX | MP145 | 31.65 | 19.2944 | 1971.1 |
| | DCCLX | 2122/263 | 74.65 | 32.0249 | 1971.1 |
| | 2118/263 | 2122/263 | 25.59 | 2.9141 | 1971.1 |
| | BV11616 | 2118/263 | 36.98 | 25.7341 | 1971.1 |
| | BV11616 | 1/265 | 51.85 | 32.2468 | 1971.1 |
| | MCDLXXXV | 1/265 | 44.66 | 53.8322 | 1971.1 |
| | MCDLXXXV | 3/504 | 65.37 | 27.3564 | 1971.1 |
| | 3/504 | 5960/504 | 14.64 | 72.5376 | 1971.1 |
| | 13/504 | 5960/504 | 13.43 | 52.0503 | 1971.1 |
| | 13/504 | 11/504 | 2.76 | 1.0371 | 1971.1 |
| II order network measured levelling data | | | | | |
| 259 | MP145 | MDL | 4.31 | 0.4473 | 1949.0 |
| 265 | BV11661 | 1/265 | 44.12 | 53.6918 | 1949.0 |
| 265 | MCDLXXXV | BV11661 | 11.20 | 0.1124 | 1949.0 |
| 266 | 2626 | MDL | 29.60 | 14.1008 | 1949.0 |
| 266 | MDXCV | 2626 | 19.50 | 11.1254 | 1949.0 |
| 266 | 2019 | MDXCV | 16.40 | 13.4856 | 1949.0 |
| 266 | 2019 | 2017 | 9.90 | 16.1834 | 1949.0 |

| II order network measured levelling data | | | | | |
|---|---|---|---|---|---|
| Line | From bench-mark | To bench-mark | $L$ [km] | $\Delta h$ [m] | Epoch [year] |
| 501 | 2005 | BV11616 | 18.10 | 21.2959 | 1946.0 |
| 501 | 2005 | 2012 | 25.50 | 9.9670 | 1946.0 |
| 501 | BV11661 | 2012 | 27.20 | 10.0778 | 1946.0 |
| 504 | 13/504 | 2075 | 1.30 | 0.6715 | 1946.0 |
| 504 | 2075 | 11/504 | 1.90 | 0.3795 | 1946.0 |
| 504 | 2017 | 3/504 | 4.70 | 11.9175 | 1946.0 |
| 508 | 2097 | 2019 | 18.07 | 0.3033 | 1949.0 |
| 508 | 2097 | 2012 | 25.40 | 11.2248 | 1949.0 |
| 523 | 1979 | 2122/263 | 27.47 | 15.0736 | 1949.0 |
| 524 | 2005 | DCCCLXIX | 20.00 | 11.7136 | 1950.0 |
| 525 | 2097 | CMLXVIII | 56.39 | 21.2719 | 1949.0 |
| 528 | MDXCV | 5960/504 | 20.45 | 87.2177 | 1949.0 |
| 529 | 2626 | 2075 | 33.29 | 24.6539 | 1949.0 |
| 558 | CMLXVIII | 2626 | 32.70 | 3.6283 | 1951.0 |
| 559 | DCCLX | CMLXVIII | 28.80 | 2.0262 | 1951.0 |
| 560 | CMLXVIII | 1979 | 44.58 | 14.9345 | 1951.0 |
| 561 | DCCCLXIX | 1979 | 29.02 | 23.1958 | 1951.0 |
| 562 | DCCCLXIX | 2118/263 | 33.65 | 35.3615 | 1951.0 |

In the area of levelling loop no. V a total of 24 II order levelling lines is surveyed, which defines the network consisting of 12 levelling loops. Scheme of network geometrical configuration is shown in Fig. 8, including nodal benchmark labels and levelling loops numeration. Table 3 lists the ellipsoidal positions of IINVT network fixed benchmarks and the II order network nodal benchmark positions (Bessel ellipsoid, Greenwich).

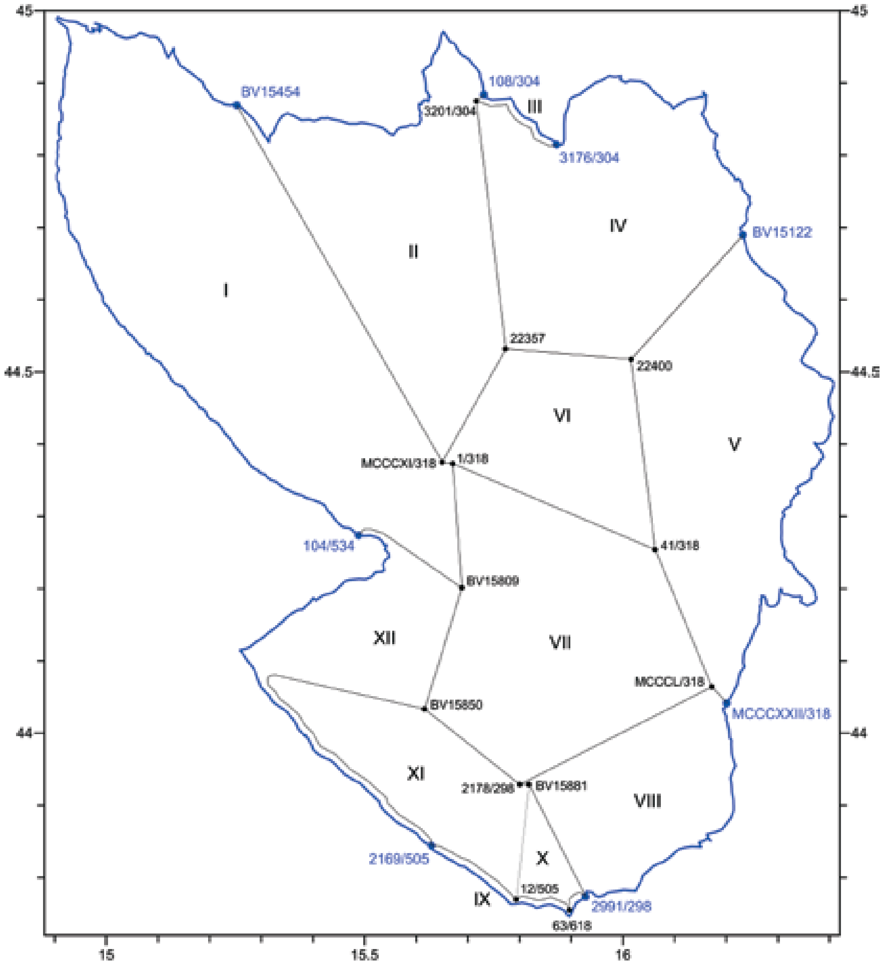


**Figure 8.** Scheme of II order levelling network geometrical configuration – loop no. V.

Table 4 provides the adjusted height differences between the fixed IINVT network benchmarks, which are forming levelling loop no. V, and directly measured height differences data of II order levelling lines. Like in earlier case length of levelling lines $L$ is expressed in kilometers, the height differences $\Delta h$ in meters and survey epoch's are expressed in years.

*Table 3. Benchmarks position in the IINVT network levelling loop no. V.*

| Fixed IINVT benchmarks | | | | | | | II order network nodal benchmarks | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Benchmark | $\varphi$ | | | $\lambda$ | | | Benchmark | $\varphi$ | | | $\lambda$ | | |
| BV15454 | 44° | 52' | 8" | 15° | 15' | 9" | 22357 | 44° | 31' | 55" | 15° | 46' | 21" |
| 108/304 | 44° | 53' | 1" | 15° | 43' | 51" | 22400 | 44° | 31' | 3" | 16° | 0' | 57" |
| 3176/304 | 44° | 48' | 53" | 15° | 52' | 18" | 1/318 | 44° | 22' | 23" | 15° | 40' | 14" |
| BV15122 | 44° | 41' | 22" | 16° | 13' | 56" | 12/505 | 43° | 45' | 54" | 15° | 47' | 36" |
| MCCCXXII | 44° | 2' | 29" | 16° | 12' | 4" | 2178/298 | 43° | 55' | 44" | 15° | 49' | 4" |
| 2991/298 | 43° | 46' | 24" | 15° | 55' | 40" | 3201/304 | 44° | 52' | 50" | 15° | 42' | 58" |
| 2169/505 | 43° | 50' | 39" | 15° | 37' | 46" | 41/318 | 44° | 15' | 14" | 16° | 3' | 44" |
| 104/534 | 44° | 16' | 26" | 15° | 29' | 18" | 63/618 | 43° | 44' | 12" | 15° | 53' | 47" |
| | | | | | | | BV15809 | 44° | 12' | 4" | 15° | 41' | 16" |
| | | | | | | | BV15850 | 44° | 2' | 0" | 15° | 36' | 57" |
| | | | | | | | BV15881 | 43° | 55' | 44" | 15° | 49' | 5" |
| | | | | | | | MCCCL/318 | 44° | 3' | 51" | 16° | 10' | 20" |
| | | | | | | | MCCCXI/318 | 44° | 22' | 30" | 15° | 39' | 50" |

*Table 4. Levelling lines data in the IINVT network levelling loop no. V.*

| IINVT network adjusted levelling data | | | | | | II order network measured levelling data | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Line | From bench-mark | To bench-mark | $L$ [km] | $\Delta h$ [m] | Epoch [year] | Line | From bench-mark | To bench-mark | $L$ [km] | $\Delta h$ [m] | Epoch [year] |
| | 104/534 | BV15454 | 165.74 | 452.1232 | 1971.1 | 307 | BV15454 | MCCCXI/318 | 82.91 | 120.1265 | 1949.0 |
| | 108/304 | BV15454 | 72.82 | 119.0795 | 1971.1 | 318 | 1/318 | 41/318 | 41.12 | 73.6228 | 1949.0 |
| | 3176/304 | 108/304 | 17.27 | 115.6729 | 1971.1 | 318 | MCCCL/318 | 41/318 | 32.79 | 281.1369 | 1949.0 |
| | 3176/304 | BV15122 | 69.64 | 340.6972 | 1971.1 | 318 | MCCCXXII | MCCCL/318 | 6.81 | 137.2697 | 1949.0 |
| | MCCCXXII | BV15122 | 124.94 | 335.3499 | 1971.1 | 318 | 1/318 | MCCCXI/318 | 0.60 | 4.7680 | 1949.0 |
| | 2991/298 | MCCCXXII | 53.11 | 149.7686 | 1971.1 | 505 | 12/505 | 63/618 | 16.93 | 7.8789 | 1947.0 |
| | 2169/505 | 2991/298 | 32.13 | 81.4446 | 1971.1 | 505 | 2169/505 | 12/505 | 15.86 | 14.2774 | 1947.0 |
| | 2169/505 | 104/534 | 84.65 | 8.4951 | 1971.1 | 505 | 2169/505 | BV15850 | 81.67 | 180.3985 | 1947.0 |
| II order network measured levelling data | | | | | | 505 | 12/505 | BV15881 | 24.00 | 103.6269 | 1947.0 |
| 298 | 2991/298 | BV15881 | 30.40 | 36.4592 | 1949.0 | 530 | 3201/304 | 22357 | 54.70 | 475.3701 | 1951.0 |
| 298 | 2178/298 | BV15881 | 0.20 | 2.8990 | 1949.0 | 531 | 22400 | 22357 | 32.45 | 264.5701 | 1951.0 |
| 298 | 2178/298 | BV15850 | 22.50 | 65.4113 | 1949.0 | 532 | BV15122 | 22400 | 33.15 | 2.0249 | 1951.0 |
| 298 | BV15809 | BV15850 | 31.10 | 177.6293 | 1949.0 | 533 | 22400 | 41/318 | 46.30 | 81.0264 | 1951.0 |
| 298 | BV15809 | 1/318 | 39.60 | 573.2010 | 1949.0 | 534 | BV15809 | 104/534 | 23.39 | 5.6947 | 1950.0 |
| 298 | 63/618 | 2991/298 | 5.90 | 59.2668 | 1949.0 | 535 | MCCCXI/318 | 22357 | 27.15 | 252.4009 | 1951.0 |
| 304 | 108/304 | 3201/304 | 1.41 | 16.1974 | 1949.0 | 565 | 2178/298 | MCCCL/318 | 37.72 | 253.5359 | 1957.0 |
| 304 | 3176/304 | 3201/304 | 24.68 | 131.8739 | 1949.0 | | | | | | |

Levelling data listed in Table 4 clearly show that in terms of relief height configuration the area of IINVT network levelling loop no. V is obviously quite intensively formed. It includes the mountainous region of the Dinarides with very emphasized height differences, facing to the Croatian north and northwest predominantly lowland area included in the levelling loop no. III. The biggest levelling line height difference is about 570 meters, and all the nodal benchmark heights are located in the range between about 1 and 835 meters above the HVRS71 reference surface. All II order levelling lines have been surveyed in a relatively short period, *i.e.* between 1947.0 and 1951.0, except the line no. 565. All levelling lines survey epoch's are about twenty years before the epoch of the IINVT network survey.

## 4. Determination of the levelling line height difference reductions

Using the II order levelling network original measurement data, listed in Tabs. 2 and 4, determination of reductions (corrections) for all measured levelling line height differences is carried out and the results are listed in Tab. 5. Reductions $r_{\Delta h}$ determination is done with the help of expressions (1) and (5), using benchmark position data listed in Tabs. 1 and 5, and using grid models of kinematic parameters, Rožić (2016), belonging to the kinematic model of the Earth's crust vertical motion presented in Figs. 3 and 4. Reduced levelling lines height differences $\Delta h_r$ are determined with the help of the expression (6). In this way systematic errors of nodal benchmarks height movements are eliminated from the original measurements and all II order levelling line height differences are reduced from their survey epoch's ($t_A$), listed in Tabs. 2 and 4, to the unique epoch of IINVT network realization ($t_B$ = 1971.1).

Calculated height difference reductions are at several millimeters to generally centimeter order of size, with different signs and seemingly of coherent and logical amounts, taking into account the locations of nodal benchmarks, lengths of levelling lines and trend of the Earth's crust height movement kinematic model in the period 1949.0–1971.1. By coincidence, that trend is quite clearly visible in Fig. 2 presented in Rožić (2015), because it relates to the specified period. However, it is a bit surprising amount of height difference reduction of 22.0 mm in levelling loop no. III for one of the sections of levelling line no. 265, or reductions in the amount of 23.3 mm and –17.8 mm for lines no. 307 and no. 533 in levelling loop no. V. They have an obviously larger amount than all other reductions. But, these reductions are like all others the result of the consistent application of the kinematic model regardless of its quality. In a way, these reductions maybe seem to indicate the existence of certain kinematic model local anomalies or "errors". By the lack of more control data these anomalies or model local lower quality, if any, cannot be seriously analyzed, verified and eliminated. Also, some reductions are very small and practically neglectable from the point of the expected kinematic model quality in general, like reductions of the 0.1, 0.3 or maybe even 0.6 mm, but consistent application of the kinematic model demands to be shown and taken into account.

Only by levelling data contained in Tab. 5 is still not possible to draw any definitive conclusion on the performed measurement reductions quality. However, it is possible to argue hypothetically, that accuracy criteria derived from levelling line reduced measurements should be favorable compared to the accuracy criteria derived from the original or unreduced measurements. Moreover, this hypothetical assertion can be empirically investigated and confirmed only if the measurement reductions are of proper quality, and this can happen only if the kinematic model of the Earth's crust relative height motion

*Table 5. Height difference reductions from survey epoch's to IINVT network survey epoch.*

| IINVT network levelling loop III | | | | | | IINVT network levelling loop V | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Line | From benchmark | To benchmark | $\Delta h$ *[m]* | $r_{\Delta h}$ *[mm]* | $\Delta h_r$ [m] | Line | From benchmark | To benchmark | $\Delta h$ *[m]* | $r_{\Delta h}$ *[mm]* | $\Delta h_r$ [m] |
| 259 | MP145 | MDL | 0.4473 | –1.4 | 0.4459 | 298 | 2991/298 | BV15881 | 36.4592 | 0.6 | 36.4598 |
| 265 | BV11661 | 1/265 | 53.6918 | 22.0 | 53.7138 | 298 | 2178/298 | BV15881 | 2.8990 | 0.0 | 2.8990 |
| 265 | MCDLXXXV | BV11661 | 0.1124 | 2.7 | 0.1151 | 298 | 2178/298 | BV15850 | 65.4113 | –2.4 | 65.4089 |
| 266 | 2626 | MDL | 14.1008 | –3.7 | 14.0971 | 298 | BV15809 | BV15850 | 177.6293 | –9.0 | 177.6203 |
| 266 | MDXCV | 2626 | 11.1254 | –5.7 | 11.1197 | 298 | BV15809 | 1/318 | 573.2010 | 8.3 | 573.2093 |
| 266 | 2019 | MDXCV | 13.4856 | –7.3 | 13.4783 | 298 | 63/618 | 2991/298 | 59.2668 | 0.3 | 59.2671 |
| 266 | 2019 | 2017 | 16.1834 | 3.1 | 16.1865 | 304 | 108/304 | 3201/304 | 16.1974 | 1.1 | 16.1985 |
| 501 | 2005 | BV11616 | 21.2959 | 6.7 | 21.3026 | 304 | 3176/304 | 3201/304 | 131.8739 | –5.9 | 131.8680 |
| 501 | 2005 | 2012 | 9.9670 | –1.9 | 9.9651 | 307 | BV15454 | MCCCXI/318 | 120.1265 | 23.3 | 120.1498 |
| 501 | BV11661 | 2012 | 10.0778 | –3.2 | 10.0746 | 318 | 1/318 | 41/318 | 73.6228 | –4.8 | 73.6180 |
| 504 | 13/504 | 2075 | 0.6715 | –0.3 | 0.6712 | 318 | MCCCL/318 | 41/318 | 281.1369 | 10.2 | 281.1471 |
| 504 | 2075 | 11/504 | 0.3795 | 0.0 | 0.3795 | 318 | MCCCXXII | MCCCL/318 | 137.2697 | 1.3 | 137.2710 |
| 504 | 2017 | 3/504 | 11.9175 | -2.7 | 11.9148 | 318 | 1/318 | MCCCXI/318 | 4.7680 | –0.1 | 4.7679 |
| 508 | 2097 | 2019 | 0.3033 | 2.5 | 0.3058 | 505 | 12/505 | 63/618 | 7.8789 | 3.0 | 7.8819 |
| 508 | 2097 | 2012 | 11.2248 | 5.2 | 11.2300 | 505 | 2169/505 | 12/505 | 14.2774 | 5.5 | 14.2829 |
| 523 | 1979 | 2122/263 | 15.0736 | –4.1 | 15.0695 | 505 | 2169/505 | BV15850 | 180.3985 | 7.1 | 180.4056 |
| 524 | 2005 | DCCCLXIX | 11.7136 | –3.4 | 11.7102 | 505 | 12/505 | BV15881 | 103.6269 | 3.9 | 103.6308 |
| 525 | 2097 | CMLXVIII | 21.2719 | –5.0 | 21.2669 | 530 | 3201/304 | 22357 | 475.3701 | –2.3 | 475.3678 |
| 528 | MDXCV | 5960/504 | 87.2177 | 1.9 | 87.2196 | 531 | 22400 | 22357 | 264.5701 | –4.5 | 264.5656 |
| 529 | 2626 | 2075 | 24.6539 | 2.3 | 24.6562 | 532 | BV15122 | 22400 | 2.0249 | –8.2 | 2.0167 |
| 558 | CMLXVIII | 2626 | 3.6283 | –5.3 | 3.6230 | 533 | 22400 | 41/318 | 81.0264 | –17.8 | 81.0086 |
| 559 | DCCLX | CMLXVIII | 2.0262 | –3.6 | 2.0226 | 534 | BV15809 | 104/534 | 5.6947 | –4.5 | 5.6902 |
| 560 | CMLXVIII | 1979 | 14.9345 | 5.6 | 14.9401 | 535 | MCCCXI/318 | 22357 | 252.4009 | 9.2 | 252.4101 |
| 561 | DCCCLXIX | 1979 | 23.1958 | –2.6 | 23.1932 | 565 | 2178/298 | MCCCL/318 | 253.5359 | 0.6 | 253.5365 |
| 562 | DCCCLXIX | 2118/263 | 35.3615 | –5.4 | 35.3561 | 298 | 2991/298 | BV15881 | 36.4592 | 0.6 | 36.4598 |

has a proper level of quality for this exact purpose. It is logical to expect that the elimination of systematic errors from height difference measurements, which are the consequences of benchmark height position changes caused by Earth's crust movements, should favorably affect on the reduced measurements accuracy in respect to the accuracy of the original measurements.

## 5. Original and reduced measurements quality determination and analysis

By using the data given in Tabs. 2, 4 and 5, and their visualization in Figs. 7 and 8, measurements quality criteria determination and comparative analysis of the original and reduced measurements of II order levelling networks in levelling loops no. III and no. V is possible. Since all levelling lines are integrated in both levelling loops into the networks of firm geometrical configuration, and these networks are fixed to the frame of previously already adjusted IINVT network, measurements accuracy criteria of the original and reduced measurements can be determined "a priori", *i.e.* before networks adjustment by using levelling loop misclosures, and can be determined "a posteriori", *i.e.* on the basis of networks adjustment. The principal difference of the "a priori" and "a posteriori" approach is the fact that in process of measurements accuracy determination a networks are treated in a very different way. In "a priori" case networks are treated as a set of mutually completely independent and unrelated levelling loops, although it is not in accordance with the reality. In "a posteriori" case networks are truly, in the geometrical sense, coherent and all levelling loops are mutually firmly and unambiguously connected. The fundamental reason for using this approach, which is quite common in the determination and analysis of the levelling measurement accuracy, lies in the fact that the comparison of "a priori" and "a posteriori" derived accuracy can indicate the presence of systematic errors contained in the levelling measurements.

Table 6 presents the results of the comparative determination of network levelling loop misclosures $W$ and $W_r$ determined using the original $\Delta h$ and reduced levelling line height differences $\Delta h_r$ in the IINVT network levelling loops no. III and no. V., as well as levelling loop lengths $F$ which are necessary for accuracy criteria calculation. During misclosure calculation, the IINVT network adjusted levelling line height differences were used, listed in Tabs. 2 and 3. Because the IINVT network is already adjusted and hierarchically superior to the II order levelling networks, the sum of misclosures within the loops no. III and no. V should be equal to zero.

Some levelling loop misclosures $W_r$ in Tab. 6, determined by a reduced measurements, are framed in accordance with the fact that they have a quite higher value compared to the misclosures $W$ determined by original measurements. Obviously measurement reductions in belonging loops led to some deterioration

*Table 6. Levelling loop misclosures.*

| IINVT network loop no. III | | | | IINVT network loop no. V | | | |
|---|---|---|---|---|---|---|---|
| Loop | $F$ [km] | $W$ [mm] | $W_r$ [mm] | Loop | $F$ [km] | $W$ [mm] | $W_r$ [mm] |
| I | 150.25 | 2.2 | $\boxed{6.8}$ | I | 312.24 | –24.6 | –14.0 |
| II | 127.06 | 13.6 | 2.4 | II | 238.99 | –39.4 | $\boxed{–73.1}$ |
| III | 175.50 | –9.4 | –7.3 | III | 43.36 | 3.6 | –3.4 |
| IV | 115.73 | –6.2 | $\boxed{–7.5}$ | IV | 214.62 | 48.2 | 43.7 |
| V | 108.73 | 45.1 | 29.6 | V | 243.99 | 5.4 | $\boxed{42.9}$ |
| VI | 200.89 | 39.2 | 38.7 | VI | 147.62 | 2.4 | $\boxed{3.0}$ |
| VII | 143.06 | 14.1 | 13.9 | VII | 204.83 | –67.0 | $\boxed{–67.7}$ |
| VIII | 87.97 | 59.6 | $\boxed{64.6}$ | VIII | 128.24 | 57.8 | 57.7 |
| IX | 5.96 | –13.9 | –13.6 | IX | 70.82 | –21.5 | –12.7 |
| X | 66.09 | -64.8 | –59.0 | X | 77.23 | 22.0 | 22.0 |
| XI | 161.84 | –13.2 | –10.4 | XI | 144.23 | –18.1 | –18.0 |
| XII | 99.98 | –28.0 | -3.3 | XII | 220.81 | 31.2 | 19.6 |
| XIII | 166.77 | –38.3 | $\boxed{–54.9}$ | Sum | | 0.0 | 0.0 |
| Sum | | 0.0 | 0.0 | | | | |

compared to the original state, which suggests the conclusion that the effects of the benchmark height movements are not successfully eliminated from the original height difference measurements. Somewhat more drastic deterioration, taking into consideration the total number of all 25 levelling loops, is present only in three levelling loops. In the loop no. XIII within the IINVT network loop no. III, Fig. 7, and in the loops no. II and no. V within the IINVT network loop no. V, Fig. 8. However, in most of the levelling loops a moderate improvement is quite recognizable, because misclosures $W_r$ are of smaller amounts and sporadically in several cases practically completely equal to the misclosures $W_r$. In IINVT network loop no. III improvement is noticeable in 9 out of 13 loops and in IINVT network loop no. V in 8 out of 12 loops. Of course, it's interesting to point to a totally unexpected incidence perceived in misclosure comparison, and that is the incidence of preservation of the misclosures sign. In fact, only in one levelling loop, out of a total 25, misclosure $W_r$ changes sign in relation to $W$, while in all other loops the sign remains compliant. It is levelling loop no. III within the IINVT network loop no. V. Moreover, it is worthwhile to emphasize the fact that the number of positive and negative misclosures $W_r$ or $W$ are in proper balance. This generally corresponds to the theoretical ideal of levelling loop misclosures interpretation as true errors, although it is quite clear that original measurements inevitably contain various systematic errors, including influence of nodal benchmark height movements. From reduced measurements only influences of benchmarks height movements are eliminated, while other systematic effects are still present.

With the misclosures $W$ and $W_r$ the “a priori” measurements accuracy criteria are determined referred to the so-called reference measurement. Reference measurement is assumed to be levelling line height difference measured by the double levelling along 1 km levelling line length. Like standard accuracy criterion in levelling the probable error $u_F$ should be used, Bratten et al. (1950). Accordingly, the measurement accuracy of the original measurements (using misclosures $W$) expressed by reference probable error, within the IINVT network loop no. III, is

$$u_F = \pm\frac{2}{3}\sqrt{\frac{\sum_1^{13} W_i^2}{\sum_1^{13} F_i}} = \pm 2.0 \text{ mm / km} \tag{7}$$

and within the IINVT network loop no. V

$$u_F = \pm\frac{2}{3}\sqrt{\frac{\sum_1^{12} W_i^2}{\sum_1^{12} F_i}} = \pm 1.8 \text{ mm / km}, \tag{8}$$

noting that the levelling loop lengths $F$ are introduced as weights. Comparatively with these values, the measurement accuracy of the reduced measurements (using misclosures $W_r$) within the IINVT network loop no. III, is

$$u_F = \pm 1.9 \text{ mm / km} \tag{9}$$

and within IINVT network loop no. V is

$$u_F = \pm 2.0 \text{ mm / km}. \tag{10}$$

Following the “a priori” measurement accuracy determination the “a posteriori” accuracy criteria are also determined. On the basis of II order networks adjustment, separately within IINVT network levelling loops no. III and no. V, determination of measurement reference probable errors is made, Pelzer (1985). In both loops the classic regular adjustment of indirect measurements, using the method of least squares, is applied, taking data from Tabs. 2, 4 and 5. The measurement weights, like the reciprocals of the levelling line lengths, are introduced. The fixed IINVT network benchmark heights from the State geodetic administration of the Republic of Croatia official database are taken. Explicit formulation of the measurement observational equations is performed in accordance with the networks geometric configuration schemes shown on Figs. 7 and 8. On the basis of the network adjustment within the IINVT network loop no. III, using the adjusted measurement corrections v and measurement weights $p$, the measurement reference probable error of original measurements is determined

$$u_\gamma = \pm \sqrt{\frac{\sum_{1}^{25} p_i v_i^2}{n_f}} = \pm 3.4 \text{ mm / km} \tag{11}$$

and within the IINVT network loop no. V

$$u_\gamma = \pm \sqrt{\frac{\sum_{1}^{24} p_i v_i^2}{n_f}} = \pm 2.3 \text{ mm / km.} \tag{12}$$

Accordingly, on the basis of reduced measurement adjustments, using adjusted measurement corrections $v_r$ and weights $p$, the measurement reference probable errors of reduced measurements are determined

$$u_\gamma = \pm 3.1 \text{ mm / km,} \tag{13}$$

$$u_\gamma = \pm 2.6 \text{ mm / km.} \tag{14}$$

noting that $n_f$ is the number of redundant measurements.

The comparatively exposed results of “a priori” and “a posteriori” accuracy determination, of original and reduced measurements respectively, raises a few principal comments. First of all, completely independent of the fact whether original or reduced measurements were analysed, it can be noted that the measurement accuracy of the II order networks in both levelling loops, mostly agrees with the prescribed Croatian standard for measurements accuracy defined by the maximum allowed amount of reference probable error at level of ± 2.0 mm/km for the effect of random errors. Since the applied “a priori” and “a posteriori” models for determining the measurement accuracy does not include the mechanism of explicit demarcation between the impact of random and systematic errors, mainly the component of measurement accuracy resulting from the impact of random errors is strongly relevant. In addition, the “a posteriori” accuracy are slightly worse than the “a priori” prescribed standard. Furthermore, the moderate difference or disagreement between “a priori” and “a posteriori” accuracy criteria points to the fact that in the original, but also noticeable in the reduced measurements, certain systemic effects or errors are contained. Of course, in the case of the reduced measurements corresponding accuracy does not include anymore the impact of systematic errors dependent on benchmark height movement, but still show the influence of some other systematic errors.

In addition to the disclosed general comments on the measurement accuracy level achieved in the observed levelling loops, it is essential to consider the relationship between derived accuracy from the viewpoint of reduced measurements impact. In this regard, in respect to the initial theoretical hypothesis that systematic effects elimination, caused by benchmarks height movement, should contribute to measurement accuracy, the inconsistent result is obtained. In levelling loop no. III turned out, on the basis of a comparison of the expression (7) and (9), and (11) and (13), that the measurement reductions introduction has had the positive impact on the measurement accuracy, while in levelling loop no. V, based on comparisons of expression (8) and (10), and (12) and (14), the opposite result is achieved. In other words, measurement reductions contributed to the elimination of the systematic impact of benchmarks height movement and increase the measurement accuracy in the loop no. III, while in the loop no. V have not contributed and resulted in a lower level of measurement accuracy. It should be noted that levels of an accuracy increase or decrease are very moderate, although in its amount is still significant, especially in respect to the prescribed state standard of II order levelling line measurement accuracy, were the impact of systematic errors is limited by the maximum allowed amount of measurement reference probable error of ±0.4 mm/km. Such contradictory and inconsistent outcome of the measurement reductions introduction indicates that the kinematic model quality is not completely adequate and sufficient for such a purpose. Of course, the question of the reasons that lead to such an outcome can be raised?

The answer to this question, but viewed from a somewhat different perspective, comes down essentially to choice one of two potential options. The first, which really makes a reasonable conclusion of an insufficient kinematic model quality for the purpose of measurements reduction determination, arising from the rather low number, characteristics and quality of the measurement levelling data; type of absolute heights; method of kinematic parameters gridding used for the kinematic model creation and maybe unrealistic hypotheses or inadequate methods used during its modeling. The second, supposing that an at least minimum quality of the kinematic model for this purpose exists, some real cause which leads to such an outcome can be identified or at least indicated. In this specific situation, a comparatively considering properties of the levelling loops no. III and no. V, the second option seems to be quite probable. The cause could be found in the characteristic behavior of systematic errors contained in the geometric levelling measurements. Specifically, empirical phenomenon of mutual compensation of different systematic errors is a well-known, because according to the different origins of the errors and modes of their action, because of the opposite direction of action (sign), various errors may completely or partly be eliminated or mutually compensated. Considering this phenomenon, inconsistency of measurement accuracy present and shown in levelling loop no. V can be explained by the lack of systematic errors mutual compensation. In fact, it means that after the elimination of systematic errors caused by benchmarks height movement the level of other systematic errors contained in measurements rose and had more negative impact on measurement accuracy. Such an explanation or hypothesis is to some extent sustainable, taking into consideration two additional facts. The fact that in both levelling loops various systematic effects are present and contained in measurements apart exclusive presence of the benchmarks height movement systematic errors. That is proved by "a priori" and "posteriori" measurement accuracy comparison and their mutual relation. Also, the fact that levelling measurements belonging to the levelling loop no. V certainly contains a quite greater impact of systematic errors, taking into account relief properties (mountains) which are significantly unfavorable for use of the geometrical levelling survey method with respect to the relief properties (lowland) of levelling loop no. III. All measurements belonging to the levelling loop no. V most probably are under a much stronger influence of all systematic errors directly correlated to the size of levelling line height differences as opposite to the measurements belonging to the levelling loop no. III, taking also into account influence of the kinematic model unreliability arrised from the normal orthometric heights usage. At the moment the second option, like quite probable and possible, has not been yet confirmed empirically, because further data and investigations are needed.

## 6. Conclusion

Based on all the data and presented results, the explicit conclusion with reasonable confidence whether the quality of the kinematic model on the total Croatian territory provides adequate quality of levelling measurement reductions determination it's not possible to make. Whatever, such outcome is appearing from the analysis and comparison of II order levelling networks accuracy determined regarding just two deliberately selected IINVT network levelling loops on the Croatian territory, chosen to be most representative. However, such outcome further leaves open the possibility that the kinematic model quality could still be satisfactory for the exact purpose of measurements reduction to unique epoch. Obviously, such outcome should be confirmed or completely rejected by the continuation of research. Primarily, research should be focused on the measurement reductions determination in several others or in all the remaining IINVT network levelling loops on the Croatian territory. Secondarily, it should be referred to the attempt of analysis of the various systematic errors contained in the levelling measurements, in order to clarify and resolve the presence of systematic errors mutual compensation. At this moment, primary line of research seems to be more realistic and pragmatic, considering the availability of the Croatian II order levelling networks measurement data on the one hand and considering the complexity of dealing with the systematic errors behavior contained in geometric levelling measurements data.

Additionally, it can be argued that the presented concept of the indirect kinematic model quality testing proved to be viable and sustainable. By this concept, the quality of the kinematic model is nevertheless quantified and qualified. In other words, it's not concluded without any doubt that the kinematic model can be used for the purpose of levelling measurements reduction to the unique epoch, because millimeter and sub-millimeter quality level of reductions is assumed. But, obviously it's shown that kinematic model quality level allows the determination of the height movement predictions of the Earth's crust discrete points at the reliable centimeter level. So, although it's not entirely certain that the kinematic model and it's quality generally should have an important role in the realization or improvement of state height reference system, though it can be used for any other purposes were reliable centimeter level of height movements is needed. In this, it's also important to underline that specified kinematic model reliability and usability are directly related just to that part of the kinematic model covered by the IINVT network area of levelling loops no. III and no. V. Anyway, taking into consideration the consistency of the kinematic model creation it's possible to generalize kinematic model quality assessment fairly reliably to a total of the Croatian territory.

During the process of reduction determination is shown that the application of the kinematic model is quite simple, fast, straight forward, unambiguous and consistent, since it is based on simple kinematic functions and kinematic parameter grid models. In fact, it turns out that the biggest problem during the determination of the height difference reductions is identification of right grid model cells containing any individual benchmark (point) and the use of bilinear interpolation in purpose to calculate exact values of height motion kinematic parameters.

Continuation of research should give a more reliable and consistent answer to the question of kinematic model quality. From the geodetic point of view it's very important because systematic errors elimination is a key point in the process of height positioning accuracy achievent.

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SAŽETAK

## Vrednovanje kvalitete kinematičkog modela visinskog gibanja Zemljine kore na teritoriju Hrvatske

*Nevio Rožić*

U članku je predočena empirijska uporaba kinematičkog modela visinskog gibanja Zemljine kore kreiranog za područje Hrvatske, Slovenije i Bosne i Hercegovine, u svrhu određivanja relativnih visinskih pomaka diskretnih točaka između različitih epoha. Također, predočena je uporaba tog modela u svrhu računanja redukcija neposrednih nivelmanskih mjerenja iz epoha izmjere u neku drugu jednoznačno odabranu epohu, a u svrhu eliminacije sistematskih utjecaja visinskog gibanja čvornih repera nivelmanskih mreža. Za teritorij Hrvatske je na indirektan način ispitana kvaliteta kinematičkog modela, temeljem komparacije kriterija ocjene točnosti nivelmanskih mjerenja mreža II. reda ustrojenih pomoću izvornih i reduciranih nivelmanskih mjerenja. Analizirani su nivelmanski vlakovi mreža II. reda na području dvije ciljano odabrane i reprezentativne nivelmanske figure hrvatske državne nivelmanske mreže I. reda, te su za njih određeni kriteriji točnosti mjerenja pomoću izvornih mjerenja i komparativno pomoću reduciranih mjerenja iz epoha izmjere mreža u epohu izmjere mreže I. reda. Komparativna usporedba kriterija točnosti izvornih i reduciranih mjerenja nije jednoznačno i nedvosmisleno potvrdila, a niti u potpunosti odbacila, dostatnost kvalitete kinematičkog modela za računanje redukcija mjerenja. Ipak, ukazala je na činjenicu da kvaliteta modela omogućuje pouzdano i kvalitetno određivanje relativnih visinskih pomaka točaka na centimetarskom redu veličine.

*Ključne riječi*: kinematički model, visinski pomaci, visinsko gibanje, kvaliteta, redukcija mjerenja, Hrvatska

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