Evaluation of agricultural yield in relation to the doses of mineral fertilizers

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Abstract

The present paper proposes a bifactorial model obtained by a generalization of the unifactorial Mitscherlich model. After analysing the graphic representation of the experimental data and the theoretical curves, one can conclude that they present good concordance. Treatments with nitrogen fertilizers within the limits of 0 – 200 kg a.i. ha^{-1} determined different optimum and maximum values for each of the four fertilization variants with phosphorus and potassium (optimum value 97 kg ha^{-1} N on P_0K_0 with yield increase of 1224 kg ha^{-1}; optimum 107 kg ha^{-1} N on $P_{50}K_{50}$, with an increase of 952 kg ha^{-1}; 111 kg ha^{-1} N on $P_{100}K_{100}$ with an increase of 968 kg ha^{-1} and 103 kg ha^{-1} N on $P_{150}K_{150}$ with an increase of 1413 kg ha^{-1}). From an economical point of view, we will maximize the benefit corresponding to production value in the hypothesis that only one fertilizer is applied, namely a complex fertilizer of the type $N_{15}P_{15}K_{15}$. For the actual price values $q = 0.2\ € \cdot kg^{-1}$, $p = 0.6€ \cdot kg^{-1}$ the solution of the equation above is $x = \ 221\ kg \cdot ha^{-1}$.

Keywords: differential equations, mineral fertilizers, Mitscherlich, model, optimal doses

Introduction

Over time, researchers have tried to describe biological, technical, social and economic phenomena and processes; as a result, there is a large number of scientific papers that present various mathematical functions, relations and models, with both theoretical and real examples (Karadavut et al. 2010).

The growth of crop plants is totally different than the growth of the same species in natural conditions, as they depend on the quantity and quality of the inputs specific for their respective production process requirements. Although the growth of crop plants can essentially be described through models that generally characterize growth processes in biology, this process has certain peculiarities (Ware et al. 1982; Hirose 1987; Fourcaud et al. 2008; Guo et al. 2011).

Plant cultivation is an economic process with certain parameters and conditions for functioning; benefit as an economic element has a minimal restrictive character for the efficiency of the respective exploitation (Matson et al. 1998; Huand et al. 2009).

Fertilizers represent one of the main inputs in the process of agricultural production, and the quantity and quality of this type of input greatly determines the quality and quantity of the yield, therefore the efficiency of the respective process (Matson et al. 1997; Cassman 1999).

Such models usually focus on the description of carbon (C) or nitrogen (N) balance and consider that plant development depends on a change of matter in different compartments, based on intake (e.g. photosynthesis) and loss (e.g. senescence) either within an individual (Heuvelink, 1996, 1999; Marcelis et al., 1998; Carvalho et al., 2006; Gayler et al., 2008) or a population (Battaglia and Sands, 1998; Gayler et al., 2006; Pretzsch et al., 2008).

Maize is widely cultivated worldwide because of its importance in feeding people and animals an in industrialization (Tagne et al. 2008, OECD-FAO, 2013, Lošák et al. 2010, 2011).

The relation of maize with fertilizers has greatly been studied for the purpose of ensuring the stability of yield quantity and quality, (Schröder et al., 1996, Douglas et al., 1998, Schröder et al. 2000, Andraski and Bundy, 2003, Vetsch and Randall, 2004, Andric et al., 2012; Hammad et al., 2012; Tajul et al., 2013; Nazli et al., 2014).

In agricultural practice, in order to obtain large yields, people fertilize their crops most commonly with mineral fertilizers containing nitrogen, phosphorus and potassium. The fertilizers used may contain only one nutrient, usually nitrogen, or they may be complex and include two or three nutrients. It is important that the mathematical relation is known between the doses of fertilizers (active substance) and the yield obtained for surface unit (ha).

In order to use all three nutrients while not stepping away from the bifactorial model, we introduce variables x for the dose of nitrogen and y for the dose of phosphorus and potassium in equal proportions, meaning y is made up of $P_{50\%}$ and $K_{50\%}$. The aim of this research was to develop mathematical models that can describe the variation of the yield and some economic indicators of the grain maize crop.

Materials and Methods

Presentation of the experimental condition

The perimeter of study and research is specific for Banat Plain, in the west of Romania. The soil in the reference area is 85% cambic chernozem.

In order to ensure the nutrients, the fertilizers used were binary complex fertilizers 0:40:30, 0:22:30, ternary complex fertilizers 15:15:15 and urea.

The biologic material is represented by maize hybrid DKC 5143 (FAO 410), recommended for the West Plain, which includes our reference area.

The average climate conditions specific for the area of the experiment are characterized by average rainfall of 600.5 mm and temperatures of 10.9 ºC, with rainfall deficit from July to August associated with high temperatures.

The unifactorial model

The most important unifactorial model is the one given by Mitscherlich. If $f(x)$ is the yield per hectare, and x is the dose of fertilizer, then the functional relation between these variables is given by the relation:

$$f(x) = a_0 + a(1 - e^{-bx}), \text{ (1)}$$

where a_0 is the initial yield, without fertilizer.

The function in relation (1) is the solution to the differential equation:

$$\frac{1}{b}\frac{df}{dx} = [f(\infty) - f(x)], \text{ (2)}$$

which means that the yield increase is proportional with the saturation deficit.

The equations were solved and the graphic representations were made with the help of MuPAD Pro 4.0.

Starting from Mitscherlich's unifactorial model (Mitscherlich, 1909, 1913), there are other researchers in the specialty literature (Harmsen, 2000, 2001), (Nijland, 2008) who have applied the function:

$$f(x) = f(0) + a(1 - e^{-bx}),$$

for one factor. In his paper, Harmsen proposes the application of Taylor series for the unifactorial model.

Nevertheless, it was relatively soon after Mitscherlich published his research that scientists felt the need to continue research in the field by expanding the model to more factors. Thus, (Baule, 1917, Mitscherlich, 1956) generalized empirically to n factors by a direct product of the type:

$$\frac{Y}{Y_x} = (1 - e^{-bN_1})(1 - e^{-bN_2}) \dots (1 - e^{-bN_n}). \text{ (3)}$$

Bifactorial model

The present paper considers a generalization of Mitscherlich function (1) to two variables, like the one in relation (4):

$$f(x,y) = a_0 + a_1\left(1 - e^{-b_1 x}\right)e^{-b_2 y} + a_2\left(1 - e^{-b_2 y}\right)e^{-b_1 x} + a_3\left(1 - e^{-b_1 b_3 x}\right)\left(1 - e^{-b_2 b_3 y}\right), \text{ (4)}$$

where a_0 is the agricultural yield obtained without fertilization, a_1, a_2 and a_3 are integral constants and b_1, b_2 and b_3 are control constants.

By developing function (4), and by grouping the terms into two functions $f_1(x,y)$ where constant b_3 is not involved and $f_2(x,y)$ which involves constant b_3, we get:

$$f_1(x,y) = a_0 + a_3 + a_2 e^{-b_1 x} + a_1 e^{-b_2 y} - (a_1 + a_2)\, e^{-b_1 x - b_2 y}, \text{ (5)}$$

$$f_2(x,y) = a_3(-e^{-b_1 b_3 x} - e^{-b_2 b_3 y} + e^{-b_1 b_3 x - b_2 b_3 y}). \text{ (6)}$$

By generalizing the left member of (2) for each of the two functions, we get:

$$\frac{1}{b_1}\frac{\partial f_1}{\partial x}+\frac{1}{b_2}\frac{\partial f_1}{\partial y}+\frac{1}{b_1 b_2}\frac{\partial^2 f_1}{\partial x \partial y}=-a_2 e^{-b_1 x}-a_1 e^{-b_2 y}+(a_1+a_2)e^{-b_1 x-b_2 y}, \quad (7)$$

$$\frac{1}{b_1 b_3}\frac{\partial f_2}{\partial x}+\frac{1}{b_2 b_3}\frac{\partial f_2}{\partial y}+\frac{1}{b_1 b_2 {b_3}^2}\frac{\partial^2 f_2}{\partial x \partial y}=a_3 e^{-b_1 b_3 x}+a_3 e^{-b_2 b_3 y}-a_3 e^{-b_1 b_3 x-b_2 b_3 y}. \quad (8)$$

When we add member and member of relations (7) and (8) we get:

$$\frac{1}{b_1}\frac{\partial f_1}{\partial x}+\frac{1}{b_2}\frac{\partial f_1}{\partial y}+\frac{1}{b_1 b_2}\frac{\partial^2 f_1}{\partial x \partial y}+\frac{1}{b_1 b_3}\frac{\partial f_2}{\partial x}+\frac{1}{b_2 b_3}\frac{\partial f_2}{\partial y}+\frac{1}{b_1 b_2 {b_3}^2}\frac{\partial^2 f_2}{\partial x \partial y}= \quad (9)$$

$$=-a_2 e^{-b_1 x}-a_1 e^{-b_2 y}+(a_1+a_2)e^{-b_1 x-b_2 y}+a_3 e^{-b_1 b_3 x}+a_3 e^{-b_2 b_3 y}-a_3 e^{-b_1 b_3 x-b_2 b_3 y}.$$

On the other hand, when we calculate the saturation deficit for $f(x,y)$ we have:

$$f(\infty,\infty)-f(x,y)= \quad (10)$$

$$=-a_2 e^{-b_1 x}-a_1 e^{-b_2 y}+(a_1+a_2)e^{-b_1 x-b_2 y}+a_3 e^{-b_1 b_3 x}+a_3 e^{-b_2 b_3 y}-a_3 e^{-b_1 b_3 x-b_2 b_3 y}.$$

Taking into account relations (9) and (10) we deduce that $f(x,y)$ verifies the differential equation:

$$\frac{1}{b_1}\frac{\partial f_1}{\partial x}+\frac{1}{b_2}\frac{\partial f_1}{\partial y}+\frac{1}{b_1 b_2}\frac{\partial^2 f_1}{\partial x \partial y}+\frac{1}{b_1 b_3}\frac{\partial f_2}{\partial x}+\frac{1}{b_2 b_3}\frac{\partial f_2}{\partial y}+\frac{1}{b_1 b_2 {b_3}^2}\frac{\partial^2 f_2}{\partial x \partial y}=f(\infty,\infty)-f(x,y). \quad (11)$$

If we particularize in relation (4) $y=0$, meaning that we only use nitrogen fertilization, then this function becomes:

$$f(x,0)=a_0+a_1 e^{-b_1 x}, \quad (12)$$

i.e. a function of type (1). The verification is the same for $x=0$.

If the second fertilizer, meaning the one based on phosphorus and potassium, is replaced by nitrogen, i.e. $a_1=a_2=a_3$, $b_1=b_2$ and $b_3=1$, practically the model becomes unifactorial, with a dose formed by the sum of the two doses. Indeed, making these particularizations in (4), we get:

$$f(x,y)=a_0+a_1\left[1-e^{-b_1(x+y)}\right], \quad (13)$$

which is a function of the form (1) with added doses.

Determining the constants

In order to determine the constants we apply the least square method in comparison with the experimental data. The experiment was made on a crop of hybrid maize

DKC 5143 (FAO 410), in the soil and climate conditions of Timisoara Didactic Station, in the period from 2006 to 2008, and the results are presented in Table 1.

We specify the fact that the doses of fertilizer used for modelling were for nitrogen as singular element, and for PK as a sum the two in equal proportions.

Table 1. Experimental data regarding the production of maize, at Timișoara Didactic Station, 2006 - 2008

PK \ N	0	50	100	150	200
P_0K_0	4585	5701	6730	7686	7547
$P_{50}K_{50}$	5514	6397	7425	7959	8381
$P_{100}K_{100}$	5806	6775	7668	8361	8865
$P_{150}K_{150}$	5901	6941	8354	9477	9665

Thus, we obtained the following coefficient values:

$$a_1 = 4022.21;\ a_2 = 1358.28;\ a_3 = 10945.52;$$
$$b_1 = 0.0077476;\ b_2 = 0.011506;\ b_3 = 0.49787.$$

Results

Experimental checking

If we represent graphically function (4) for every row and respectively column in Table 1, as well as the corresponding experimental data, we get the graphs in the figures below:

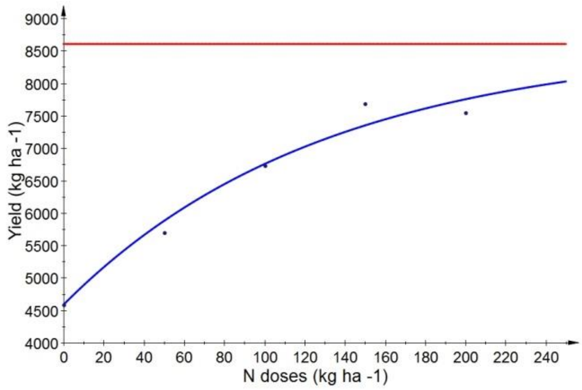

Figure 1. Yield in relation to N, case P_0K_0

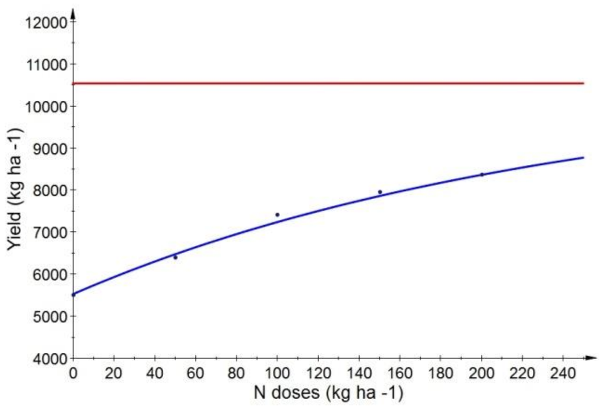

Figure 2. Yield in relation to N, case $P_{50}K_{50}$

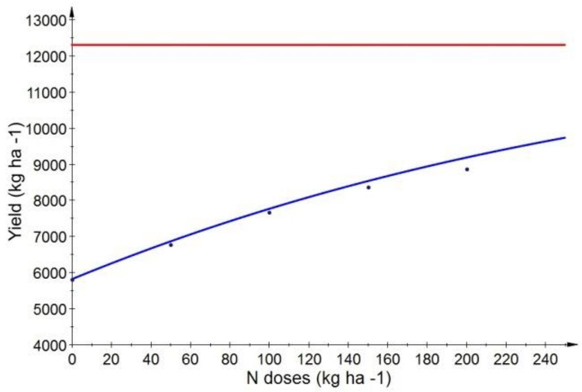

Figure 3. Yield in relation to N, case $P_{100}K_{100}$

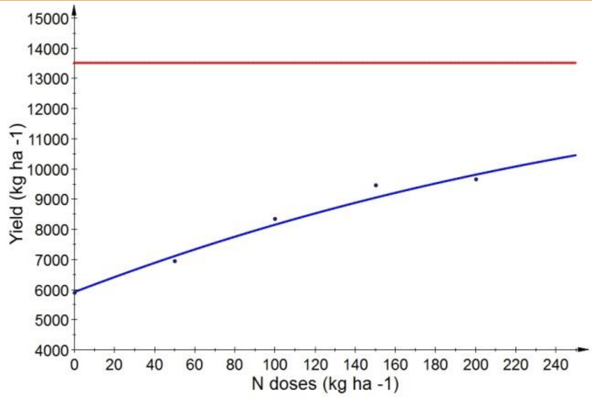

Figure 4. Yield in relation to N, case $P_{150}K_{150}$

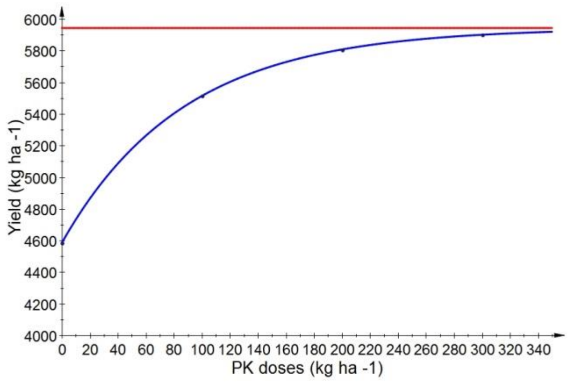

Figure 5. Yield in relation to PK, case N_0

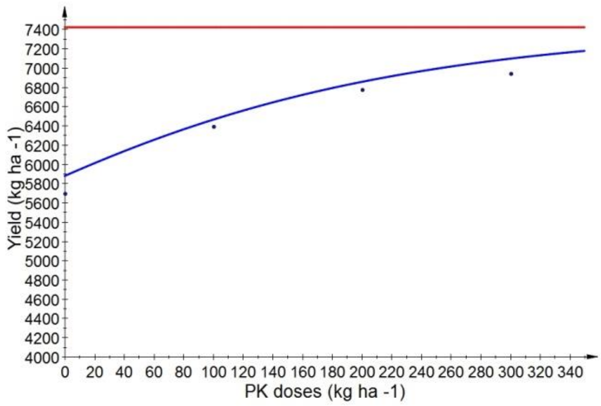

Figure 6. Yield in relation to PK, case N_{50}

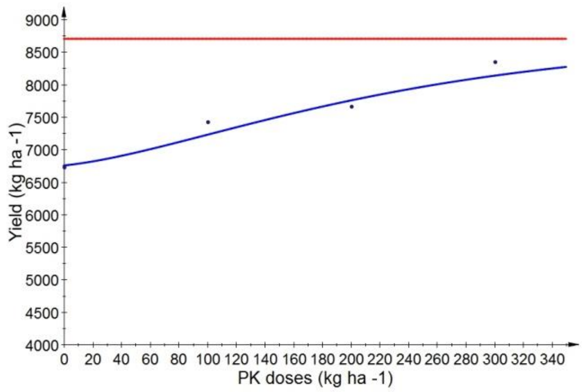

Figure 7. Yield in relation to PK, case N_{100}

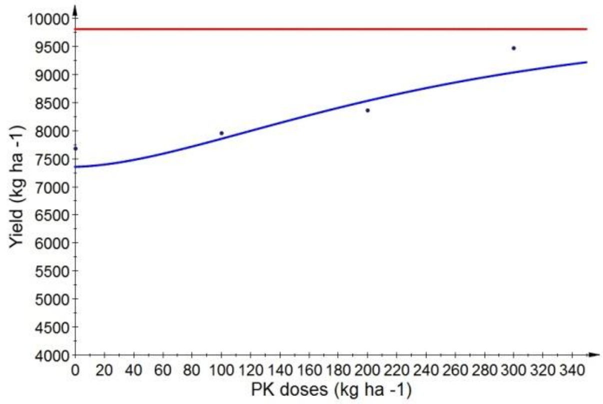

Figure 8. Yield in relation to PK, case N_{150}

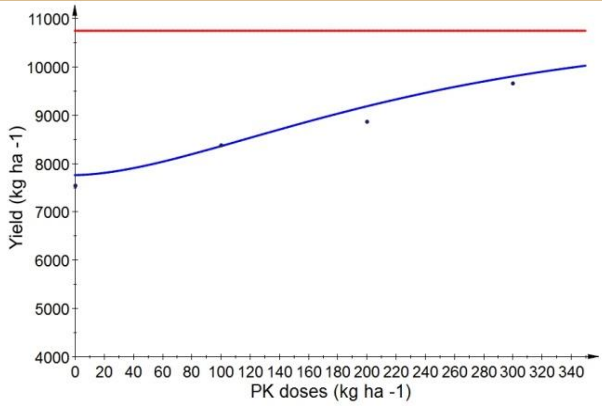

Figure 9. Yield in relation to PK, case N_{200}

The figures above emphasize the good concordance between the experimental data and the theoretical curves resulted from (4).

Yield increase for doses of fertilizers

For introducing the next section, the one about economic considerations, we need to tackle the issue of determining the production maximum. Thus, as it can be seen from the graphic representations 10 – 13 that represent production increases, we will have the following optimum doses and increases for fertilizers supplements: optimum 97 kg ha^{-1} nitrogen on P_0K_0, with production increase 1224 kg; optimum 107 kg ha^{-1} nitrogen on $P_{50}K_{50}$ with production increase 952 kg; optimum 111 kg ha^{-1} nitrogen on $P_{100}K_{100}$ with production increase 968 kg; optimum 103 kg ha^{-1} nitrogen on $P_{150}K_{150}$ with production increase 1413 kg.

In addition, every graphic representation presents the cumulative increases given by the doses of nitrogen fertilizers with different PK combinations.

The method used for determining the optimum values corresponding to figures 10 to 13 is that of annulling first order derivatives.

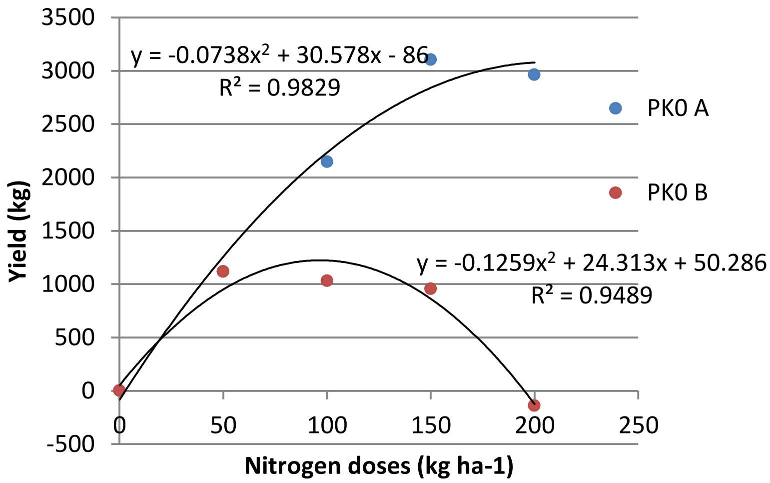

Figure 10. Production increase given by the doses of nitrogen with PK0; A – increase from the control variant; B – increase given by a larger dose than the previous one.

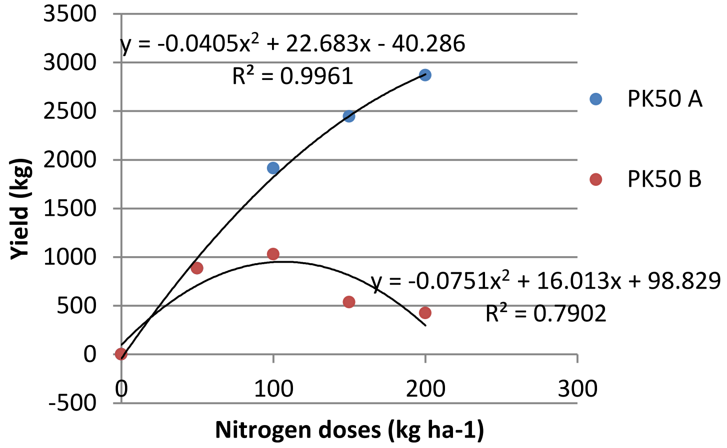

Figure 11. Production increase given by the doses of nitrogen with PK50; A – increase from the control variant; B – increase given by a larger dose than the previous one.

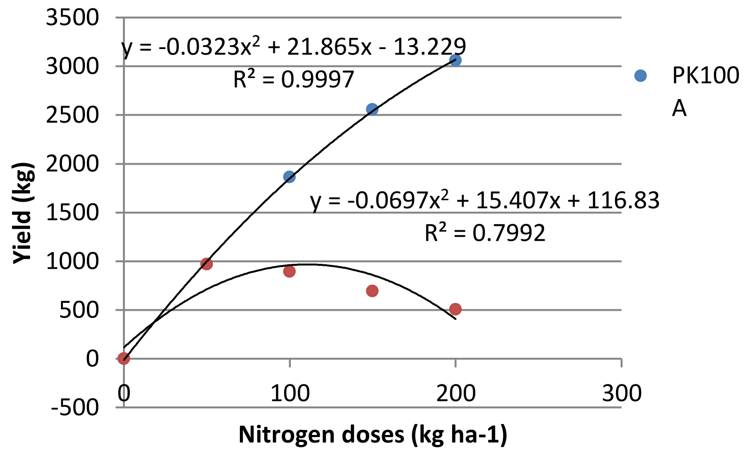

Figure 12. Production increase given by the doses of nitrogen with PK100; A – increase from the control variant; B – increase given by a larger dose than the previous one.

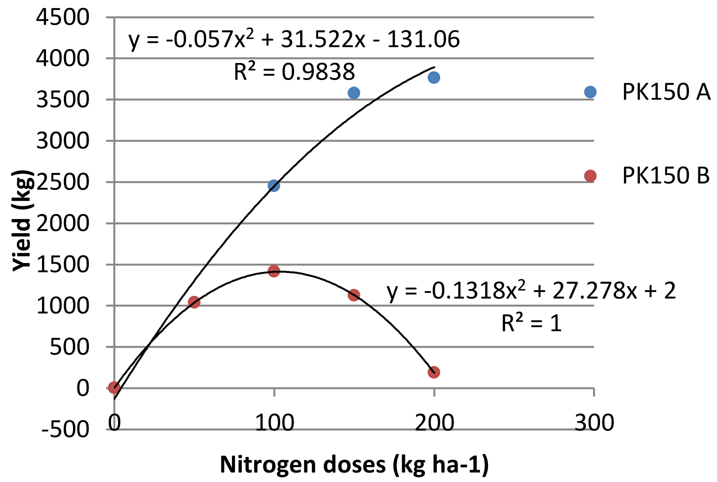

Figure 13. Production increase given by the doses of nitrogen with PK150; A – increase

Economic considerations

It is a known fact that in agricultural practice farmers frequently use complex fertilizers of the type nitrogen - phosphorus - potassium in various set proportions. There are familial exploitations in the frame of subsistence agricultural systems in which farmers, for financial reasons, reduce certain technological stages, and fertilization is one of the seriously affected features. In these cases, instead of a balanced, more expensive, fertilization, a compromise is made, consisting either in applying only nitrogen fertilizers, or in applying complex NPK fertilizers with set percentages (as the example dealt with in the paper), but in insufficient doses that lack balance for that particular crop.

On the other hand, the firms that produce and/or distribute fertilizers provide that type of fertilizer for economic reasons. One of these types is that in which nitrogen, phosphorus and potassium are brought in equal percentages: 15% (NPK 15:15:15). In order to optimize the yield benefit in the case of this fertilizer, we will consider the yield value to be of the type (4), while noting that the only fertilizer available is a complex fertilizer of the type $N_{15}P_{15}K_{15}$.

If we assume the capitalizing price for the maize yield $q = 0.2\,€ \cdot kg^{-1}$, then the yield value is, by notation, $V = qf(x,y)$, meaning:

$$V = q\left(a_0 + a_1(1 - e^{-b_1 x})e^{-b_2 y} + a_2(1 - e^{-b_2 y})e^{-b_1 x} + a_3(1 - e^{-b_1 b_3 x})(1 - e^{-b_2 b_3 y})\right). \quad (14)$$

Under the hypothesis that we only apply fertilizers of the type $N_{15}P_{15}K_{15}$, we will also accept the fact that ,from the point of view of the quantity, $y = 2x$.

If $p = 0.6\,€ \cdot kg^{-1}$ is the selling price of the complex we considered, then the actual per hectare cost of the fertilizer is $C = \frac{px}{0.15} + c$, where c represents fixed costs.

We define benefit, B, as the difference between yield value V and costs C, $B = V - C$ or:

$$B = q\Big(a_0 + a_1(1 - e^{-b_1 x})e^{-2b_2 x} + a_2(1 - e^{-2b_2 x})e^{-b_1 x} + a_3(1 - e^{-b_1 b_3 x})(1 - e^{-2b_2 b_3 x})\Big)\left(\frac{px}{0.15} + c\right).$$

But B is maximum if the derivative is zero, meaning: $\frac{dB}{dx} = 0$.

For the actual price values $q = 0.2\,€ \cdot kg^{-1}$, $p = 0.6€ \cdot kg^{-1}$ the solution of the equation above is $x = 221\,kg \cdot ha^{-1}$, which solution represents the optimal dose of nitrogen active substance used for obtaining the maximum benefit.

Graphically, this optimal solution is obtained as being the abscissa of the tangent between the graph of the curve given by yield value and the parallel to the cost line, represented in R^2 as in Figure 14.

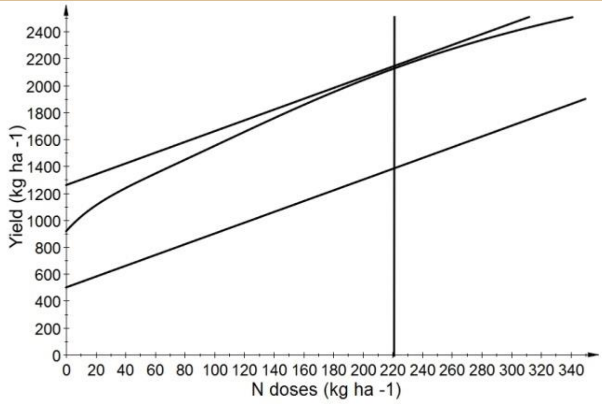

Figure 14. Graphical determination of the optimal solution

In this case, of the graphical interpretation, we also observe that the optimal solution is on the same level with the theoretical one, namely $x = 221\ kg \cdot ha^{-1}$.

These results create a general model for predicting the grain maize yield in relation to fertilizer type and dosage under the conditions presented above; however, it can be expanded. Moreover, the proposed model makes it possible to estimate certain economic efficiency indicators for the crop under analysis.

These technical and economic models can be put into practice through applications, even on mobile devices (smartphone, tablet), which will render them more accessible to agriculturists.

What makes the model presented here new and different from other, classic models, presented by Mitscherlich (1909, 1013), Harmsen (2000, 2001) and Nijland (2008) is the fact that it ensures good concordance between the experimental data and the theoretical behaviour, as well as the fact that it is highly adaptable to different crops and experimental conditions. Although it was developed on experiments with fertilizers, its high flexibility makes it adaptable to other variables as well (irrigation norms, phytosanitary treatments, etc).

Conclusions

The bifactorial model is obtained by generalizing the Mitscherlich unifactorial model in the form of relation (4). If we represent the corresponding functions graphically in an orthogonal axes system, together with the experimental data in Table 1, we observe good concordance between them, which means that the model under consideration does a good job in evaluating the yield in relation to the doses of fertilizers.

From an economic point of view, we maximized the benefit associated to the value function given by relation (14) in the hypothesis that we apply some complex fertilizers of the type $N_{15}P_{15}K_{15}$. For the example considered in the paper, we obtain the same optimal value both theoretically and descriptively, namely the optimal quantity of complex $N_{15}P_{15}K_{15}$ that has to be applied in order to obtain the maximum benefit.

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