# THREE-STATE MARKOV MODEL OF USING TRANSPORT MEANS

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***Abstract***

Guarantee of the high level of tasks execution in enterprise is a proper organization of processes and provision of necessary resources for their implementation. Particularly important, especially from the distribution company point of view, is reliability of held transport means. It depends on rational fleet management, adherence to service intervals, proper their use, as well as even workload and avoidance of unnecessary mileage, which contributes to accelerated wear.

The analysis presented in this article showed that exploitation of transport means is also affected by factors not directly related to them, such as personnel decisions, which strongly determined the degree and manner of their use in the investigated company. Incompetent employees of the customer service department caused that there were generated unavoidable mileage, which could be avoided. They contributed not only to the increase of process costs, but also increased the degree of transport means consumption, unnecessarily reducing their efficiency and effectiveness.

In the study, Markov models were proposed in both discreet and continuous physical time. It concerned two stages of the process, before and after implementation of changes. An analysis of the process evolution over time was also made, calculating boundary probabilities. The obtained models not only allowed for a description of the analyzed system and prediction of selected logistic indicators, but also indicated the directions for possible improvements.

**Key words:** Markov models, exploitation of vehicles, fleet management, vehicles usage

## 1. INTRODUCTION

Studies of exploitation systems, i.e. their elements, which are technical objects and relations between them, are described very widely in the literature. They mainly focus on three main pillars related to their use, diagnosis and operation (Będkowski & Dąbrowski, 2000). Presented analyses concern both the whole exploitation systems, where e.g. combinations of vehicles, machines (Kolator & Niziński, 2004, 2006; Migawa, 2013) are considered, as well as individual components, which consist of technical objects. Many mathematical tools and methods are used for such research, which enabled to obtain satisfactory results. Among them there are also methods of chains and Markov processes (used in this article), as well as semi-Markov, which use the analysis of random processes (also called methods of state space). Their usefulness is demonstrated when it is not possible to make an assumption of events independence and random variables (Iwanik & Misiewicz, 2015). They allow predicting (with a specific probability) the next state, and their reliability is determined by the measures of analytical and prognostic usefulness of the exploitation process model, which are errors in estimating the model parameters and errors in forecasts. They will depend on a number of factors, which limit their use and which, for transport systems, can be classified as such:

1. Exploitation diversification of the consumption of studied samples and vehicle populations. Populations of old types and non-renewed vehicles are usually more diverse than those of new types and regularly renewed.
2. Quality of service and support, which may be worse for new types than for old types in the completion phase (tools, workshop equipment, spare parts), training and development of staff skills.
3. Differentiation of tasks performed by the vehicles influencing different times of preparation for carrying them out. Special vehicles, for example, require additional treatment, such as the need to seal the tank against water crossing or to supplement the extinguishing agents in the fire-fighting vehicle.
4. Numbers and uniformity of studied samples of the same type vehicles. It may turn out that models estimated from too small sample have little analytical and prognostic usefulness. On the other hand, even large samples from the population of old vehicles may prove to be a mixture of many subpopulations, depending on individual vehicle history of operation.

Most often Markov processes are used in modelling of phenomena related to the reliability and readiness of various systems and the effectiveness of their functioning (Nowakowski, 2011; Restel, 2014; Jodejko-Pietruczuk & Werbińska-Wojciechowska, 2014). They are also perfect for tests associated with diagnostics of technical objects, machines and devices (Sugier & Anders, 2013; Smarsz, 1990; Niziński & Wierzbicki, 2002) as well as individual, separated structural elements (Chatys, 2015; Cheng, Sun, Zhao & Gu, 2016; Drożdżel & Krzywonos, 2009; Chojnacki & Stępień; 2007). They are even used in research related to human health and condition (Zygulska & Sokołowski 2008; Leviton, Schulman, Kammerman, Porter, Slack, & Graham, 1980).

This versatility and universality, as well as the ability for modelling even very complex systems intrigued the author, encouraging him to wonder whether it is possible to describe the relations between seemingly independent processes in a company using Markov models. It is obvious that proper and effective fleet management is connected with taking care of appropriate diagnostics, maintenance and repair of vehicles, as well as their rational use. However, it is wondering whether decisions made in other areas of the company, not directly related to operating processes, may affect their accelerated wear and tear. The possibility to obtain data from the company distributing spare parts and subassemblies gave the author an opportunity to answer his questions.

The surveyed company is a wholesaler of spare parts for motor vehicles. It offers a wide range of products for cars, accessories, workshop tools and necessary fluids and consumables. One of the company's branches dealing with direct delivery of orders to local car workshops was provided with study. Distribution is carried out in its own vehicles on the basis of previously planned routes assigned to individual drivers. Orders are placed via the customer service. On their basis, sellers prepare a document of issue and deliver it to the warehouse, where purchases are completed and prepared for transport, and then picked up by the drivers and provided to the recipient. The company has been struggling with the problem of multiple returns for a long time. These were mainly due to the fact that products delivered to the workshop did not fit to repaired vehicle despite the fact that the year of manufacture, brand and vehicle model was the same. In such situations, the mechanics returned the wrong part and ordered a new one (sometimes in another company). The whole process generated additional costs, forced unplanned journeys, and above all had a negative impact on the company's image and reduced customer satisfaction. It also forced additional, unplanned exploitation of transport means. Therefore, an initiative was taken in the company to identify the cause of problem and to streamline the implemented processes, which were successful. In the article, an additional study was carried out, showing how the decisions made influenced on exploitation of transport means. Markov processes were used for this purpose.

## 2. CHARACTERISTICS OF STUDIED PROCESS

The factor most strongly affecting the quality perceived by the customer is time of order delivery and its correctness. The necessary parts are defined on the basis of special catalogues dedicated to each brand. Orders are most often placed by phone, the customer calls with the brand, model, engine power and manufacture year of the given car, and on this basis individual items are searched. Then the party accepting an order assigns it to a given route and sends information about the need of order completion to the warehouse. The prepared packages are collected by the driver, who delivers them to the customer. After delivering all orders, the driver returns to base where receives another delivery list. It has often happened that before the driver returned from route, the distribution centre had already been informed that part of delivered assortment was inappropriate, did not meet the customer's needs and had to be picked up. Depending on the customer's expectations, such purchases are taken up as a matter of urgency (especially if they concern a large or very expensive assortment) and then the driver instead of going with deliveries first goes to collect the defective goods and only then carries out subsequent orders. If there is no need for immediate collection, all returns are collected at the end of the day, after all deliveries have been made, which is the driver's last route. As far as possible, returns are also received during the processing of essential orders if the driver receives such information in advance and if time and distance allow. In this case, no additional mileage is generated.

As the number of returns was worrying for the management, exposed to financial losses and damaged the company's image, it was decided to investigate the problem thoroughly. In the first place, mistakes were seek in the warehouse (release of goods not in accordance with the list provided by the customer service department) or on the driver's side (mistakes in the delivered packages). However, it turned out that the scale of problem in these two cases is negligible. Therefore, the analysis covered the stage when the customer along with the sales representative, determines the specification of required assortment. It turned out that after checking the completed and issued orders and returns from the last quarter for the eight working sales representatives, in as many as six cases very large quantities of returned assortment were found, especially from the spare parts department for vehicles and motorcycles. Only two salesmen were successful. Returns in their case were occasional. The next step was to check the level of knowledge in the field of mechanics and knowledge of the program for handling orders. In this case, the results were unfavorable for the majority of sellers, as six of them proved their knowledge only at a sufficient level, which did not ensure professional performance of their duties. Even if the computer and software handling responsible for ordering parts was at a good level, the lack of theoretical knowledge of mechanics, as well as practical experience in carrying out repairs resulted in erroneous indications of necessary parts in the majority of accepted orders.

This is because it is not an easy and obvious process. The complexity of this order will be illustrated by an example of Mazda model 6 passenger car manufactured in 2005 with a spark ignition engine with a capacity of 2000 cubic centimeters and a power output of 147 KM. The mechanic needed a brake component, namely grinding inserts and front petal brake discs. As in the recalled calendar year the manufacturer decided to manufacture, apart from traditional version, also a modernized one (changes concerned both the appearance of body, as well as engine and braking system), it resulted in two types of braking systems produced for this type of car, each equipped with different discs and grinding inserts. In addition to original parts, replacements from other companies are also available. As a result, after opening the catalogue, it turns out that according to specification quoted by the customer there are dozens of different items to choose from. The VIN number of vehicle could be helpful, but it is not always available to the mechanic when ordering, and not all manufacturers include this information in parts description. In such a situation, an experienced sales representative knows that in the modernized version, which has a higher engine power, brake discs with a larger diameter are installed, and in the standard version a smaller is needed, so selects the right one. The inexperienced staff very often suggested only the stock of a given assortment and issued a part which was available at the moment or chose the most frequently ordered one, which unfortunately was often not the right decision.

After analyzing the results of audit, the company's management decided to implement changes in the customer service department concerning the staffing of direct sales positions. So far, the salesmen working in the company have been randomly recruited from the employment agency. The situation revealed during the audit resulted in a decision to change the source of personnel. For that purpose, the market was re-analyzed and companies providing employment services were selected, recruiting only those which offer trained employees in accordance with the company's expectations. The basic criterion was knowledge of mechanics and vehicle construction, confirmed by experience and practice in the profession related to such issues. In addition, direct recruitment was carried out in parallel, looking for salespeople, who met exactly the requirements.

Gradually, sales representatives were replaced by qualified staff and it turned out that such a change directly reduced the number of returns, improved the company's image and increased customer satisfaction. This has been influenced by several factors. Firstly, there were no more casual people in retail positions. Their place was taken by specialists, who were able to advise the customer during the telephone order picking what exactly assortment should be ordered and what additional parts are needed for the proper repair or overhaul. Secondly, each sale representative was assigned to a specific number of regular customers, which over time turned out to be a great success, because the customer working with a dedicated sale representative, could get to know him well, appreciate his knowledge, experience and trust, which greatly facilitated and accelerated the process of placing orders. For the representatives of mechanical workshops (mostly small, few-person enterprises), such a change was particularly beneficial in terms of streamlining their work. Previously, any improper delivery caused unplanned downtime, caused costs resulting from suspension of repairs and overhauls, and had a negative impact on customer relations due to delays in the acceptance of repaired equipment.

It was also decided to check how this change translated into the exploitation of transport means used by the company. Since on-board computers are used in cars, in which the driver records every time the activity is performed, it was decided on the basis of these records to analyze the process before and after implemented change in terms of exploitation of transport means. Such an analysis is presented in this article. A comparison was made of two stages of the exploitation process, carried out before and after implemented changes, using the Markov processes for that purpose.

## 3. MARKOV MODEL IN THE DISCREET TIME

Markov processes are such a group of stochastic processes $\{X_t\}_t$ that meet the property of Markov, i.e. assumption that there is no event (Filipowicz, 1996). This means that for each moment $t_0$ the probability of change in any system position for $t > t_0$ depends only from its position at the moment of $t = t_0$, and does not depend on how this process proceeded in the past. Therefore, a random process $\{X(t_0 + \tau), t\epsilon T\}$ is calling the Markov process, when for any finite time sequence $t_1 < t_2 < \cdots < t_n$ $(t_1, t_2 \dots t_n \,\epsilon\, T)$ and any real numbers $(x_1, x_2, \dots, x_n)$, an equality occurs (1):

$$P[X(t_n) < t_n | X(t_{n-1}) = x_{n-1},\ X(t_{n-2}) = x_{n-2}, \dots, X(t_1) = x_1] = P[X(t_n) = x_n | X(t_{n-1}) = x_{n-1} \quad (1)$$

which means that the conditional probability distribution of a random variable $X(t_n)$ at the moment of $t_n$ depends only on the probability distribution of a random variable $X(t_{n-1})$ at the moment of $t_{n-1}$, and does not depend on probability distributions of a random variable that the process assumed at the moments of $t_1, t_2, \dots, t_{n-2}$.

The construction of Markov model began with identification of possible exploitation states in which vehicles carrying out the deliveries are. A minimum number of 3 separate states have been distinguished, which enabled to analyze the delivery process before and after implementation of changes. These were states as follows:
S1 – performing the task according to the schedule – delivery;
S2 – handling operations;
S3 – completion of returns.

The operational functions of the facility and system performed in individual states are as follows:
S1 – driving in connection with the transport task, i.e. delivery of ordered goods to the customer.
S2 – handling activities covering loading and unloading of goods, completing documents necessary to release goods, settlement of money for goods delivered to the customer, if the customer does not have a signed delivery contract with a deferred payment period and is settled in cash.
S3 – returns – refers to additional, unplanned journeys, which result from deliveries of the wrong assortment. They take place as a matter of urgency when the customer demands immediate collection of goods (which also results from the provisions of contract with the workshops), or otherwise at the end of working day. Then, after all deliveries have been made, the driver goes to workshops and collects returned products, generating an additional route. If it is possible to receive returns during scheduled deliveries, e.g. in case of replacement of goods, this does not result in an additional route.

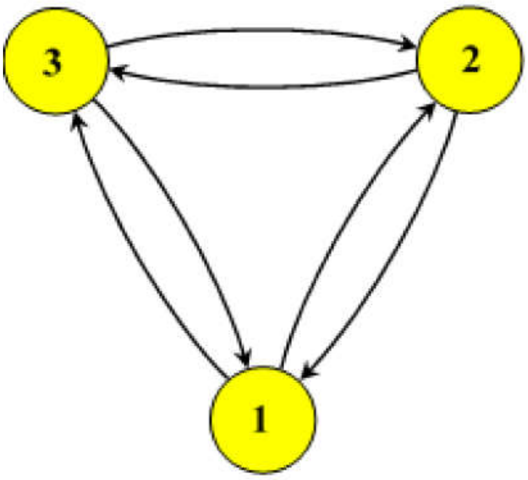
**Table 1.** Transitions matrix between states

| ↓Si→Sj | S1 | S2 | S3 |
|---|---|---|---|
| S1 | 0 | 1 | 1 |
| S2 | 1 | 0 | 1 |
| S3 | 1 | 1 | 0 |

Source: own study

After defining the possible exploitation states, the next step was to analyze the possible transitions between states. In studied process all interstate transitions are allowed, as described in a form of matrix (Table 1) and with a graph on Figure 1.

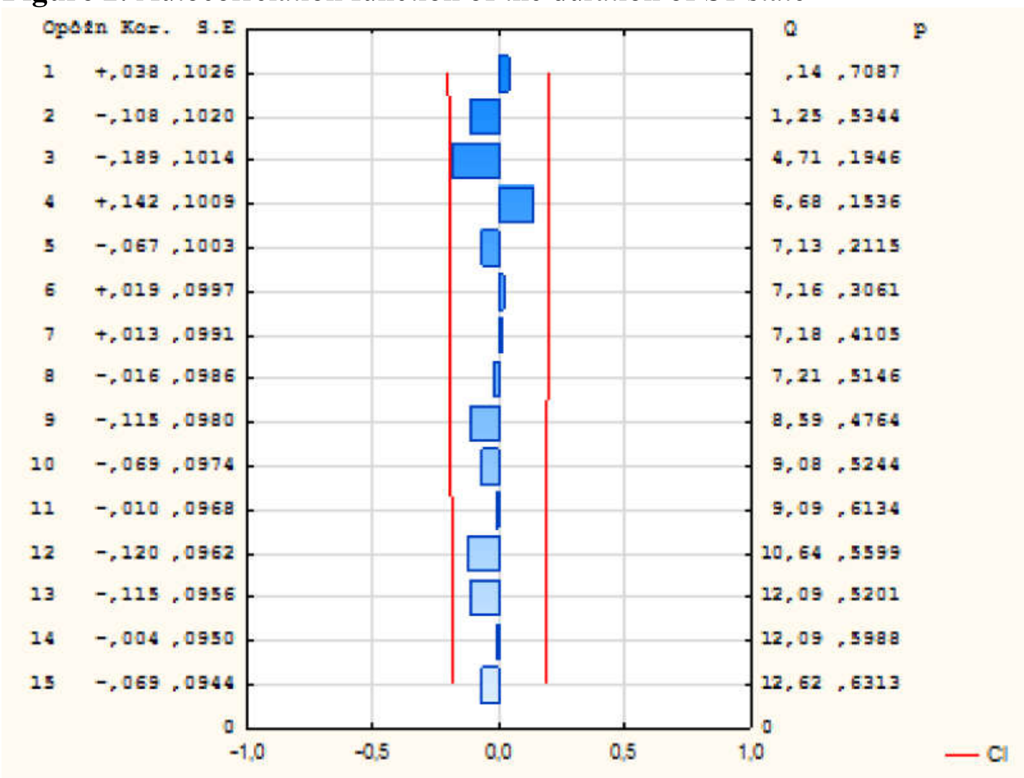
**Figure 1.** Graph of transitions between states




Source: own study

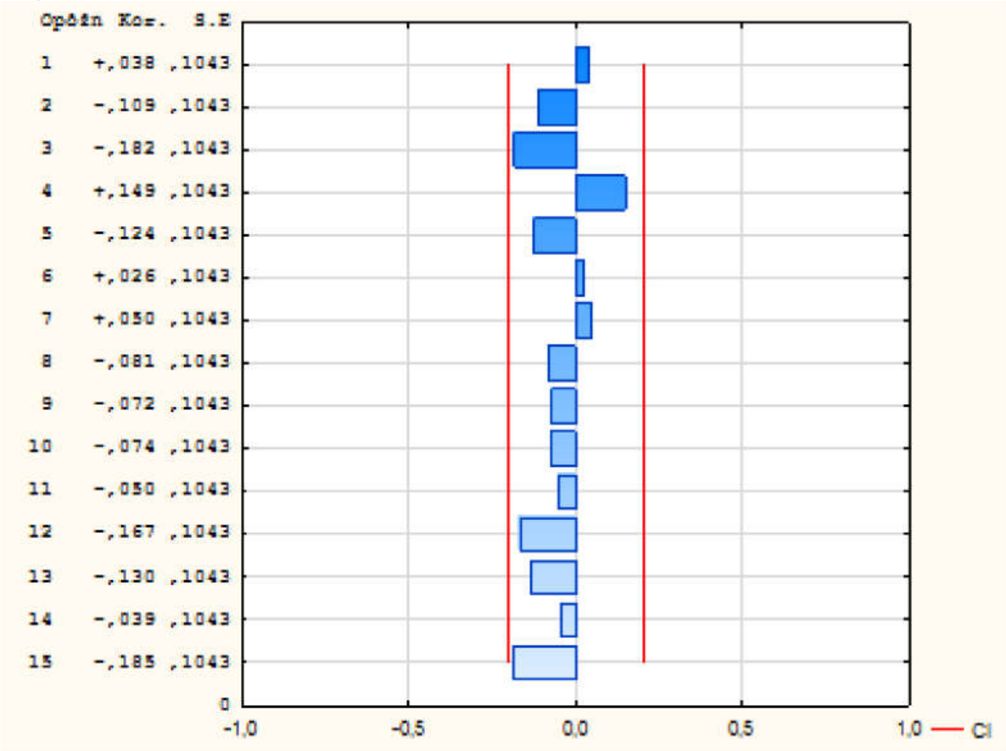
Before beginning the estimation, it is necessary to examine aforementioned Markov property, i.e. the property of memory lack, which means that the probability of each event depends only on the previous result and not on the previous history of process (Iwanik & Misiewicz, 2015; Kałuski, 2007). For this purpose, the autocorrelation function and partial autocorrelation function is used, which graphs for example duration of S1 state were presented on Figure 2 and Figure 3.

**Figure 2.** Autocorrelation function of the duration of S1 state




Source: own study

**Figure 3.** Partial autocorrelation function of the duration of S1 state




Source: own study

The above correlograms do not show the value of autocorrelation function and partial autocorrelation function significantly different from zero, which proves that the history of investigated process is negligible in further analyses.

Markov processes with discreet time are called Markov chains. Their mathematical description is a stochastic matrix $P$, which elements $p_{ij}$ are probabilities of transitions from $X_n = i$ state to $X_{n+1} = j$ state (Iwanik & Misiewicz, 2015; Kałuski, 2007; Niziński & Żurek, 2011; Woropay et al., 2003). Therefore, in the next research stage, the database containing trajectories phase of objects was processed to a form of estimator element values $p_{ij}$ of matrix transitions probability $P$. The values of these estimators from test are transitions frequency $w_{ij}$ from $S_i$ state to $S_j$ state: They are necessary to draw the model of Markov chain for studied process. In this way, a matrix of transition probabilities was obtained for two analyzed periods of the exploitation process – before implemented changes (stage 1) and after their implementation (stage 2). Obtained values of estimators of both stages are presented in Table 2 and Table 3, respectively.

**Table 2.** Element values $p_{ij}$ of matrix $P$ of the first research stage

| $p_{ij}$ | S1 | S2 | S3 |
|---|---|---|---|
| S1 | 0 | 0.87 | 0.130 |
| S2 | 0.789 | 0 | 0.212 |
| S3 | 0.294 | 0.706 | 0 |

Source: own study

**Table 3.** Element values $p_{ij}$ of matrix $P$ of the second research stage

| $p_{ij}$ | S1 | S2 | S3 |
|---|---|---|---|
| S1 | 0 | 0.951 | 0.049 |
| S2 | 0.935 | 0 | 0.065 |
| S3 | 0.25 | 0.75 | 0 |

Source: own study

The reliability and dispersion of estimated estimators were then investigated by calculating their standard deviations, also called maximum estimation errors for the central confidence intervals, and the coefficients of variation of the estimators, called relative estimation errors. Satisfactory results allowed the research to continue and to compare two periods. It was investigated how the individual probability estimators changed, determining the percentage difference between the period before and after implementation of change in the seller's position. The results obtained are presented in Table 4.

**Table 4.** Probabilities value ratio for transitions of two process stages

| % | S1 | S2 | S3 |
|---|---|---|---|
| S1 | 0 | 9.4 | -62.4 |
| S2 | 18.5 | 0 | -69.1 |
| S3 | -15 | 6.3 | 0 |

Source: own study

Among the interstate changes that have taken place in the analyzed system, it is worth to note, first of all, the relation between driving and handling operations. It turned out that both the number of transitions between S1-S2 states (9%) and in the opposite direction S2-S1 states (18%) have increased. This has been influenced by several factors. First of all, acceptance of orders by specialists significantly reduced the time of their preparation, as well as eliminated long browsing through catalogues and searching for appropriate parts. This shortened the total preparation time and enabled the driver to go on the road faster and return earlier, resulting in more journeys. In addition, the professional approach and knowledge of the subject resulted in better advice, which resulted in limited in some situations subsequent deliveries, as all the necessary elements related to a given repair were completed in a single order. This was the case when the party accepting an order pointed out the necessity of purchasing parts related to replacement of a given component, which are usually also needed and the buyer did not pay attention to them. An example is replacement of gasket underneath the engine head, which in many cases also forces replacement of screws in the head, or replacement of camshaft, at which the sealing rings would need to be replaced. Proposal of such a comprehensive purchase, suggested by the sales representative, limited further routes and allowed to serve more customers at that time, which also contributed to the increase in number of interstate transitions. Moreover, it had a very positive impact on the company's image. There was a clear customer satisfaction resulting from reliable service and professional advice. However, between S1 state (delivery) and S3 state (returns) there was a significant decrease in the transition frequency of 62% and 15%, respectively, which results from the fact that total number of transitions in the total sum of performed tasks decreased. This became a highly satisfactory situation for the company, as it proved the achievement of its objectives.

Models of the Markov chain, estimated for both analyzed stages turned out to be ergodic, which enabled a long-term forecast of the evolution of analyzed system (Niziński & Żurek, 2011; Woropay et al., 2003). For this purpose, the limit probabilities $p_j$ allowing for prediction of the implementation process after a long period of time were estimated. The calculations were made using Solver addition to Microsoft Excel and confirmed in Mathematica program dedicated to carry out calculations in a mathematical environment. The obtained results were compared with each other as shown in Table 5 and Figure 4.

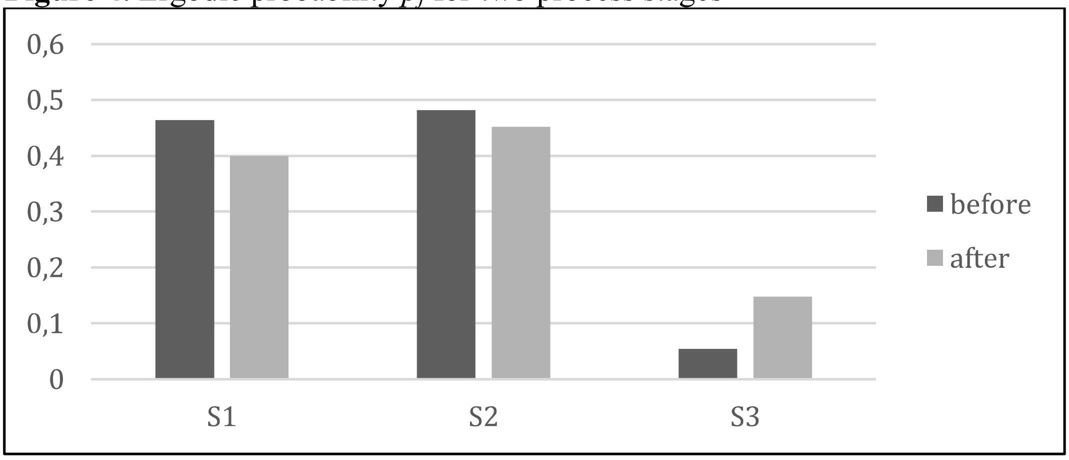
**Table 5.** Ergodic probability $p_j$ for two process stages

| $p_j$ | | S1 | S2 | S3 |
|---|---|---|---|---|
| before | $p_j$ | 0.4 | 0.452 | 0.148 |
| | $p_{j\%}$ | 40 | 45.2 | 14.8 |
| after | $p_j$ | 0.464 | 0.482 | 0.054 |
| | $p_{j\%}$ | 46.4 | 48.2 | 5.4 |

Source: own study

According to the Markov model, in a discreet time the system primarily aims to maintain in two states, in S1 state performing the delivery and in S2 state including handling operations, at a very similar level. However, the system's limit probabilities after changes are higher for these states than for the values before changes by 16% and 6.6%, respectively. This is due to increased number of initial returns and high S3 probability limit of 14.8%. On the other hand, after changes, the limit probability of return completion was 5.4%, i.e. it decreased by as much as 63%. Long-term forecasts for the Markov chain are therefore satisfactory from an entrepreneurial point of view.

**Figure 4.** Ergodic probability $p_j$ for two process stages




Source: own study

In order to check whether the process was far from the limit of equilibrium state at the time of data collection, differences in frequency and ergodic probabilities were calculated, which are presented in Table 6.

**Table 6.** Deviations % of $w_j$ frequencies from ergodic probability $p_j$ of S1 – S3 states

| Deviation % | S1 | S2 | S3 |
|---|---|---|---|
| before | -0.43 | 0.52 | -0.43 |
| after | 0.53 | -0.42 | -0.83 |

Source: own study

It turned out that for both processes all percentage deviations from the hypothetical equilibrium were satisfactory small, thus the system at the research stage was not far from asymptotic equilibrium for the Markov chain.

Average lead times for individual activities were also compared and the results are presented in Table 7. Again, they are satisfactory and indicate a decrease in the average time of handling (by 11%) and returns (by 23%), i.e. states that do not translate into profit, but even generate costs for the company. However, the average duration of the most important condition of supply of the purchased assortment from the point of view of the activity increased by 3%. This means an increase in the transport volume and thus an increase in the number of transactions, thus also in the company's revenue.

**Table 7.** Comparison of average durations of S1 – S3 states

| State →<br>Time in minutes ↓ | S1 | S2 | S3 |
|---|---|---|---|
| before | 111.46 | 26.5 | 41.24 |
| after | 114.98 | 23.44 | 31.75 |
| change % | 3.16 | -11.53 | -23 |

Source: own study

## 4. MARKOV MODEL IN CONTINUOUS PHYSICAL TIME

Since the analysis of phenomenon only in selected moments of time (as for the Markov chain) is often insufficient to fully describe it (Żółtowski & Niziński, 2002), it was decided to build the Markov model in a continuous physical time, which can provide more complete information about the process. In such a case, the Markov model describes transition intensity matrix $\Lambda$ with elements $\lambda_{ij}$ (Table 8, Table 9), which is a function characterizing the rate of changes in transition probabilities $pij(t)$ (Filipowicz, 1996).

**Table 8.** Element values $\lambda_{ij}$ of matrix $\Lambda$ of the first research stage

| $p_{ij}$ | S1 | S2 | S3 |
|---|---|---|---|
| S1 | 0 | 0.87 | 0.13 |
| S2 | 0.789 | 0 | 0.212 |
| S3 | 0.294 | 0.706 | 0 |

Source: own study

**Table 9.** Element values $\lambda_{ij}$ of matrix $\Lambda$ of the second research stage

| $p_{ij}$ | S1 | S2 | S3 |
|---|---|---|---|
| S1 | 0 | 0.951 | 0.049 |
| S2 | 0.935 | 0 | 0.065 |
| S3 | 0.25 | 0.75 | 0 |

Source: own study

On the basis of estimated values of transition intensity, ergodic probabilities were estimated for two process stages and obtained results were presented in Table 10.

**Table 10.** Ergodic probability $p_j$ for two process stages in continuous physical time

| $p_j$ | | S1 | S2 | S3 |
|---|---|---|---|---|
| before | $p_j$ | 0.600884 | 0.150823 | 0.248294 |
| | $p_{j\%}$ | 60.1 | 15.1 | 24.9 |
| after | $p_j$ | 0.641631 | 0.143636 | 0.214732 |
| | $p_{j\%}$ | 64.2 | 14.4 | 21.5 |

Source: own study

The rate of changes in transition probability increased significantly for the first state, which means that during the hour of lasting the process of such changes there was almost 7% more, whereas it decreased for the second state by almost 5% and for the third state by 14%, which indicates a decrease in the number of transitions per hour. For both research stages, the received results show that studied system aims to maintain in more than 60% of the total delivery time (S1), about 15% in organizational terms (S2) and more than 20% in returns. While the asymptotic probability for S3 state has changed after implemented changes, but it is still relatively high.

In the last stage, it was studied how far from limit equilibrium the system was after changes during continuous physical time. Frequencies of states in the time domain were calculated and compared with limit probabilities to obtain the results presented in Table 11.

**Table 11.** Deviations in empirical frequencies and limit probabilities

| | S1 | S2 | S3 |
|---|---|---|---|
| $p_j$ % in time | 64.16 | 14.36 | 21.47 |
| $w_j$ % in time | 80.39 | 17.02 | 2.59 |
| Deviation $w_j$ from $p_j$% | 25.03 | 18.1 | -86.9 |

Source: own study

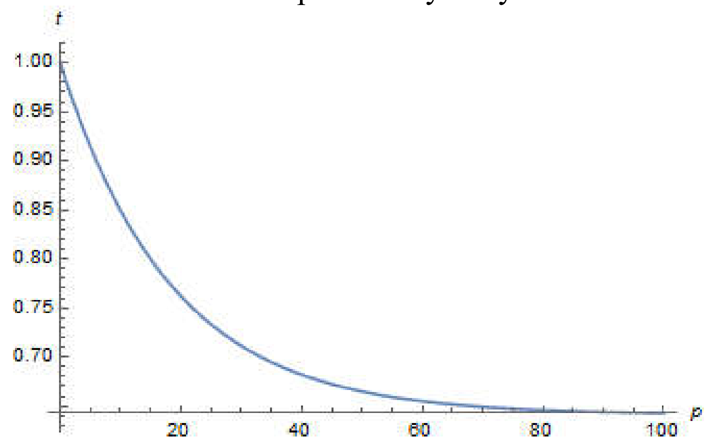
It turns out that the system during the study was far from equilibrium limit. The probabilities limit for the first and second states are lower than calculated frequencies, but these are not drastic differences. Differently it is in case of the third state, where deviation is very high and negative, what suggests that after some time the reduced number of returns will increase again, but to a lower level than before implemented changes. The reason for such a situation may be some kind of routine and less care of employees, which comes as long as they are employed in the company.

The last stage of the study was to determine the characteristic time of object's reaching stationary state after a given set of initial states. For that purpose, it is necessary to solve the Chapman-Kolmogorov-Smirnov equations, which for the process under study have the following matrix form:

$$\begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \end{bmatrix} \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & -\lambda_{33} \end{bmatrix} = \begin{bmatrix} p_1'(t) \\ p_2'(t) \\ p_3'(t) \end{bmatrix} \quad (2)$$

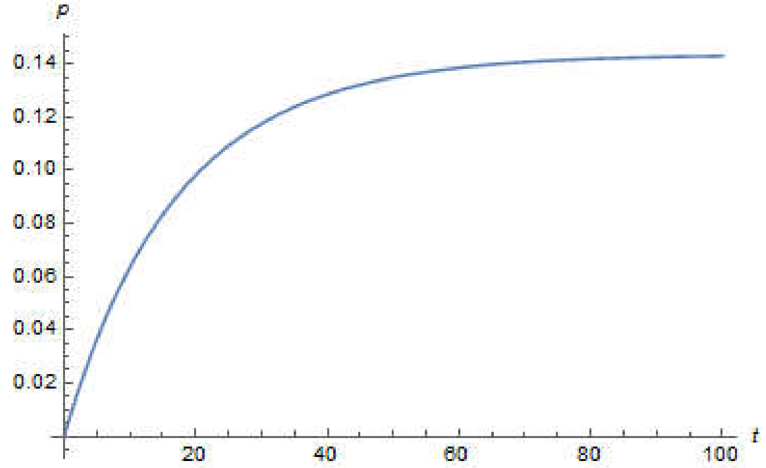
Correct analytical solution of CH-K-S equations with restrictions, standardization, was determined using the Mathematica Markov continuous module. For the purpose of study, it was assumed that at the initial moment $t = 0$ of the process $X(t)$ was in S1 state. This enabled to draw the function showing the process of reaching the limit state, respectively for S1 state (Figure 5), S2 state (Figure 6) and S3 state (Figure 7).

**Figure 5.** Evolution of the probability of system in S1 state



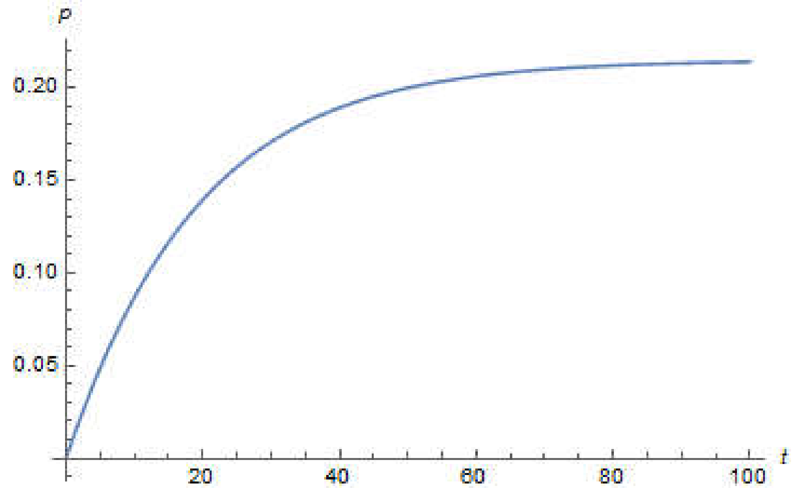

Source: own study

**Figure 6.** Evolution of the probability of system in S2 state




Source: own study

**Figure 7.** Evolution of the probability of Star cars in the system state of S3 state




Source: own study

The probability of S1 state (delivery completion) decreases to the limit value in about 60 minutes. Asymptotic values for the remaining states are determined in the same time. Therefore, one hour after forcing the initial S1 state, the system achieves an equilibrium state.

## 5. SUMMARY AND CONCLUSIONS

The survey showed how simple modifications in a company can affect its overall functioning. The change in staff responsible for sales, seemingly insignificant from the exploitation of transport means point of view, turned out to be a strong determinant of the level of vehicle use. Incompetent employees of the customer service department, due to their poor knowledge of vehicle mechanics, contributed to generation of additional journeys, resulting from an increased number of returns. This resulted in many over-planned routes and contributed to increased consumption of transport means, and as a result, caused additional costs incurred by the company, which were generated by more frequent maintenance repairs of braking, steering, wheels and suspension systems, as well as replacement of filters and operating fluids.

After implemented changes, when purchasing advice increased to a high level, the time previously devoted to handling of returns, now was largely spent on subsequent deliveries. Therefore, it became from an activity generating costs to one which is profitable and vehicle usage became a normal part of the logistic process and was fully justified. Calculated forecasts according to the Markov model for discrete and continuous time differ. Both models generate the same forecast of the most probable limit values, but with different values of limit probabilities. According to the forecast for Markov chain (for abstract discrete time of subsequent changes of states) and the empirical forecast according to the frequency of observation, the system after changes aims to maintain mainly in S1 state (delivery) and S2 state (handling activities), while maintaining in S3 state regard to only 5% of observation. Furthermore, the analysis of system's deviation from the asymptotic balance showed that it is not significant, and therefore the presented model can be considered as reliable.

According to the Markov model in a continuous period of time, the system after changes aims to maintain in S1 state for 64% of the time, which is a value higher than the first research stage by almost 7%. A similar increase (5%) was recorded for handling operations. However, the decrease in limit maintain during lead time of returns was not as spectacular as for the chain and was only 14%. The conclusion from this forecast should be to focus the management on long-term maintenance of the effect of replacing the sales department staff, whose activity and involvement may decrease after a longer period of employment. Therefore, it is worth to consider special bonus mechanisms, which will enable to maintain success in a longer term.

To sum up, it should be emphasized that even a simple, three-state Markov model creates a possibility to describe the system of transport means functioning. It is sufficient for an overall survey that indicates the main directions for improvement in the company. Positive results of the analysis also encourage a deeper diagnosis of the analyzed exploitation system, which may be carried out by defining the exploitation states of vehicles in more detailed way. These can include, e.g. states of use, standby duty, maintenance, repair and stops in repair. The analysis of such definite phase trajectories and time characteristic for time series will allow to:

- examine the impact of objects diversity on the course and results of process,
- determine the impact of indeterminate (random) circumstances on the exploitation indicators and reliability indicators of operated objects and sets of objects,
- real forecast indicators and results of the process of objects operation, taking into account their diversity and the influence of random factors on the process.

Models that enable detection of the phenomenon nature, and to forecast, will be valuable not only for the surveyed company, but also – thanks to the use of established pattern – for other exploitation systems. They can be used particularly effectively in systems where the readiness of technical objects is a very important parameter, and therefore not only in companies providing distribution services, but also e.g. in public institutions (health care, fire brigade). However, their use is limited by the possibility of obtaining detailed information on the times of particular activities performed by vehicles, which in many companies are not measured and registered. For systems that guarantee accurate recording of start-up and completion times (e.g. via on-board computers), the best results will be obtained, which allow the exploitation processes to be shaped in a highly reliable way. Currently, more entrepreneurs choose such solutions. The increases in their popularity cause that methods presented in this article are future-proof.

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