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Analytical Control of the Mass of Injected Fuel into Diesel Engine Cylinder

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ABSTRACT

In real operating conditions, the engine indicator diagram is the most common source of data for analyzing its current technical condition. Despite this, it is not possible to determine all diagnostic parameters directly from it, for example, the mass of fuel injected into the cylinder. To determine the fuel supply, it is possible to use the calculation of heat release in the cylinder or by solving a system of differential equations describing the working process in the cylinder. However, in those cases, the approximate value of the average temperature of the cylinder walls is unknown, as well as which of the empirical formulas for calculating the heat transfer from the gases to the cylinder walls should be used. Therefore, an additional method for calculating the mass of fuel injected into the cylinder was developed, in which the actual working process, under certain assumptions, was represented by a calculated cycle with isochoric and isobaric sections of fuel combustion. As a result, two systems of algebraic equations were compiled, the solutions of which can be used to find the mass of injected fuel. During the research, the process of solving equations was modernized, which allowed for a significant increase in accuracy.

1 Introduction

In real operating conditions, the engine indicator diagram is the most common data source for analyzing its current technical condition. Usually, the maximum cycle pressure, maximum compression pressure and, after calculating the diagram area, the engine indicated power and mean indicated pressure are determined directly from it. The mentioned parameters allow us to evaluate the technical condition of the friction pair "piston rings - cylinder", the power distribution between the engine cylinders, and also, having the dependence of the pressure in the cylinder on the crank angle, it is possible to

evaluate the dynamics of fuel combustion. However, not all diagnostic parameters can be determined directly from the performance diagram. One of them is the mass of fuel injected into the cylinder. Knowing this mass, one can calculate, for example, such engine performance indicators as engine efficiency and specific fuel consumption. In addition, by calculating the mass of actually injected fuel, it is possible to integrally assess the condition of the engine's high-pressure fuel equipment (pump, injectors). Thus, it can be argued that calculating the mass of injected fuel is a relevant task that allows for a more complete assessment of the current technical condition of the engine.

Nomenclature

a_0, a_1, a_2 – coefficients, the values of which are determined depending on the composition of gases in the cylinder;

A_c – work of compression received with proposed methodology [kJ];

A'_c – work of compression received in simulated data (using complexes AVL-Boost or Blitz-PRO) [kJ];

A_{rw} – area of expansion work [J];

b_i – specific indicated fuel consumption received with proposed methodology [g/(kW·h)];

b'_i – specific indicated fuel consumption received in simulated data (using complexes AVL-Boost or Blitz-PRO) [g/(kW·h)];

C, H, O, S – mass fraction, respectively of carbon, hydrogen, oxygen, sulfur in fuel;

$c_{cp}(T)$ – temperature-dependent molar isochoric heat capacity of the PCP (if $\alpha = 1$) [J mol⁻¹ K⁻¹];

c_{vm} – temperature-dependent mass isochoric heat capacity of gases in the engine cylinder [J kg⁻¹ K⁻¹];

$c_{CO_2}(T), c_{H_2O}(T), c_{SO_2}(T), c_{N_2}(T), c_{air}(T)$ – temperature-dependent isochoric molar heat capacities, respectively of the carbon dioxide, water, sulfur dioxide, nitrogen and air [J mol⁻¹ K⁻¹];

$c_v(T_c), c_v(T_y), c_v(T_z)$ – temperature-dependent isochoric molar heat capacity of gases, respectively at points c, y, z of the cycle [J mol⁻¹ K⁻¹];

D – cylinder diameter [m];

F_w – area of the current heat exchange surface, determined depending on the current volume of the cylinder [m²];

G – total mass of gases in the cylinder [kg];

G_{in} – mass of air passing through the intake valves/ports [kg];

G_{out} – mass of gases passing through the exhaust valves [kg];

i_{in} – specific enthalpy of air in the receiver [J kg⁻¹];

i_{out} – specific enthalpy of gases passing through the exhaust valves [J kg⁻¹];

k – coefficient characterizing the relative increase in the amount of gaseous combustion products;

$k_{hr}(T)$ – temperature dependant specific heat ratio;

L – amount of pure air in the cylinder charge (per 1 kg of fuel) [mol kg⁻¹];

L_0 – theoretically required amount of air for combustion of 1 kg of fuel [mol kg⁻¹];

L_{cr} – connecting rod length [m];

m_{fb} – mass of burned fuel [kg];

M_r – amount of residual gases in the cylinder charge (per 1 kg of fuel) [mol kg⁻¹];

n_1, n_2 – polytropic coefficient, respectively of compression and expansion strokes;

N_c, N_y, N_z – amount of gases respectively at points c, y, z of the cycle [mol];

N_i – indicated power received with proposed methodology [kW];

N'_i – indicated power received in simulated data (using complexes AVL-Boost or Blitz-PRO) [kW];

p – current average gas pressure in the cylinder [Pa];

p_a, p_b, p_c, p_z – pressure, respectively at points a, b, c, z of the cycle [Pa];

p_s – pressure of supercharged air [kPa], [Pa];

p'_b – calculated pressure at which the conditional opening of the exhaust valve occurs [Pa];

Q_f – heat obtained from fuel combustion [J];

Q_L – lower calorific value of the fuel [J kg⁻¹];

Q_w – the amount of heat received/given off by gases as a result heat exchange with walls cylinder [J];

q_{in} – mass of injected fuel received with proposed methodology [kg, g];

q'_{in} – mass of injected fuel received in simulated data (using complexes AVL-Boost or Blitz-PRO) [kg, g];

$R \approx 287$ [J kg⁻¹ K⁻¹] – gas constant;

$R_\mu = 8.314$ [J mol⁻¹ K⁻¹] – universal gas constant;

S_p – piston stroke [m];

T – current average absolute temperature of gases in the cylinder [K];

T_a, T_c, T_y, T_z – temperature respectively at points a, c, y, z of the cycle [K];

T_r – temperature of residual gases in the cylinder before the start of compression [K];

T_s – temperature of the charge air in the receiver [K];

T_w – conditional averaged temperature of the heat exchange surface over the cycle [K];

U_c, U_y, U_z – internal energy of gases, respectively at points c, y, z of the cycle [J];

V – current cylinder volume [m³];

V_a, V_b, V_c, V_z – cylinder volume, respectively at points a, b, c, z of the cycle [m³];

V_g – the volume of the cylinder in BDC [m³];

x – fraction of injected fuel that has burned to date;

α – excess air coefficient;

α_w – coefficient of heat transfer from gases to the cylinder walls [J m⁻² K⁻¹ s⁻¹];

γ_r – coefficient of residual gases;

$\Delta_b = (b_i - b'_i)/b'_i \cdot 100\%$ – relative error in calculating the specific indicated fuel consumption [%];

$\Delta_c = (A_c - A'_c)/A'_c \cdot 100\%$ – relative error in calculating the compression work [%];

$\Delta_N = (N_i - N'_i)/N'_i \cdot 100\%$ – relative error in calculating the indicated power [%];

$\Delta_\eta = (\eta_i - \eta'_i)/\eta'_i \cdot 100\%$ – relative error in calculating the indicated efficiency [%];

$\Delta_q = (q_{in} - q'_{in})/q'_{in} \cdot 100\%$ – relative error in calculating the mass of injected fuel [%];

Δm_v – mass of fuel that burns in the isochoric section cy [kg], [g];

Δm_p – mass of fuel that burns in the isobaric section yz [kg], [g];

ΔT_s – heating of scavenging air from the cylinder walls [K];

ΔM – increase in the amount of gaseous combustion products (per 1 kg of fuel) [mol kg⁻¹];

ΔQ_{cy} – the amount of heat supplied to gases in section cy due to the combustion of fuel with mass Δm_v [J];

ΔQ_{yz} – the amount of heat supplied to gases in section yz due to the combustion of fuel with mass Δm_p [J];

δQ_w – amount of heat that is lost/acquired by gases during heat exchange with the cylinder walls in elementary time dt [J];

ε – geometric compression ratio;

η_i – indicated efficiency received with proposed methodology [%];

η'_i – indicated efficiency received in simulated data (using complexes AVL-Boost or Blitz-PRO) [%];

φ – crank angle, radian.

BDC – Bottom Dead Center;

CFD – Computational Fluid Dynamics;

PCP – “Pure Combustion Products”, consisting of carbon dioxide, water, sulfur dioxide, water and nitrogen;

QD – quasi-dimensional;

TDC – Top Dead Center.

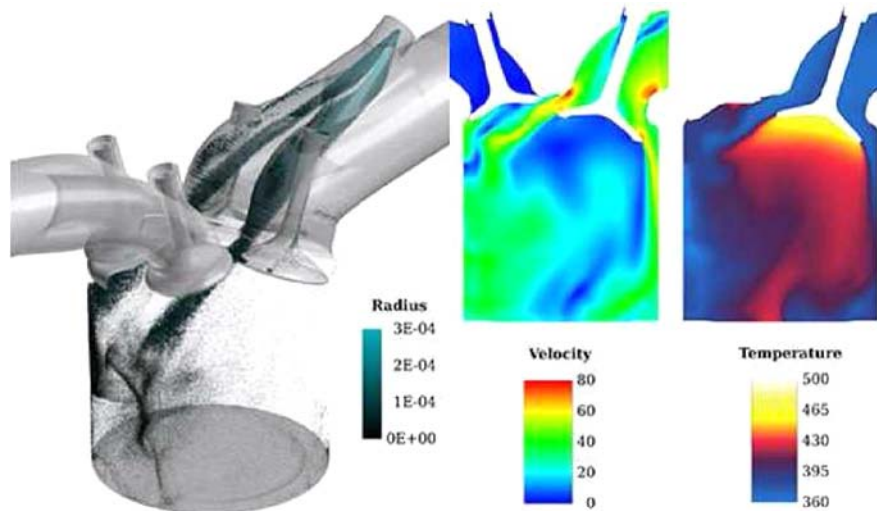


Figure 1 Visualization of the cylinder filling process in the CONVERGE CFD software package.

At present, modeling of processes in engine cylinders (including diesel engines) is mainly carried out using following main approaches: CFD, single-zone and QD. Modern CFD models allow solving complex design problems taking into account real gas movement, fuel combustion dynamics and heat transfer. Examples are presented in Figure 1.

However, along with the complication of mathematical models, the requirements for computer technology and the amount of necessary initial data required to carry out calculations increase. On the other hand, for solving practical diagnostic problems, the calculation of detailed pressure and temperature fields is excessively cumbersome.

Single-zone models (and their further development QD models) are simpler. Their use allows solving a fairly wide range of problems in modeling engine operating processes without resorting to detailed calculations of pressures and temperatures throughout the entire cylinder volume. In QD models, calculations of processes in a cylinder are carried out by dividing the volume of the cylinder into several zones (examples of two-zone models from [1] are shown in Figure 2).

In [2] and [3] it is shown, that single-zone model, despite its relative simplicity, gives good results in terms of accuracy, while being very fast.

The mass of fuel injected into the cylinder can be found by calculating the heat release rate in the cylinder.

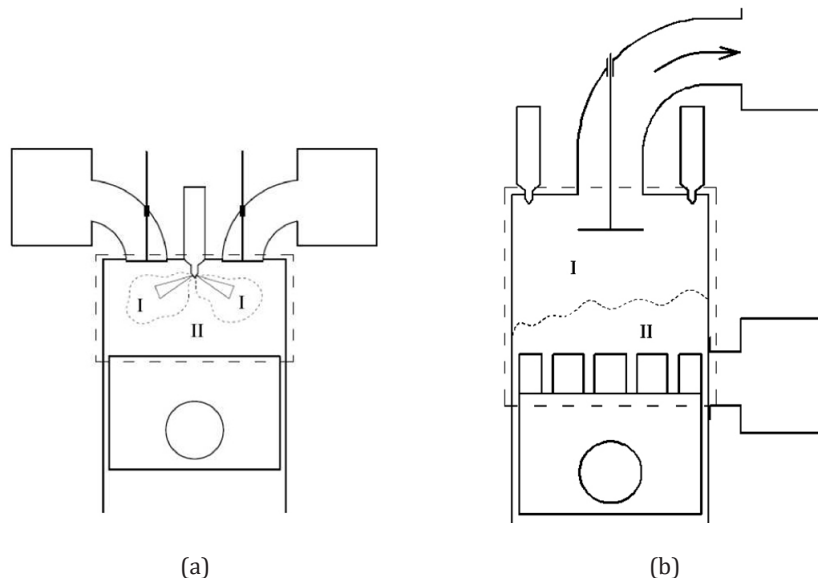


Figure 2 Application of the two-zone model to the combustion process in a diesel engine (a) and scavenging in a two-stroke engine (b)

For example, in [2, 4, 5] authors use the following formula to calculate the heat release in the cylinder:

$$\delta Q_{hr} = \frac{k_{hr}(T)}{k_{hr}(T)-1} p dV + \frac{1}{k_{hr}(T)-1} V dp + \delta Q_w. \quad (1)$$

In works [2, 6, 7] it is indicated that specific heat release k_{hr} has a great influence on the peak heat release and on the shape of the heat release curve. In [2] a fifth-order logarithmic polynomial function is used to obtain the dependence of k_{hr} on temperature.

However, calculating the mass of fuel injected into the cylinder based on calculating heat release using Equation (1) or something similar does not seem very convenient due to the lack of taking into account the change in the mass of gases in the cylinder as the fuel burns.

Another approach to calculating the mass of fuel injected into the cylinder was used in [8]. In this case, the calculated dependencies are derived from a well-known system of differential equations, consisting of the equation of the first law of thermodynamics, the equation of state (in this case, the Clapeyron equation) and the law of conservation of mass.

$$\begin{cases} \delta Q_f + \delta Q_w = c_{vm} \cdot d(G \cdot T) + \\ + p dV - i_{in} \cdot dG_{in} + i_{out} \cdot dG_{out}; \end{cases} \quad (2)$$

$$\begin{cases} d(p \cdot V) = R \cdot d(G \cdot T); \end{cases} \quad (3)$$

$$\begin{cases} dG = q_{in} \cdot dx + dG_{in} - dG_{out}. \end{cases} \quad (4)$$

The result of the solution will be the dependence of pressure on the crank angle $p(\varphi)$. During compression and after burning completion $dG = dx = \delta Q_f = 0$. In the last systems of equations, the heat obtained from the combustion of fuel can be expressed as

$$\delta Q_f = q_{in} \cdot Q_L \cdot dx. \quad (5)$$

In operating conditions, the indicator diagram is the dependence of the pressure p on the crank angle φ – $p(\varphi)$ (in other words, on the cylinder volume V), expressed in tabular form. Therefore, knowing $p(\varphi)$, it is possible to find the law of change in the mass G of gases in the cylinder depending on the crank angle φ . Thus, the calculation of the mass of injected fuel can be considered as a problem inverse to the calculation of the pressure in the cylinder. To do this, it is necessary to integrate a system of two differential equations, which are derived from the original system of Equations (2) – (4):

$$\begin{cases} dG = \frac{(2c_{vm} + a_0 \cdot T^2 - a_2) \cdot d(p \cdot V) - R \cdot \delta Q_w + p \cdot R \cdot dV}{R(Q_L + T(c_{vm} + a_0 \cdot T^2 - a_2))}, \end{cases} \quad (6)$$

$$\begin{cases} dT = T \left(\frac{d(p \cdot V)}{p \cdot V} - \frac{dG}{G} \right), \end{cases} \quad (7)$$

where

$$c_{vm} = a_0 \cdot T^2 + a_1 \cdot T + a_2. \quad (8)$$

In [8] it is shown that solving system of Equations (6) – (7) gives results that are generally acceptable in accuracy (error less 5%). However, when finding the mass of fuel injected into the cylinder using the latter system of equations, difficulties arise, listed below.

Integration of the above system of differential equations requires the presence of known initial conditions, the search for which requires additional efforts. It should also be noted that the system of Equations (6) – (7) as well as Equations (2) – (4), includes the component δQ_w – the amount of heat that is lost/acquired by gases during heat exchange with the cylinder walls during time $d\tau$:

$$\delta Q_w = \alpha_w \cdot F_w (T_w - T) d\tau. \quad (9)$$

In case of using simple single-zone and QD models α_w is determined by the selected empirical dependence proposed by Woschni, Hohenberg, Annand or others. When using these empirical dependencies, it should be remembered that it is not known in advance which of them gives the result closest to the actual one.

In addition, to calculate the value of T_w , it is necessary to have data on the conditions of heat exchange of the cylinder walls, piston and cylinder cover with cooling water. It means to know the heat transfer coefficient from the crank parts to the water; the values of the thermal conductivity coefficients of the materials, the thickness of the parts, and the temperature and flow rate of the cooling water. In the case of using QD models, the volume of required initial data will be additionally increased, since in this case it is necessary to calculate the interaction between zones. If collecting such a large array of data is difficult, then the intuitive choice of initial data for performing calculations can lead to serious errors.

2 Material and Method

2.1 Assumptions Accepted

In connection with the above-mentioned shortcomings, a method for calculating the mass of injected fuel was developed, for which unknown data related to heat exchange are not required. In this case, the actual working process was represented by a calculation cycle with isochoric and isobaric sections of fuel combustion (the Sabaté cycle) (see Figure 3). The unknown mass of injected fuel is found based on the parameters of the calculation cycle.

As a result mass of burned fuel m_{fb} can be considered as sum of two values Δm_v and Δm_p

$$m_{fb} = \Delta m_v + \Delta m_p. \quad (10)$$

In the most general case, the mass of injected fuel q_{in} and the fuel that burns in the cylinder m_{fb} differ slightly due to the presence of losses during fuel combustion.

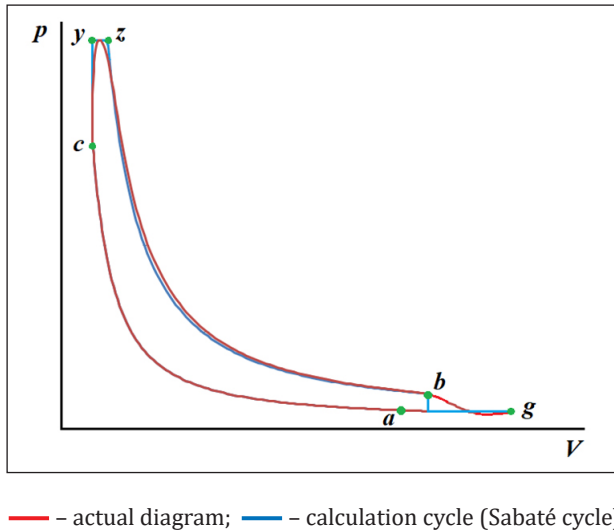


Figure 3 Dependence of pressure on engine cylinder volume

When developing the calculation method, it was assumed that the same amount of heat is supplied to the actual and calculated cycles, which is released during combustion of the same fuel supply for both cycles. Besides, the area of expansion work A_{rw} of the idealized and actual cycles is the same (A_{rw} is determined from the actual diagram taken from the engine). Also equal for the idealized and actual cycles are the compression pressure p_c and the maximum pressure p_z . In addition, the actual value of pressure p_b at the moment of opening of the exhaust valves is used in the calculations (point b theoretical cycle in which the volume is equal to V_b , m³). Other parameters were determined as a result of solving a system of algebraic equations. In this case, the following assumptions were made.

- 1) At the moment of the start of compression (at point a of the calculation cycle), the temperature of the air charge is estimated using the heat balance equation:

$$T_a = (T_s + \gamma_r \cdot T_r + \Delta T_s) / (1 + \gamma_r), \quad (11)$$

where

$$\gamma_r = M_r / L. \quad (12)$$

- 2) Compression of the air charge occurs with a constant polytropic coefficient n_1 .
- 3) The fuel burns equally evenly in the isochoric and isobaric sections.
- 4) Isochoric combustion of fuel occurs at the TDC, i.e. it is assumed that fuel combustion begins at TDC at the end of compression (i.e. at point c).
- 5) By the end of the isobaric combustion section (upon reaching point z calculation cycle) the portion of fuel injected into the cylinder burns completely, i.e. there is no losses during burning of fuel and

$$q_{in} = m_{fb} = \Delta m_v + \Delta m_p. \quad (13)$$

- 6) In the section of isobaric combustion of fuel (between points y and z calculation cycle) there is no heat exchange between the cylinder walls and gases.
- 7) Expansion of combustion products occurs with a constant polytropic coefficient n_2 .
- 8) It is believed that at the elementary level, fuel consists only of carbon, hydrogen, oxygen and sulfur. The mass fractions of these elements, depending on the composition of the original oil and the technology of its processing, are usually within the following limits: carbon $C = 0.84 \dots 0.87$; hydrogen $H = 0.1 \dots 0.14$; oxygen $O = 0.001 \dots 0.01$; sulfur $S = 0.0001 \dots 0.005$. The lower calorific value Q_L of the fuel can be taken from its passport data or, for example, determined using the Dulong's or (in our case) Mendelev's formula.

2.2 Description of the Calculation Procedure

2.2.1 Initial Data

Calculation of the mass of injected fuel must begin with the following calculations:

- 1) lower calorific value Q_L of the fuel (Equation A1);
- 2) theoretically required amount L_0 of air for combustion of 1 kg of fuel (Equation A2);
- 3) increase in the amount of gaseous combustion products ΔM (per 1 kg of fuel) (Equation A3);
- 4) coefficient k characterizing the relative increase in the amount of gaseous combustion products (Equation A4);
- 5) polytropic compression index n_1 (Equation A5);
- 6) temperature T_a (Equation 11);
- 7) gas temperature T_c at the end of compression i. e. at point c of the cycle (Equation A6);
- 8) quantity of gases N_a (N_c) at a point a (c) (Equation A7);
- 9) write down the formula for the isochoric molar heat capacity of PCP (Equation A12), based on Equations (A8) – (A10) and a given elemental composition of the fuel.

2.2.2 Creating Calculation Formulas

In order to find the volume V_z at point z and the polytropic expansion index n_2 we use the condition of equality of the expansion work of the real and calculated cycle. In this case we obtain a system of two equations with two unknowns:

$$\begin{cases} A_{rw} = p_z \cdot (V_z - V_c) + (p_z \cdot V_z - p_b \cdot V_b) / (n_2 - 1); & (14) \\ p_b \cdot V_b^{n_2} = p_z \cdot V_z^{n_2}. & (15) \end{cases}$$

Now, taking into account the assumptions that were stated above, we will write down the necessary equations for isochoric and isobaric sections of the calculation cycle in order to find the rest of unknowns.

For the section cy we write the equation of the first law of thermodynamics for the transition of the working gases from point c to the point y , as well as the Clapeyron equation for these same points. In this section, part of injected fuel is burned, so at the point y the mass of the gases will increase by the value Δm_v – by that part of the fuel that burns in the isochoric section. The equation of the first law of thermodynamics:

$$\Delta Q_{cy} = U_y - U_c; \quad (16)$$

$$\Delta m_v \cdot Q_L = N_y \cdot c_v(T_y) \cdot T_y - N_c \cdot c_v(T_c) \cdot T_c. \quad (17)$$

The values of $c_v(T_c)$ and $c_v(T_y)$ are calculated using Equations (A11) – (A14) for points c and y respectively.

During this part of the cycle the fraction of injected fuel that is burned when the process reaches point y is:

$$x_y = \Delta m_v / (\Delta m_v + \Delta m_p). \quad (18)$$

The expression for the excess air coefficient α is:

$$\alpha = L/L_0 = N_c / (L_0(1 + \gamma_r)(\Delta m_v + \Delta m_p)) \quad (19)$$

Quantity of gases at a point y :

$$N_y = N_c + \Delta m_v \cdot \Delta M. \quad (20)$$

Clapeyron equation for point c :

$$p_c \cdot V_c = N_c \cdot R_\mu \cdot T_c. \quad (21)$$

Clapeyron equation for point y :

$$p_y \cdot V_c = N_y \cdot R_\mu \cdot T_y. \quad (22)$$

Subtract Equation (22) from Equation (21). We obtain:

$$V_c(p_z - p_c) = R_\mu(N_y \cdot T_y - N_c \cdot T_c). \quad (23)$$

For the yz section we will do the same: we will write down the equation of the first law of thermodynamics when the gases pass from point y to point z , and also subtract the Clapeyron equation for point z from the Clapeyron equation for point y . In this section, the combustion of the injected fuel occurs, therefore, at point z the mass of the gases additionally increase by the value Δm_p . The equation of the first law of thermodynamics taking into account assumption (6) will be written as:

$$\Delta Q_{yz} = U_z - U_y + p_z(V_z - V_c); \quad (24)$$

$$\begin{aligned} \Delta m_p \cdot Q_L &= N_z \cdot c_v(T_z) \cdot T_z - \\ &- N_y \cdot c_v(T_y) \cdot T_y + p_z(V_z - V_c). \end{aligned} \quad (25)$$

At this point, the value $c_v(T_z)$ is also determined depending on the temperature T_z using Equations (A11), (A12), (A15).

The fraction of injected fuel that is burned when the process reaches point z (see assumption 5):

$$x_z = 1. \quad (26)$$

Quantity of gases at a point z :

$$N_z = N_c + (\Delta m_v + \Delta m_p) \cdot \Delta M. \quad (27)$$

Clapeyron equation for point z :

$$p_z \cdot V_z = N_z \cdot R_\mu \cdot T_z. \quad (28)$$

Let us subtract Equation (28) from Equation (22):

$$p_z \cdot (V_z - V_c) = R_\mu(N_z \cdot T_z - N_y \cdot T_y). \quad (29)$$

As a result, we obtain a system of four equations:

$$\left\{ \begin{aligned} \Delta m_v \cdot Q_L &= N_y \cdot c_v(T_y) \cdot T_y - N_c \cdot c_v(T_c) \cdot T_c; \end{aligned} \right. \quad (30)$$

$$\left\{ \begin{aligned} V_c(p_z - p_c) &= R_\mu(N_y \cdot T_y - N_c \cdot T_c); \end{aligned} \right. \quad (31)$$

$$\left\{ \begin{aligned} \Delta m_p \cdot Q_L &= N_z \cdot c_v(T_z) \cdot T_z - N_y \cdot c_v(T_y) \cdot T_y + \\ &+ p_z(V_z - V_c); \end{aligned} \right. \quad (32)$$

$$\left\{ \begin{aligned} p_z \cdot (V_z - V_c) &= R_\mu(N_z \cdot T_z - N_y \cdot T_y). \end{aligned} \right. \quad (33)$$

In this system unknown are T_y , T_z , Δm_v , Δm_p . The required mass of fuel injected into the cylinder (see assumption 5 – no fuel underburning)

$$q_{in} = \Delta m_v + \Delta m_p. \quad (34)$$

Finally, we have 2 systems: Equations (14) – (15) and (30) – (33). To solve them, it is necessary to specify the values of T_r , γ_r , ΔT_s . In [9, 10] it is recommended taking the following limits for these values (depending on the engine type): $\gamma_r = 0.01...0.08$; $T_r = 700...800$ K; $\Delta T_s = 5...30$ K.

Both systems are solved numerically. To do this, the authors used ready-made solution “fsolve” included in the “MatLab” package. The procedure for solving a system of nonlinear algebraic equations using the built-in function “fsolve” usually involves using the default solution settings, as was done when solving this problem.

In function has been used the Levenberg–Marquardt method as the solution algorithm, which assumes that the number of unknowns is equal to the number of equations used. The value of the “Function Tolerance” parameter, which represents the value of the last change in the sum of squares of the function values, also did not change compared to the default value (10^{-6}).

3 Results and Discussion

3.1 Results of Initial Calculations

In order to conduct preliminary testing of proposed method for calculating the mass of fuel injected into the cylinder, the data obtained during the simulation of diesel engine operating processes has been used. For this purpose, the “AVL – Boost” [11] and “Blitz – PRO” [12] packages were taken (the latter is open for free use, both links are given in references).

The processes for 4 engines were simulated: 2 two-stroke (“AVL – Boost” package) and 2 four-stroke (“Blitz – PRO” package).

The simulation results were obtained as a series of crankshaft rotation angle values and the corresponding cylinder pressure values. The basic data for the calculation that can be obtained under real operating conditions were also used: the pressure and temperature of the boost air p_s and T_s , as well as the elemental composition of the fuel, i.e. the content of carbon, hydrogen, oxygen

and sulfur in the fuel. It was assumed that due to purification, the water content in the fuel can be neglected. In fuel, combustion whom calculated during modeling, lower heating value composed $Q_L = 42800$ [kJ/kg].

Preliminary calculations showed, that the results contained an unacceptably high error in calculating the mass of fuel injected into the cylinder. For the MAN 6K90 MC-C engine, the calculated mass of injected fuel was 141.226 g (error $\Delta_q = +9\%$), MAN 6G70 ME-C – 81.736 g ($\Delta_q = +9.6\%$), Wartsila 6L20 – 6.103 g ($\Delta_q = +412.6\%$), MaK 8M43C – 12.598 g ($\Delta_q = +12.6\%$). Also, unacceptably large deviations (up to 10%) were obtained when calculating the indicated efficiency and specific indicated fuel consumption.

It is assumed that the reason for this may be the following: in the proposed calculation method, it is considered that the pressure drops sharply at the moment of opening of the exhaust valves (see Figure 3 – point *b* of calculation cycle). Consequently, the piston stroke, during which work is performed, is limited by the angle that corresponds to the opening of the exhaust valve.

Table 1 Initial engine data.

Parameter	Engine brand			
	MAN 6K90 MC-C	MAN 6G70 ME-C	Wartsila 6L20	MaK 8M43C
Engine stroke	2	2	4	4
D , m	0.9	0.7	0.2	0.43
S_p , m	2.3	3.256	0.28	0.61
L_{cr} , m	3.165	3.256	0.3655	1.22
Rotation speed, min^{-1}	104	70	1200	500
ε	19	22.4	15	18
Start of compression (after BDC)	70°	58°	25°	15°
Start of exhaust (after TDC)	130°	130°	100°	140°

Table 2 Diesel engine operation process data obtained from simulation.

Parameter	Engine brand & model used			
	MAN 6K90 MC-C (AVL – Boost)	MAN 6G70 ME-C (AVL – Boost)	Wartsila 6L20 (Blitz – PRO)	MaK 8M43C (Blitz – PRO)
p_s , kPa	300	188	414	299
T_s , K	318	316	317	312
p_z , kPa	12826.20	15607.09	20557.31	18135.57
p_c , kPa	11406.95	12087.87	17427.95	16124.38
p_b , kPa	850.54	511.29	2009.51	1156.62
A_{rw} , kJ	4391.1	2943.5	43.1	383.9
q_{in}' , g	129.6	74.6	1.1906	11.1712
N_i' , kW	4636.6	1844.1	243.6	957.4
A_c' , kJ	1716.1	1362.9	19.2	153.8
b_i' , g/(kW·h)	174.417	169.908	175.972	175.024
η_i' , %	48.225	49.505	47.8	48.057

Table 3 Initial data for calculation according to the proposed method.

Parameter	Engine brand			
	MAN 6K90 MC-C	MAN 6G70 ME-C	Wartsila 6L20	MaK 8M43C
γ_r	0	0	0	0
ΔT_{gr} , K	20	14	53	31
T_{gr} , K	338	330	370	343
n_1	1.3727	1.3674	1.3721	1.3745
T_c , K	917	981	1003	1008
$N_a(N_c)$, mol	121.656	86.845	1.314	10.032

However, since in reality the gases cannot leave the cylinder immediately after the opening of the exhaust valve, as a result, they still perform some work **after** the valve opening. Thus, in the proposed method, the necessary work is performed during a shorter piston stroke, which leads to a significant error in calculating the mass of injected fuel towards an increase.

3.2 Modification of the Calculation Procedure and Updating the Results

To reduce errors, calculation methodology was little modified. On the first stage polytropic expansion index n_2 and the volume V_z were determined as before, from the system of Equations (14) – (15).

Then, based on the found values of n_2 and V_z , a certain calculated pressure p_b' was determined at which the conditional opening of the exhaust valve occurs. It was assumed that this opening occurs at the BDC (see Figure 3 – point g of calculation cycle):

$$p_b' = p_z (V_z/V_g)^{n_2}, \quad (35)$$

where V_g is the volume of the cylinder in BDC

$$V_g = V_c + 0.25 \cdot \pi \cdot D^2 \cdot S_p. \quad (36)$$

After this, the system of Equations (14) – (15) was solved again, but this time the value p_b' was substituted into it instead of p_b , and V_g instead of V_b :

$$\begin{cases} A_{rw} = p_z \cdot (V_z - V_c) + (p_z \cdot V_z - p_b' \cdot V_g)/(n_2 - 1); \\ p_b' \cdot V_g^{n_2} = p_z \cdot V_z^{n_2}. \end{cases} \quad (37) \quad (38)$$

The updated values of n_2 and V_z found from the last system of equations were substituted into the Equations (30) – (33) from which the unknowns Δm_v and Δm_p were found. The results of the adjusted calculation are given in the Table 4.

Table 4 Results adjusted calculation according to the proposed methodology.

Parameter	Engine brand			
	MAN 6K90 MC-C	MAN 6G70 ME-C	Wartsila 6L20	MaK 8M43C
Preliminary volume V_z , m ³	0.162264	0.090333	0.001199	0.01022
Preliminary value of n_2	1.2751	1.3288	1.3549	1.2941
p_b' , kPa	725.037	445.950	1249.9	1029.662
Recalculated volume V_z , m ³	0.152926	0.087449	0.001006	0.009807
Recalculated index n_2	1.2424	1.3129	1.2513	1.2704
Δm_v , g	10.221	18.891	0.179	0.948
Δm_p , g	114.706	56.833	0.971	10.611
q_{in} , g ($\Delta_{q'}$, %)	124.927 (-3.6%)	75.7 (+1.5%)	1.150 (-3.4%)	11.559 (+3.5%)
N_p , kW ($\Delta_{N'}$, %)	4702.4 (+1.4%)	1943.3 (+5.4%)	244.6 (+0.4%)	1007.6 (+5.2%)
A_c , kJ ($\Delta_{c'}$, %)	1678.7 (-2.3%)	1278.3 (-6.2%)	18.7 (-2.6%)	148.4 (-3.5%)
b_p , g/(kW·h) ($\Delta_{b'}$, %)	165.776 (-4.9%)	163.659 (-3.7%)	169.217 (-3.8%)	172.063 (-1.7%)
η_p , % ($\Delta_{\eta'}$, %)	50.738 (+5.2%)	51.395 (+3.8%)	49.706 (+4.0%)	48.884 (+1.7%)

Below in one axis are given the dependence of pressure on volume, obtained using models and the proposed method.

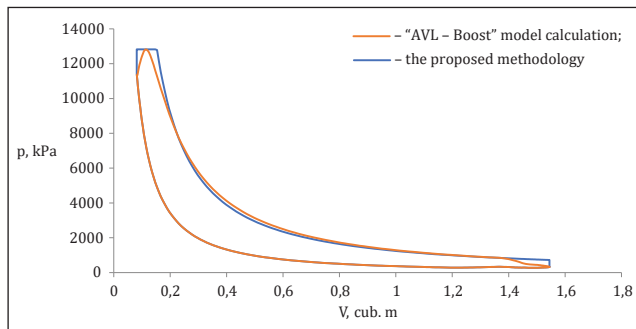


Figure 4 Dependence of pressure on cylinder volume of MAN 6K90MC-C engine

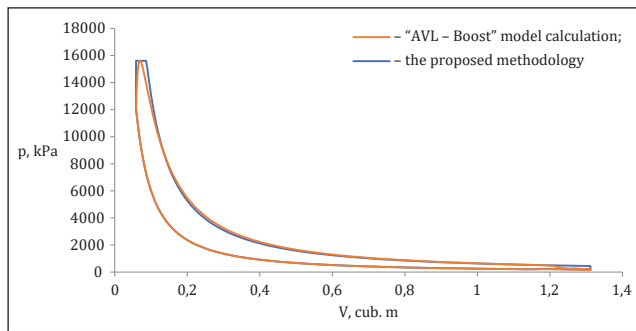


Figure 5 Dependence of pressure on cylinder volume of MAN 6G70ME-C engine

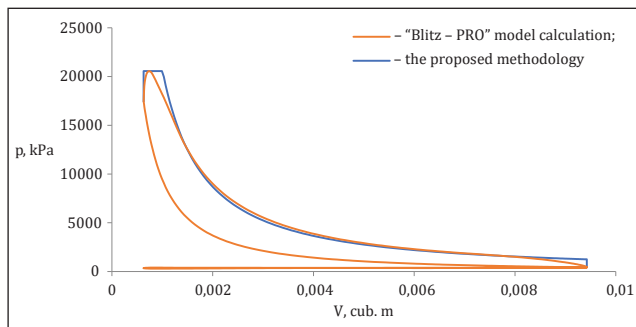


Figure 6 Dependence of pressure on cylinder volume of Wartsila 6L20 engine

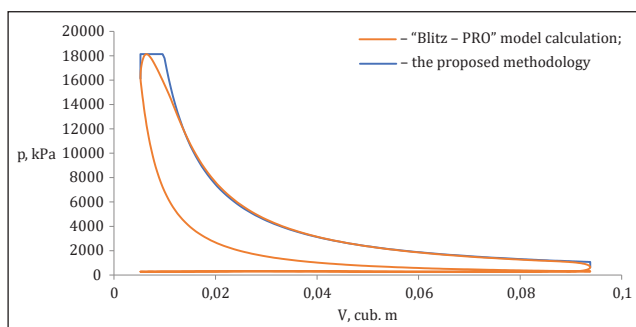


Figure 7 Dependence of pressure on cylinder volume of the MaK 8M43C engine

From the presented diagrams it is clear that the $p(V)$ dependence constructed by the proposed method fairly accurately reflects the original curve. The slight discrepancy between the expansion lines of the calculated and initial cycles is explained by the fact that processes in the real cycle cannot occur with a constant polytropic coefficient due to a combination of various factors.

4 Conclusion

The data provided show that the accuracy of calculating the mass of fuel injected into the cylinder increases significantly with correction. The fact that the error in this case is less than 5% (which is sufficient for many engineering calculations) allows us to hope that after improvement and experimental verification, the proposed calculation method can be used in practical diagnostic tasks.

The advantages of the proposed method include: 1) the initial data for its application is quite easy to collect, since there is no need to separately calculate heat exchange; 2) the calculation of unknown values occurs significantly faster compared to methods that require step-by-step integration of a system of differential equations (as, for example, for single-zone or QD methods). This is potentially useful in diagnosing internal combustion engines under real operating conditions. For the same reasons, the proposed method can be applied in diagnosing the operating process of marine diesel engines using portable diagnostic equipment (e.g., those manufactured by IMES GmbH, Baewert, Lemag Premet, etc.).

It is also worth noting the following. When comparing the idealized and original cycles, it was checked: mass of injected fuel, power, specific indicated fuel consumption, indicated efficiency and compression work. The acceptable error achieved with the proposed method allows us to assert, that the idealized cycle adequately reflects the real work process. It is known that the value of the volume V_z correlates with actual duration of fuel combustion (with an increase in the duration of combustion, the V_z also increases, with a decrease, it decreases). This makes it possible to estimate the duration of the actual combustion of fuel, which expands the range of diagnostic conclusions.

Besides, the authors believe that the proposed method for calculating the mass of injected fuel will be useful for an integral assessment of the technical condition of the diesel fuel equipment. To do this, it is necessary to compare the value of the geometric fuel supply (based on the fuel pump rack index) and the calculated mass of fuel that burned in the cylinder.

However, it is additionally necessary to check how the accuracy of the calculation changes over the entire operating range of engine loads. For this reason, it will be necessary to clarify how the coefficient of residual gases γ_r , the temperature of residual gases T_r and heating of the scavenging air from the cylinder walls ΔT_s af-

fect the accuracy of the calculation, since at partial loads the quality of scavenging of the cylinder changes significantly. It should also be noted, that the calculations given were made for diesel engines with a simple combustion chamber shape. Therefore, the question of the applicability of the given method for engines with pre-chamber mixture formation is also relevant.

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Appendix A

$$Q_L = 33.9 \cdot C + 102.974 \cdot H + 10.9(S - O). \quad (A1)$$

$$L_0 = \frac{1}{0.21} \left(\frac{C}{0.012} + \frac{H}{0.004} + \frac{S}{0.032} - \frac{O}{0.032} \right). \quad (A2)$$

$$\Delta M = (8 \cdot H + O)/0.032. \quad (A3)$$

$$k = \Delta M / L_0. \quad (A4)$$

$$n_1 = \ln \left(\frac{p_c}{p_a} \right) / \ln \left(\frac{V_a}{V_c} \right). \quad (A5)$$

$$T_c = T_a (V_a / V_c)^{n_1 - 1}. \quad (A6)$$

$$N_c = N_a = \frac{p_c \cdot V_c}{R_\mu \cdot T_c} = \frac{p_a \cdot V_a}{R_\mu \cdot T_a}. \quad (A7)$$

Temperature dependant isochoric molar heat capacities of carbon dioxide, water vapor, nitrogen, and pure air (proposed in [1]; formulas are available at <http://blitz-pro.zeddmalam.com/extra/Tutorial/Help.pdf> (p. 11)):

$$c_{CO_2}(T) = -3.5902 \cdot 10^{-19} \cdot T^6 + 4.11717 \cdot 10^{-15} \cdot T^5 - 1.9643 \cdot 10^{-11} \cdot T^4 + 5.14242 \cdot 10^{-8} \cdot T^3 - 8.2174 \cdot 10^{-5} \cdot T^2 + 0.082126 \cdot T + 10.333; \quad (A8)$$

$$c_{H_2O}(T) = -4.897476 \cdot 10^{-20} \cdot T^6 + 3.447949 \cdot 10^{-17} \cdot T^5 + 2.656956 \cdot 10^{-12} \cdot T^4 - 1.366663 \cdot 10^{-8} \cdot T^3 + 2.443866 \cdot 10^{-5} \cdot T^2 - 0.00550647 \cdot T + 25.104; \quad (A9)$$

$$c_{N_2}(T) = 4.062159 \cdot 10^{-19} \cdot T^6 - 4.487468 \cdot 10^{-15} \cdot T^5 + 1.993208 \cdot 10^{-11} \cdot T^4 - 4.470124 \cdot 10^{-8} \cdot T^3 + 5.053716 \cdot 10^{-5} \cdot T^2 - 0.0208783 \cdot T + 23.56; \quad (A10)$$

$$c_{air}(T) = 5.52411 \cdot 10^{-19} \cdot T^6 - 5.726799 \cdot 10^{-15} \cdot T^5 + 2.375597 \cdot 10^{-11} \cdot T^4 - 4.951694 \cdot 10^{-8} \cdot T^3 + 5.1761 \cdot 10^{-5} \cdot T^2 - 0.019099 \cdot T + 23.005. \quad (A11)$$

$$c_{cp}(T) = \frac{1}{L_0 \cdot (1+k)} \left(\frac{C}{0.012} \cdot c_{CO_2}(T) + \frac{H}{0.002} \cdot c_{H_2O}(T) + \frac{S}{0.032} \cdot c_{SO_2}(T) + 0.79 L_0 \cdot c_{N_2}(T) \right); \quad (A12)$$

$$c_v(T_c) = \frac{c_{air}(T_c) + \gamma_r \cdot c_{cp}(T_c)}{1 + \gamma_r}. \quad (A13)$$

$$c_v(T_y) = \frac{(\alpha - x_y) \cdot c_{air}(T_y) + (\alpha \cdot \gamma_r + x_y(1+k)) \cdot c_{cp}(T_y)}{k \cdot x_y + \alpha \cdot (1 + \gamma_r)}. \quad (A14)$$

$$c_v(T_z) = \frac{(\alpha - 1) \cdot c_{air}(T_z) + (\alpha \cdot \gamma_r + 1 + k) \cdot c_{cp}(T_z)}{k + \alpha(1 + \gamma_r)}. \quad (A15)$$