RGNZ/MGPB

A new technique based on Ant Colony Optimization for designing mining pushbacks in the presence of geological uncertainty

Rudarsko-geološko-naftni zbornik
(The Mining-Geology-Petroleum Engineering Bulletin)
UDC: 622
DOI: 10.17794/rgn.2023.1.10

Preliminary communication

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Abstract
An essential task in the open-pit mine optimizing process is determining the extraction time of material located in the ultimate pit, considering some operational and economic constraints. The proper design of pushbacks has a significant impact on the optimum production planning. On the other hand, some uncertainty sources such as in-situ grade cause both deviations from production and financial goals. This paper presents an extension of a multi-stage formulation for risk-based pushback designing that utilizes the ant colony optimization (ACO) algorithm to solve it. For more detailed studies, two different strategies were developed according to statistical and probabilistic issues. The data of Songun copper mine located in NW Iran was used to evaluate the ability of the proposed approach in controlling the risk of deviation from production targets and increasing the project value. The results indicated the effectiveness of the proposed approach in pushback designing based on geological uncertainty. Examining different strategies showed that the technique based on multiple probability produces better solutions.

Keywords:
open pit mine; pushback design; stochastic optimization; geological uncertainty; ant colony optimization

1. Introduction

Long-term open pit mine production scheduling (OPMPS) aims to identify the extraction time and destination of extracted material. Due to a large number of blocks inside the ultimate pit limit, the pit divides into a series of sub-pits termed “pushbacks”, “cutbacks” or “phases” such that the net present value (NPV) is maximized (**Hustrulid et al., 2006**). Pushbacks greatly affect the meeting of production goals, postponing waste production, providing a minimum mining width, and guaranteeing safe pit slopes. A popular procedure to design pushbacks starts with the generation of some nested pits, which is done by modifying the block economic value and repetitions of the ultimate pit limit algorithms, such as Lerchs-Grossman graph methodology. The resulting nested pits are then used as a good guide to choose a possible order of pushbacks (see **Figure 1**). This approach suffers from production of pits sizes and the probability of being disconnected by pushbacks (**Meagher et al., 2014**). **Nancel-Penard et al. (2021)** propose an integer linear programming model (ILP) to maximize profit while respecting geospatial and design constraints, such as minimum width at the bottom of the mine and between successive pushbacks. **Yarmuch et al. (2021)** proposed an integer programming (IP) model to generate mineable pushbacks using a closeness factor that compares two designs. Another weakness of all conventional approaches is ignoring any kind of uncertainty, which leads to create unrealistic pushbacks in terms of operational requirements. The major uncertainties in mining projects are classified as geological, technical, and economic uncertainties (**Dimitrakopoulos, 1998**). According to recent studies, geological uncertainty is considered as the main source of uncertainty due to the 70% deviation from production and financial goals. (**Vallee, 2000**). A 10~25% NPV increment has been reported due to using stochastic optimizers (**Godoy et al., 2004; Leite et al., 2009**). Stochastic optimization methods based on the utilization of some ore body simulations called “Realization” have attracted much more attention over the past decade to overcome the inherent shortcomings of the conventional approaches. Originally, risk analysis in mine planning based on realizations was discussed by **Ravenscroft (1992)**. **Gholamnejad et al. (2007)** introduced a new method to integrate the grade uncertainty in the pushback design process based on the conventional approach with a little difference in the concept of parametric analysis. Unlike the regular decrement of the block value in the conventional nested pit design, the economic value of each block would be

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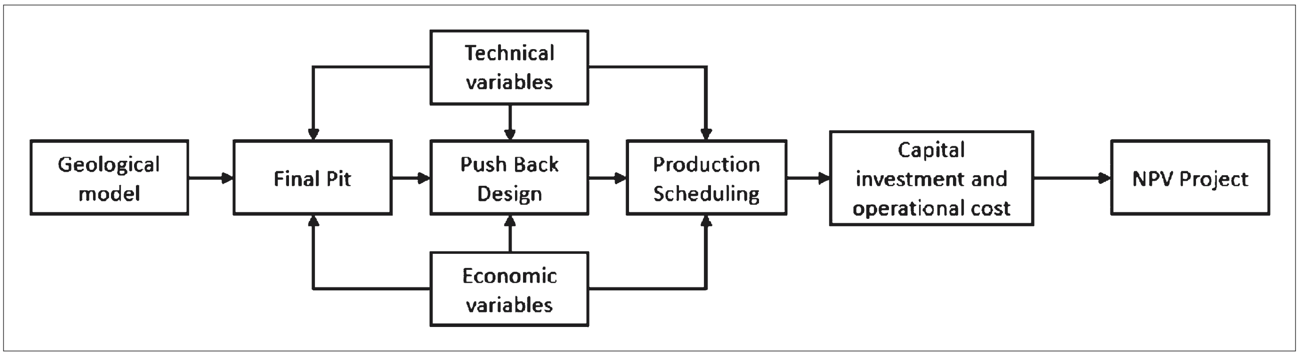

Figure 1: Long-term mine planning process and including pushback design

reduced in each step proportional to the block uncertainty in the proposed algorithm. This means that if there are two blocks with the same grade and different degrees of uncertainty, the economic value of the highly uncertain block will decrease more than the others. **Consuegra et al. (2010)** developed a stochastic integer programming (SIP) model to incorporate the grade uncertainty into the pushback design process. The approach involves generating nested pits, combining them to a certain number of pushbacks based on the obtained NPVs, and finally, generating a long-term production planning based on a different number of pushbacks using the SIP formulation (**Dimitrakopoulos et al., 2008; Ramazan et al., 2012**). **Meagher et al. (2009)** developed a parametric maximum-flow/minimum-cut approach to quantify the value of management flexibility in the production planning process in the presence of geological and economic uncertainties. A parametric minimum cut algorithm is then applied to design the pushback that meets the production goals in the early stages by exploring blocks with a high probability of being ore, not only high value. Another stochastic pushback design approach based on parametric maximum flow algorithm was developed by **Asad et al. (2013)** to control the sizes of pushbacks. **Goodfellow et al. (2013)** developed a stochastic pushback design algorithm for complex multi-process open-pit mines under geological uncertainty that perturbs an initial design using the simulated annealing algorithm. **Bai et al. (2018)** developed a new algorithm to generate pushbacks that takes to account complex geometric requirements such as slopes, phase bench and bottom width, smoothness, and continuity. It is shown that almost all frameworks suffer from explicit integration of grade uncertainty to create an optimal single solution, especially in large orebodies. The current study introduces a stochastic model to design pushbacks considering the geological uncertainty which is solved by the Ant Colony Optimization (ACO) algorithm. This method is able to optimize large deposits with multiple processing destinations and zones with variable slope angles. In this research, first, the modeling procedure with two different strategies is outlined, and then the difference between obtained and deterministic solutions is discussed.

2. Formulation of the general problem

Like OPMPS, the pushback design could also be efficiently modeled as an integer programming (IP) formulation to maximize the NPV and subject to some technical constraints, i.e. Equations 1 to 9.

$$Maximize\ \ Z = \sum\nolimits_{n=1}^{N} \sum\nolimits_{p=1}^{P} V_n x_{n,p} \tag{1}$$

Subject to:

$$x_{n,p} \epsilon \left(0,1\right), for\ n = 1\ to\ N, p = 1\ to\ P \tag{2}$$

Slope constraint: The extraction of each block depends on the earlier extraction of its predecessors.

$$x_{n,p} - \sum\nolimits_{\tau=1}^{p} x_{m,\tau} \leq 0, for\ m = 1\ to\ N, p = 1\ to\ P \tag{3}$$

Where (the predecessor blocks of given block)

Reserve constraint: A block extracts only once.

$$\sum\nolimits_{p=1}^{P} x_{n,p} \leq 1, for\ n = 1\ to\ N \tag{4}$$

Processing capacity: The total ore processed during each pushback should be within the allowable range.

$$\sum\nolimits_{n=1}^{N} o_n \times w_n \times x_{n,p} \geq \underline{O}, for\ p = 1\ to\ P \tag{5}$$

$$\sum\nolimits_{n=1}^{N} o_n \times w_n \times x_{n,p} \leq \bar{O}, for\ p = 1\ to\ P \tag{6}$$

Mining capacity: The total material mined during each pushback should be within the allowable range.

$$\sum\nolimits_{n=1}^{N} w_n \times x_{n,p} \geq \underline{M}, for\ p = 1\ to\ P \tag{7}$$

$$\sum\nolimits_{n=1}^{N} w_n \times x_{n,p} \geq \bar{M}, for\ p = 1\ to\ P \tag{8}$$

Average grade constraint: Controls the minimum allowable average grade of material mined during each pushback.

$$\frac{1}{N_p} \sum\nolimits_{n=1}^{N} g_n x_{n,p} \geq \underline{G}, for\ p = 1\ to\ P \tag{9}$$

Where:

- N is the total number of blocks,
- n is the block number,
- P is the total number of pushbacks,

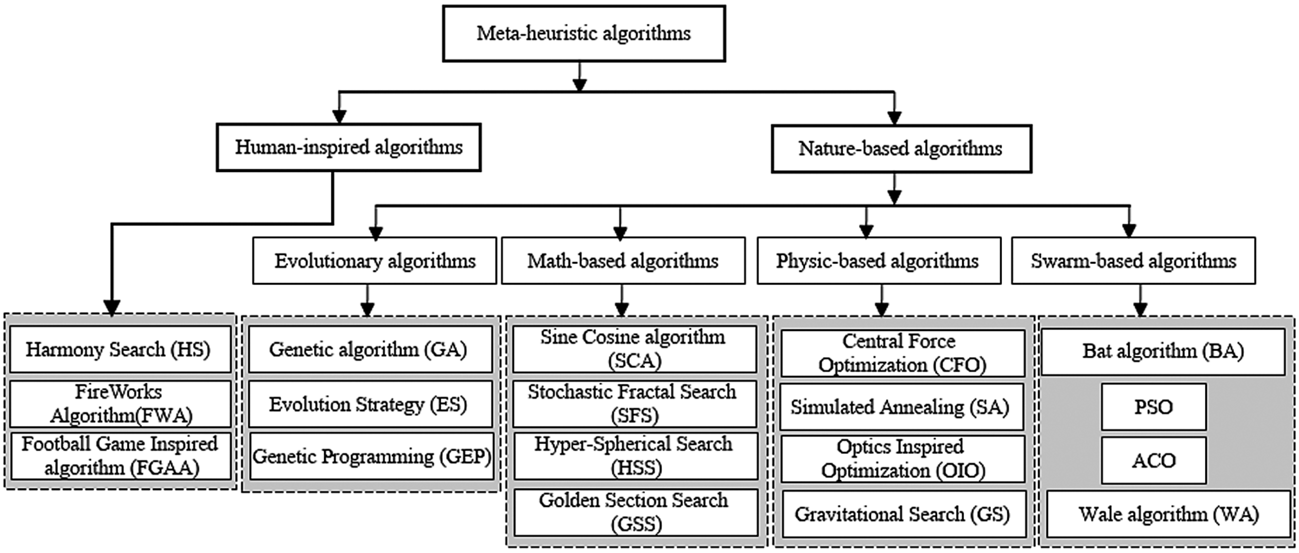

Figure 2: The classification of meta-heuristic optimization algorithms

- p is the pushback number,
- V_n is discounted value of the block,
- $x_{n,p}$ is a binary variable that represents the extraction of the n^{th} block in p^{th} pushback,

$$x_{n,p} = \begin{cases} 1 \\ 0 \end{cases} \textit{if extraction is done in pushback p otherwise}$$

- o_n indicates the material type of block,

$$o_n = \begin{cases} 1 \\ 0 \end{cases} \textit{if } n^{th} \text{ block is an ore block } \textit{otherwise}$$

- w_n shows the weight of the block,
- $\overline{O}$ and $\underline{O}$ indicate the maximum and minimum of processing capacity,
- $\overline{M}$ and $\underline{M}$ denote the maximum and minimum of mining capacity,
- N_p shows the total number of blocks extracted in pushback,
- g_n denotes the average grade of block,
- $\underline{G}$ indicates the lower limit of average grade.

3. Modification of pushback design algorithm considering the geological uncertainty

The formulation and the solving process in the proposed approach is similar to the one used for mine production scheduling using the ACO algorithm by **Gilani et al. (2016)**. In the same way, goals, and constraints such as maximizing the NPV, meeting production goals, minimizing the stripping ratio, and ensuring the safety slope requirements for each phase have been considered. To specify the initial goals of each phase, an initial pushback design is required, which can be created via conventional algorithms. Then, the near-optimal pushback design yielding the mentioned goals is defined by the new proposed ACO algorithm. The final solution is a pushback design that mimics the same average tonnages of each pushback in the starting design and simultaneously minimizes the variability of the tonnages sent to the various destinations over the set of geological realizations. Integration of geological uncertainty in the algorithm was done using two new block models named "E-Type block model" and "Risk block model". The E-Type block model is the average of all realizations. The Risk block model was created by calculating the extraction probability of each block in each pushback using all realizations. The general IP formulation has been slightly modified to account the variability of metal grade, which will be solved via the new proposed ACO algorithm, both of which are described below.

3.1. Modified formulations

The modified formulation seeks to find a solution that has minimum deviation from a tonnage target over all realizations. In this way, a new probabilistic factor is added to **Equation 1** as follows:

$$\textit{maximize } Z = \frac{1}{S}\left\{\sum_{s=1}^{S}\sum_{n=1}^{N}\sum_{p=1}^{P} V_{n,s}.x_{n,p} - S.C_{pr}Prob_n^p\right\} \quad (10)$$

subject to constraints (2) to (9),

Where:

- $Prob_n^p$ represents the extraction probability of the n^{th} blockin the p^{th} pushback.
- C_{pr} is a coefficient cost related to $Prob_n^p$ as follows:

$$C_{pr} = 100 - Prob_n^p,\ pr = 1,\ldots,100 \textit{ and } (C_{pr} < C_{pr-1})$$

- S is the total number of realizations,
- s is the realization number.

The values of $Prob_n^p$ are the constituents of the Risk block model and are obtained using all realizations and their created pushbacks by conventional algorithms (**Equation 11**).

```
Input:
    Import simulations, E-Type block model and their created pushback design.
    Import economical and technical parameters.
Initialization:
    Initial solution ← created pushback design for E-Type block model
    Encode the initial solution.
    Create risk block model based on the created pushback design for all simulations.
    Initialize pheromone value based on initial solution and risk block model
Iterations:
    while (I < the maximum number of iterations or the stopping criterion is met)
        for all artificial ants do
          for all pushbacks do
              New solution ← found non-normal pit using equations 13-14.
              Normalize the new solution.
              Decode the normalized pit.
              Update pheromone locally using equation 17.
          endfor
          Calculate the fitness value of created pushback.
        endfor
        Find the best of iteration (BOI) solution.
        Update the best so far (BSF) solution.
        Update pheromone globally using equations 18-19.
    Endwhile
Final solution ← BSF
```

Figure 3: Pseudo code of the ACO Algorithm to pushback design considering geological uncertainty

$$Prob_n^p = \frac{100}{S}\sum_{s=1}^{S} x_{n,s}^p \qquad (11)$$

Where $x_{n,s}^p$ is a binary variable that takes 1 if the n^{th} block of s^{th} realization locates in p^{th} pushback, and otherwise, takes 0. According to the objective function (**Equation 10**), the blocks with high values of $Prob_n^p$ will be located in the earlier pushbacks.

In the large-scale mines with millions of blocks, in other words millions of variables, solving the problem is very difficult or costly. On the other hand, the non-linearity of the objective function complicates the situation. So, meta-heuristics algorithms such as the ACO will be efficient and can simplify the formulation by implicitly obeying slopes and various other constraints (**Gilani et al., 2016**).

3.2. The implementation of the ACO procedure

The meta-heuristic algorithms as summarized in **Figure 2** can be classified into two classes: nature-based and human-inspired algorithms. In the first class, the searching mechanism is inspired by biological or physical phenomena. However, in the second class, the human phenomena are used to explore the search space. The nature-based algorithms could be classified into evaluation-based, physic-based, math-based, and swarm-based algorithms (**Poormirzaee et al., 2021**).

The ACO is a swarmed-based algorithm that has been successfully applied to solve many engineering problems such as assignment problems, vehicle routing, and traveling salesman due to its efficiency (**Dorigo et al., 2019**). The proposed technique uses the ACO algorithm to solve the modified IP formulation. The application of the ACO approach in OPMPS was been introduced by **Sattarvand et al. (2009, 2013)**. The ACO algorithm developed by **Dorigo et al. (2004)** was inspired by the search behaviour of ants. Naturally, ants wander randomly to find food, then return to their colony after finding it, and finally deposit a chemical substance called a "pheromone". The pheromone trails help other ants to travel non-randomly by transmitting a message to them. The amount of the pheromone in the paths depends on the their lengths, the number of ants passing through them, and the rates of deposition and evaporation of the pheromone. In long distances, where the reduction of the pheromone is greater than its deposition, eventually no trace of the pheromone remains. However, in shorter routes, the pheromone deposition overcomes its reduction and this leads to the attraction of other ants. **Figure 3** illustrates the pseudo-code of the proposed ACO approach to design the mining pushbacks considering the grade uncertainty. The basic principle in ACO is to start with a sub-optimal solution and gradually improve it in several steps during its own specific process.

3.2.1. Initial solution and pheromone initialization

The primary process of the algorithm is assigning some initial pheromones to blocks. In this regard, the pheromone values (τ_n^p) are initialized based on the initial solution, which represents the attractiveness of the n^{th} block to locate on the pit limit of the p^{th} pushback. The primary pushback design stores as an array of blocks (X_N^0) whose elements will take a pushback number (1 *to* *p*). The initial values of pheromone (τ_0) assigned in a way that the pit limits or an imaginary layer just above the topography surface take the higher values (**Sattarvand, 2009; Sattarvand et al., 2013**). Two new rules are considered to apply geological uncertainty: creating the initial pushback design by the conventional approach for the E-Type block model, and further pheromone deposition based on the $Prob_n^p$ values (**Equation 12**):

$$\tau_{n\ initial}^p = C_\tau \tau_0 \frac{Prob_n^p}{100} \tag{12}$$

Where C_τ is a coefficient to control the impact of $Prob_n^p$ on pheromone initialization. The experiments indicate that the uniform initial pheromone leads to increase the running time (**Dorigo et al., 2004**). Thus, the initial pushback design was improved by utilizing the deterministic version of ACO (Dtrm-ACO) in order to generate the initial solution.

3.2.2. Schedule encoding and decoding

Due to the special structure of the ACO which is based on path searching, the initial pushback design should be represented in the same shape. In this way, the process of searching the depth of pushbacks will be done in the columns of block instead of a 3D block model, and any subsequent process would be carried out on these columns. Consequently, any 3D pushback design ($X_{i,j,k}^p$) was encoded by a 2D array ($pitdepth_{(i,j,p)}$), which indicates the depth of pushback along column (*i,j*).

The data stored in $pitdepth_{(i,j,p)}$ represents the deepest surface of p^{th} pushback, so a decoding process should be done to determine the blocks of p^{th} pushback. In this regard, a transformation procedure is performed from the first pushback, and all blocks of column (*i,j*) located between $pitdepth_{(i,j,p)}$ and $pitdepth_{(i,j,p-1)}$ are considered as p^{th} pushback. It should be noted that the topography is considered as the previous pit depth for the first pushback.

3.2.3. Iterations

In each iteration, the pit depth determination process is performed only for ore-containing columns. The pheromone value of each block reflects its attractiveness to be selected as the deepest block of pushback in the column under investigation. In the next step, the pheromone updation process is performed so that the blocks around the previous non-optimal pit give higher values based on the quality of the previous solutions. Determining the depth, normalizing, and updating the pheromone are described in the following sub-sections as the three main steps of each iteration.

3.2.3.1. Depth determination

This process involves determining the depth of ores containing columns based on their pheromone values and some other heuristic information, such as the economic value of the blocks, in order to create a non-normalized pushback. To do that, a probabilistic rule named "random proportional rule" is used by artificial ant to choose the block as the pit floor of the given column:

$$P_n^k = \frac{[\tau_n]^\alpha [\eta_n]^\beta}{\sum_{l\epsilon N_n^k} [\tau_l]^\alpha [\eta_l]^\beta} \tag{13}$$

Where τ_n and η_n are the pheromone value and heuristic information of the n^{th} block, respectively. The relative influence of the pheromone value and the heuristic information is represented by two parameters, α and β. The N_n^k is the set of feasible choices of ant *k*. In this study, by choosing the Ant Colony System (ACS) as the appropriate variant, its special selection rule called "pseudorandom proportional rule" is used by the ant *k* to choose the pit depth as follows:

$$j = \begin{cases} \arg\max_{l\in N_n^k} \left\{ [\tau_n]^\alpha [\eta_n]^\beta \right\} \\ J, \end{cases}, \text{ if } q \le q_0 \ otherwise \tag{14}$$

Where q is a random variable uniformly distributed in [0,1], is a parameter $(0 \le q_0 \le 1)$, and J is a random variable selected according to **Equation 13** (with $\alpha = 1$).

Since this process is applied only in columns containing ore blocks, the depth of completely waste columns is defined based on the adjacent columns during the normalization process. Experiments show that limiting the depth determination process is more effective. In this regard, the search process at each stage is limited to the biggest possible pit and the depth of the previous pushback (or topography).

3.2.4. Constraint handling

Like other meta-heuristic algorithms, there is no explicit mechanism for managing constraints in the ACO algorithm. Here, an innovative approach was used to handle the slope constraint by changing the pit depth in each column and normalizing it. In addition to satisfying the slope constraint, the normalized pit also covers the previous non-normal pit (**Gilani et al., 2015**). A penalty function is added to the objective function to control the rest of constraints as follows:

$$maximize\ Z = \frac{1}{S}\left\{ \sum_{s=1}^{S} \sum_{n=1}^{N} \sum_{p=1}^{P} V_{n,s}.x_{n,p} - S.C_{pr} Prob_n^p - P(x) \right\} \tag{15}$$

subject to constraints (2), (3) and (4) and:

$$P(x) = \left(C_p^{m-}.d_{p,s}^{m-} + C_p^{m+}.d_{p,s}^{m+} + C_p^{o-}.d_{p,s}^{o-} + C_p^{o+}.d_{p,s}^{o+} + C_p^{g-}.d_{p,s}^{g-} \right)$$

$$\forall s = 1, \ldots, S \ and \ \forall p = 1, \ldots, P \tag{16}$$

Where:

- $C_p^{m+} = c^{m+} / \left(1+d\right)^p$: Unit surplus cost for mining more than $\bar{M}$ in p^{th} pushback (c^{m+} is the undiscounted unit surplus cost).
- $C_p^{m-} = c^{m-} / \left(1+d\right)^p$: Unit shortage cost for mining less than $\underline{M}$ in p^{th} pushback (c^{m-} is the undiscounted unit shortage cost).
- $C_p^{o+} = c^{o+} / \left(1+d\right)^p$: Unit surplus cost for ore extracting more than $\bar{O}$ in p^{th} pushback.
- $C_p^{o-} = c^{o-} / \left(1+d\right)^p$: Unit shortage cost for ore extracting less than $\underline{O}$ in p^{th} pushback.
- $C_p^{g-} = c^{g-} / \left(1+d\right)^p$: Unit shortage cost for average grade less than $\underline{G}$ in p^{th} pushback.
- $d_{p,s}^{m-}$ and $d_{p,s}^{m+}$, represent the shortage and the surplus of rock materials mining during p^{th} pushback in scenario s, respectively.
- $d_{p,s}^{o-}$ and $d_{p,s}^{o+}$, denote the shortage and the surplus of ore mining during p^{th} pushback in scenario s, respectively.
- $d_{p,s}^{g-}$, denotes the shortage in average grade of ore sent to plant during p^{th} pushback in scenario s.

As seen in the penalty function $P(x)$, the pushback design of all realizations is utilized to calculate the incurred shortages and surpluses.

3.2.5. Pheromone update

The pheromone values are updated during two stages of evaporation and deposition. The first stage decreases the amount of pheromone at a uniform rate so that the wrong solutions are ignored. In the next stage, some pheromone is reaccumulated in the blocks that participated in pushback construction. There are different strategies to update the pheromone values, such as ant system (AS), elitist ant system (EAS), ranked-based ant system (AS_{rank}), max-min ant system (MMAS), and ant colony system (ACS). The main difference between pheromone update systems is in the way blocks are selected for the update process and the amount of pheromone accumulation by them. In AS, all generated pushbacks (solutions) are allowed to participate in pheromone deposition. However, in EAS, MMAS and ACS, only the best-so-far solution allows the accumulation of excess pheromones. MMAS uses special constraints on pheromone deposition to avoid falling into local optima. Unlike the previous ones, the AS_{rank} strategy allows a number of reasonable solutions to accumulate pheromones.

In this research, the ACS method was used due to its running time and required computational resources (**Soleymani Shishvan et al., 2015**) and its more potent search process (**Dorigo et al., 2004**).

The pheromone updation in the ACS is done by two rules "local and global pheromone update" (**Equations 17-18**). The first is performed immediately after building each pushback, and the second one is done after each iteration only by the best-so-far ant (**Dorigo et al., 2004**).

$$\textit{local update: } \tau_n^p = \left(1-\xi\right)\tau_n^p + \tau_0, \left(0<\xi<1\right) \quad (17)$$

$$\textit{global update: } \tau_n^p = \left(1-\rho\right)\tau_n^p + \rho\Delta\tau_n^{best}, \left(0<\rho<1\right) \quad (18)$$

Where ξ and ρ are the local and global rates of evaporation, respectively. τ_0 is the initial pheromone value, and $\Delta\tau_n^{best}$ is the deposited pheromone by the best-so-far ant on the n^{th} block.

Another pheromone update rule is performed based on the degree of uncertainty of the blocks in order to apply geological uncertainty in the algorithm (**Equation 19**). In this regard, the Risk block model is used and blocks with high values of $Prob_n^p$ receive more pheromone and have a higher chance of being located in the initial pushbacks.

$$\tau_n^p \leftarrow \tau_n^p + C_{prob}\tau_0 \frac{Prob_n^p}{100} \quad (19)$$

In order to further investigate, two different uncertainty-based strategies named Single risk-based (ACO-SRB) and Multiple risk-based versions of ACO (ACO-MRB) were developed for updating the pheromone values according to the value of $Prob_n^p$. The pheromone update in the first strategy is based on a fixed probability of $Prob$, and the n^{th} block of the p^{th} pushback is allowed to receive pheromone values if $Prob_n^p \geq Prob$. In this case, pushbacks are created by blocks with a risk lower than the threshold. While the update process in the second strategy is performed on all blocks according to their $Prob_n^p$. In this case, blocks with probability values of $Prob_n^p$, or in other words, low-risk blocks, will have more chances to be mined in the initial pushbacks, and vice versa.

3.2.6. Algorithm termination

The algorithm terminates after a certain iteration, or if the solution is not improved after several iterations.

4. Numerical results and discussion

The proposed approach was used for pushback designing of Songun mine in the presence of geological uncertainty. Songun is a large open-pit copper mine with a truck-and-loader mining system with a mining capacity of 46 Mt per year and an average grade of 0.661%. Songun deposit is classified into two categories: supergene and hypogene. The Supergene region, containing 182 Mt of ore material with a grade of 0.62%, makes up 12% of the total ore body and is considered the primary feed of concentrators in the early years. The ore tonnage of hypogene is 1300 Mt with an average grade of 0.44%. The deposit includes 2,515,968 blocks with dimensions of 25 × 25 × 12.5 meter. Five areas with different stable slope angles according to geotechnical studies have been considered, as shown in **Table 1**.

20 realization of the ore body containing copper grades, recovery, tonnage, and type of materials are sim-

Table 1: Different geotechnical regions

Region	Azimuth (degree)	Slope angle
a	0	38°
b	90	38°
c	130	30°
d	235	30°
f	275	36°

ulated by Sequential Gaussian Simulation (SGSim) approach. The initial pushback designs that prepared for all realizations and the E-Type block model are used to obtain the Risk block model and initial solution. The number of pushbacks for all ore bodies is considered four, based on experience and the total number of blocks. The economic parameters used in the model are prepared based on actual data, as shown in **Table 2**.

Table 2: Economic parameters used to pushback generation

Parameter	Value	Unit
Price	5,500	$/ton (metal)
Cost of sale	20	$/ton (metal)
Mining cost of waste materials	1.56	$/ton
Mining cost of ore materials	1.75	$/ton
Processing cost	11.85	$/ton
Expenses cost	2.57	$/ton
Dilution	8	%
Mining recovery	95	%
Discount rate	10	%

As mentioned before, some penalty functions, as summarized in **Table 3** are considered to handle the constraints.

A series of primary tests were conducted in order to determine the appropriate parameters of ACO as follows:

- Maximum number of iterations = 1000;
- Maximum number of consecutive iterations without improvement = 250;
- Number of artificial ants = 10;
- Global evaporation rate (ρ) = 0.1;
- Local evaporation rate (ξ) = 0.15;
- Upper and lower bounds are considered as 3 and 0, respectively;
- α and β are set to 1 and 0.15, respectively;
- C_{prob} = 5;
- Initial pheromone (τ_o) = 0.1;
- Minimum pheromone (τ_{min})= 0.001;
- Pseudorandom factor (q_0) = 0.7.

Firstly, the initial sub-optimal pushback designs generated for 20 realizations and the E-Type block model were improved by the deterministic version of ACO (ACO-Dtrm) to create the initial solution and Risk block model. Then, ACO-SRB and ACO-MRB strategies were implemented on the initial solution to improve it. As shown in **Figures 4, 5,** and **6**, the improvement in fitness values by ACO-SRB and ACO-MRB was 1.48% and 1.76%, respectively. The time required for this improvement was 60 minutes. It can be seen that ACO-SRB has led to better and more effective results in some cases, especially in cases with a lower value. It seems that ACO-SRB has more potential to fall into local optima as well as incomplete search of the entire domain. However, it is a good option for planning with fixed risk. On the other hand, ACO-MRB has a higher chance of finding better solutions due to searching unexplored or less explored domains with low values of *Prob*.

A general comparison is made between the initial pushback design and the improved ones by the ACO-Dtrm, ACO-SRB, and ACO-MRB for the estimated, realizations, and E-Type block models. **Figure 7** shows that all three versions of ACO can significantly increase the fitness value of the original pushback designs. The ACO-Dtrm resulted in an increment in the fitness value of the initial pushback designs for the estimated, all realizations, and E-Type block models by 37.32%, 31.74%, and 30.86%, respectively.

A more detailed comparison between ACO-SRB and ACO-MRB is presented in **Table 4** to indicate their efficiency in providing the final solution in two different modes. In the first one, the initial solutions are created by a conventional approach for estimated and E-Type block models. Nevertheless, the improved solution by ACO-Dtrm for E-Type block model is considered as the initial solution in the second mode. The apparent result is that ACO-MRB is slightly better than ACO-SRB (in 90% of the cases), especially when the value of is high. Considering s from 0 to 100%, using ACO-SRB, an av-

Table 3: Penalty functions considered to handle the constraints

Constraint	Condition of violation the constraints	Penalty value
Mining capacity	less than 37 Mt ($\underline{M}$)	10% increasing the overhead and mining costs
	more than 46 Mt ($\bar{M}$)	20% increasing in mining cost
Milling capacity	less than 12 Mt ($\underline{O}$)	10% increasing the overhead and mining costs
	more than 14 Mt ($\bar{O}$)	Waste of extracted ore
Average grade	less than the 0.55% ($\underline{G}$)	Change the plant recovery as: $Recovery(\%) = G_{cu} \times \frac{R(\%)}{\underline{G}(\%)}$

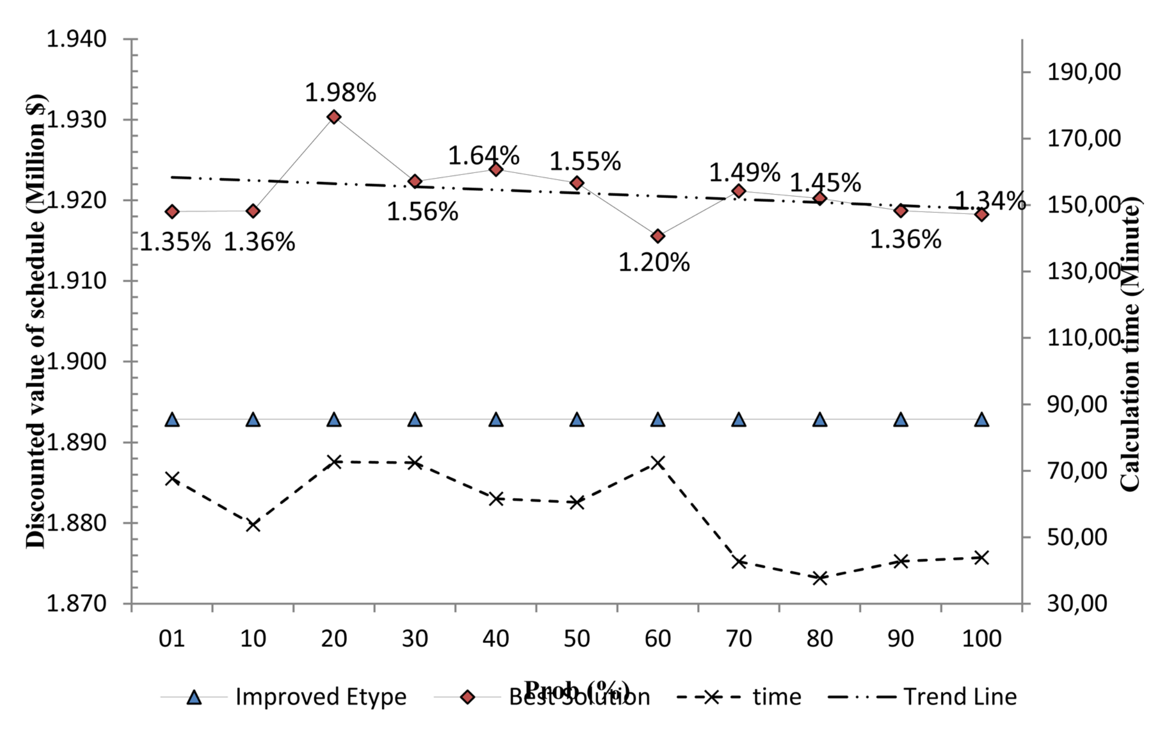

Figure 4: The performance of ACO-SRB in pushback design with different probabilities

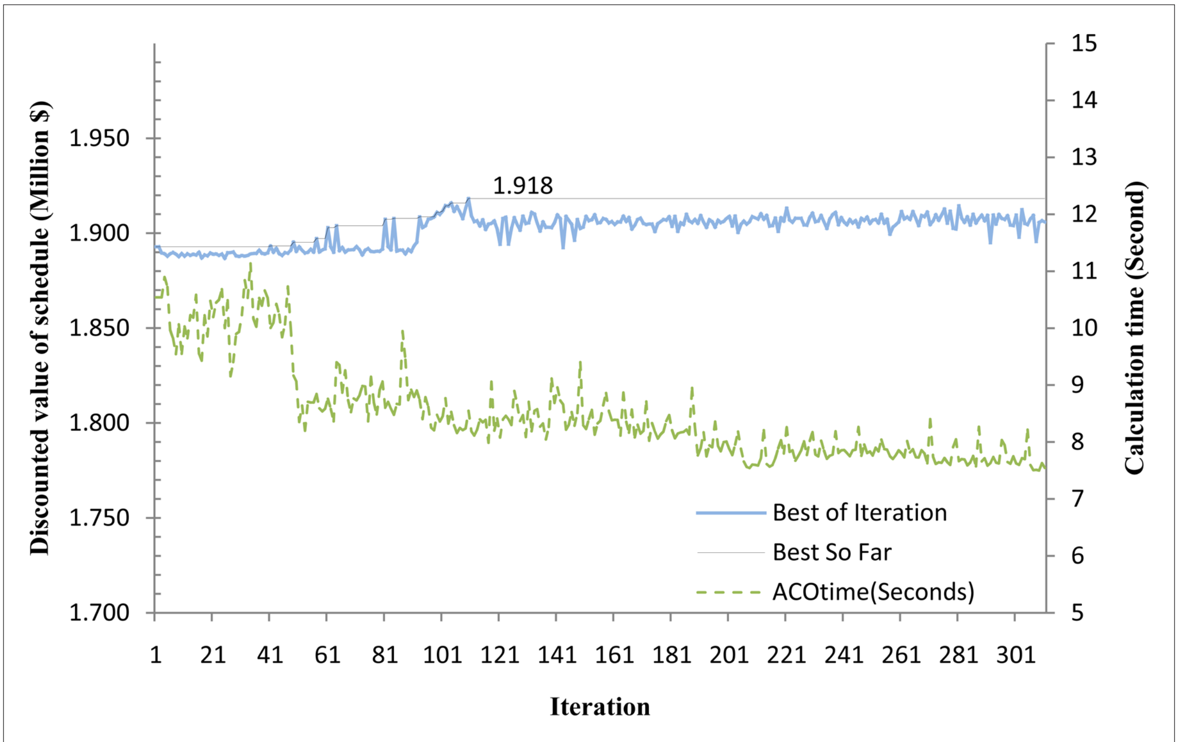

Figure 5: The performance of ACO-SRB in pushback design with Prob=100%

erage improvement of 1.48% was achieved in the fitness value (ACO-MRB resulted in a 1.76% improvement). **Table 5** shows the material distribution, average grade, and lifetime of each pushback provided by ACO-MRB approach. A more detailed comparison is performed in terms of material quantity, average grade, and obtained fitness value for each pushback. **Figures 8, 9, 10** and **11** show a clear view of changes in the final solution created by the ACO-MRB strategy compared to the conventional and the ACO-Dtrm methods.

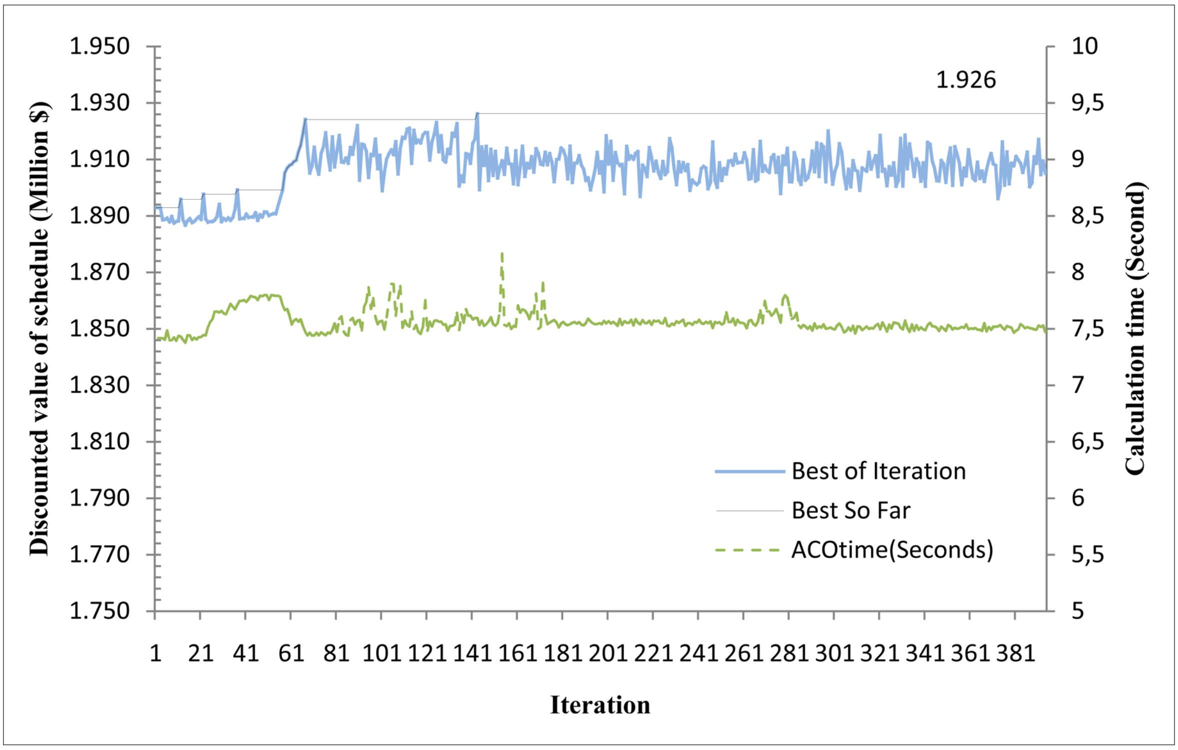

Figure 6: The performance of ACO-MRB in pushback design

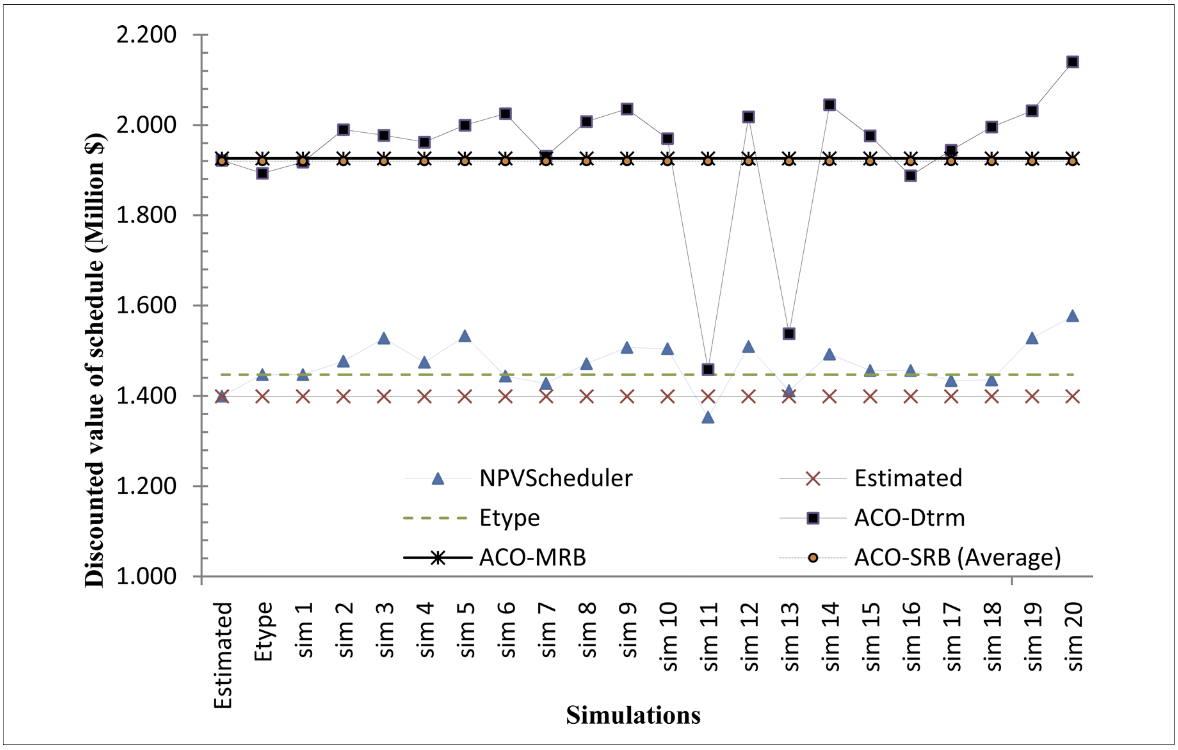

Figure 7: Comparison between conventional and proposed approaches

As can be seen in **Tables 6** and **7**, although the NPV is increased in all pushbacks, this increase is relatively higher in the first and last pushbacks. The approach led to a reduction in the stripping ratio for all pushbacks and an increment in the ore amount for the first pushback. The average grade of ore material has increased in all pushbacks, and this increase is relatively higher in the last pushbacks in order to overcome the risk associated with the grade uncertainty. From a physical size point of view, the created pushbacks have almost equal size.

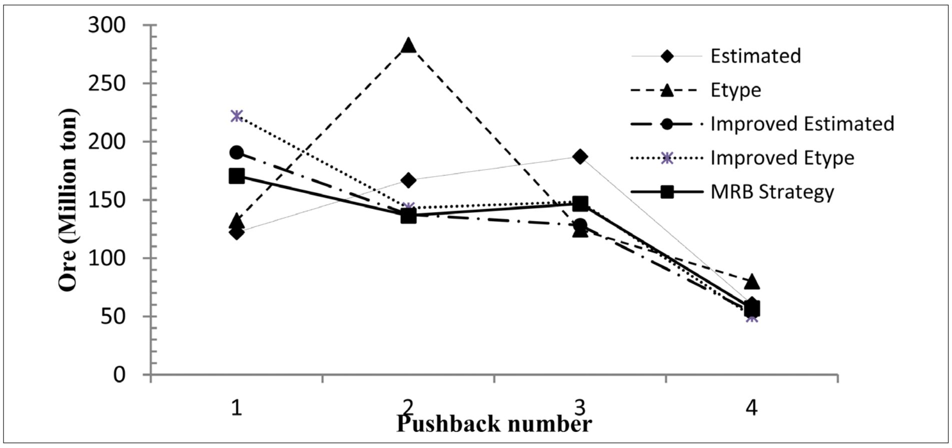

Figure 8: Comparison between the ACO-MRB, conventional, and ACO-Dtrm methods in terms of ore production

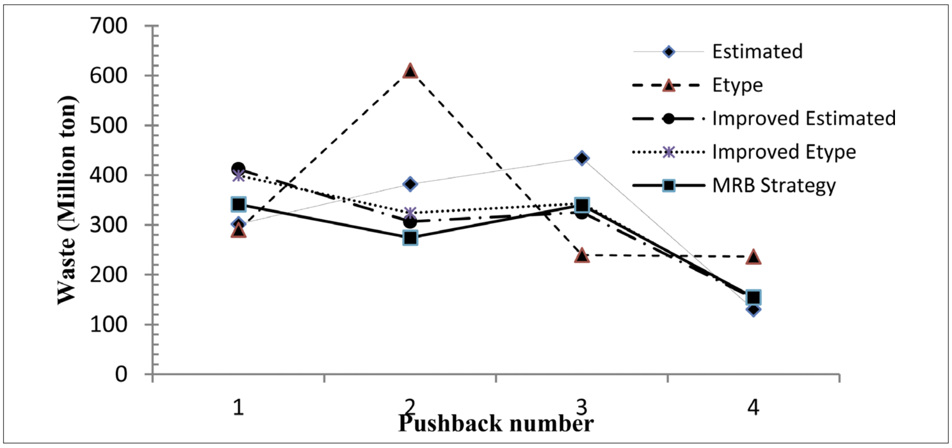

Figure 9: Comparison between the ACO-MRB, conventional, and the ACO-Dtrm methods in the aspect of striping

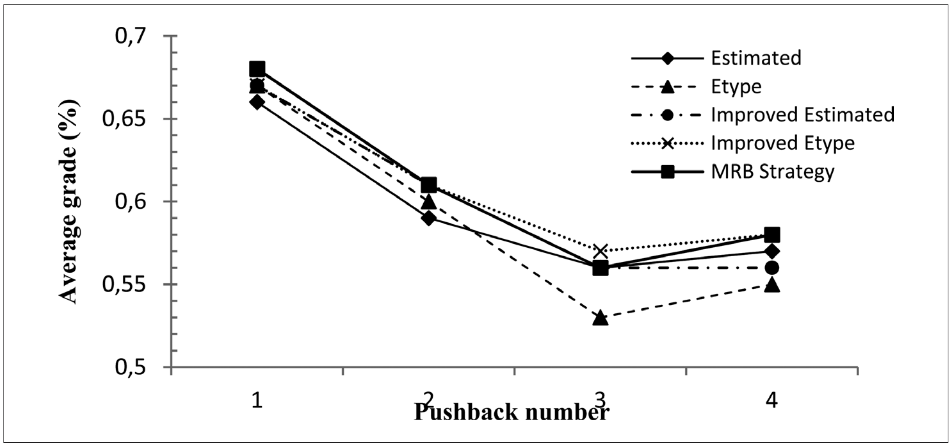

Figure 10: Comparison between the ACO-MRB, conventional, and the ACO-Dtrm methods in term of metal content

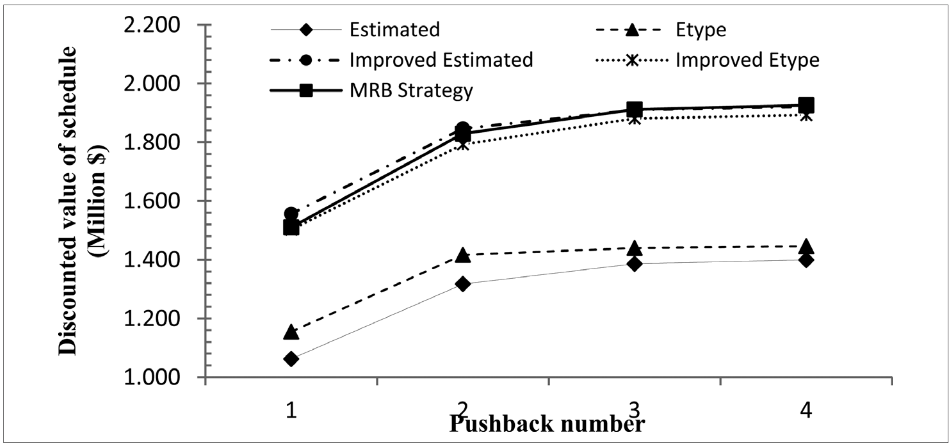

Figure 11: Comparison between the ACO-MRB, conventional, and the ACO-Dtrm approaches in term of NPV

Table 4: Comparison between the ACO-MRB, and ACO-SRB methods

Improvement (%)	ACO-MRB	ACO-SRB (%)										
		100	90	80	70	60	50	40	30	20	10	1
related to Estimated	37.64	37.07	37.10	37.21	37.28	36.88	37.35	37.47	36.36	37.94	37.10	37.10
related to E-Type	33.16	32.61	32.64	32.75	32.81	32.43	32.88	33.00	32.89	33.45	32.64	32.63
related to Improved E-Type	1.76	1.34	1.36	1.45	1.49	1.20	1.55	1.64	1.56	1.98	1.36	1.35
Time (Hour)	0.83	0.73	0.71	0.63	0.71	1.21	1.01	1.03	1.21	1.21	0.90	1.13

Table 5: the summary of created pushback considering geological uncertainty by ACO-MRB

Pushback number	Total rock (ton)	Ore (ton)	Stripping ratio	Average grade (%)	Lifetime (Year)
1	511,739,482	170,537,080	2.00	0.68	11
2	410,794,199	136,520,625	2.01	0.61	9
3	486,654,189	146,789,961	2.32	0.56	10
4	211,368,008	56,941,230	2.71	0.58	4
total	1,620,555,879	510,788,896	2.26	0.61	34

Table 6: The efficiency of ACO-MRB strategy related to the conventional approach

Pushback number	Efficiency (%)				
	Total rock	Ore	Average grade	NPV	Lifetime
1	21.07%	28.89%	1.49%	30.78%	22.22%
2	-54.00%	-51.74%	1.67%	21.80%	-52.63%
3	33.69%	17.84%	5.66%	253.70%	25.00%
4	-33.22%	-28.89%	5.45%	122.53%	-28.57%
total	-18.82%	-17.59%	3.40%	33.16%	-18.60%

Table 7: The efficiency of ACO-MRB strategy related to ACO-Dtrm

Pushback number	Efficiency (%)				
	Total rock	Ore	Average grade	NPV	Lifetime
1	-17.63%	-23.22%	1.49%	0.53%	-17.6%
2	-12.04%	-4.54%	0.00%	9.86%	-12.0%
3	-1.00%	-1.11%	-1.75%	-6.37%	-1.0%
4	4.54%	13.23%	0.00%	18.91%	4.5%
total	-9.06%	-9.41%	0.00%	1.76%	-9.1%

5. Conclusions

In the current study, a meta-heuristic approach based on Ant Colony Optimization (ACO) is proposed to integrate geological uncertainty in the pushback designing process using some realizations of the deposit. In the first step, the design process was done by conventional methods using the created realizations, which was partially improved using the ACO-Dtrm approach in the next stage. To improve the fitness value of the initial solutions created by the ACO-Dtrm procedure, two different strategies were considered based on a single predefined probability value (*Prob*) and multiple probability values , respectively. The procedure was tested in a case study of a large copper mine. The results indicate that the proposed approach leads to a single pushback design with improved NPV while incorporating grade uncertainty. The results revealed that the multiple probability strategy appears to produce better results. However, in situations with a high degree of flexibility, the single probability strategy is more practical. The provided pushback design by the second strategy has an improvement of 37.64% and 33.16 compared to the pushback design created by the conventional approach for the estimated and the E-Type block models, respectively. In addition, the improvement compared to the ACO-Dtrm approach for the E-Type block model was also 1.76%.

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SAŽETAK

Nova metoda za projektiranje faznoga razvoja kopa pri geološkim nesigurnostima bazirana na algoritmu optimizacije kolonijom mrava

Uzimajući u obzir neka operativna i ekonomska ograničenja, u procesu optimizacije površinskoga kopa bitan je zadatak određivanje vremena eksploatacije materijala koji se nalazi na najdubljoj etaži. Pravilan dizajn veličine zahvata etaže ima znatan utjecaj na optimalno planiranje proizvodnje. S druge strane, neki izvori nesigurnosti, kao što su terenske nepoznanice, uzrokuju odstupanja od proizvodnih i financijskih ciljeva. Ovaj članak predstavlja proširenje višesegmentnoga modeliranja za projektiranje faznoga razvoja kopa temeljenoga na riziku koji se za rješavanje koristi algoritmom optimizacije kolonijom mrava (eng. *ant colony optimization*, ACO). Za detaljnije proučavanje razvijene su dvije različite strategije prema statističkim i probabilističkim načelima. Za procjenu sposobnosti predloženoga pristupa u kontroli rizika odstupanja od proizvodnih ciljeva i povećanja troškova projekta korišteni su podatci iz rudnika bakra Songun koji se nalazi u sjeverozapadnome Iranu. Rezultati su pokazali učinkovitost predloženoga pristupa u projektiranju faznoga razvoja kopa kod geološke nesigurnosti. Ispitivanje različitih strategija pokazalo je kako metoda višestruke vjerojatnosti daje bolje rezultate.

Ključne riječi:
površinski kop, fazni razvoj kopa, stohastička optimizacija, geološka nesigurnost, algoritam optimizacije kolonijom mrava

Author’s contribution

Seyyed-Omid Gilani (Assistant Professor of Mining Engineering) proposed the idea and performed the modeling, programming and analysis of the results. **Sayfoddin Moosazadeh** (Assistant Professor of Mining Engineering) collaborated in the literature review and data collection. **Rashed Poormirzaee** (Assistant Professor of Mining Engineering) performed the data curation.