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Prediction of Rock Tensile Fracture Toughness: Hybrid ANN-WOA Model Approach

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Abstract

Various techniques are used in rock engineering to evaluate tensile fracture toughness, which is a critical parameter in assessing and designing stable rock structures. These methods typically involve laboratory investigations and statistical analysis. Nevertheless, artificial neural networks can also establish correlations among different data sets. Artificial intelligence approaches are becoming increasingly essential in all engineering fields, including the ones that study rock fracture mechanics. In this work, an artificial neural network with a hidden layer and eight neurons as well as a hybrid artificial neural network with a whale optimization algorithm were utilized to determine the tensile fracture toughness of rocks. In order to develop accurate models, this study has carefully selected four fundamental parameters to serve as inputs. These parameters include radius, thickness, crack length, and mean tensile strength of specimens. Also, 113 rock datasets were collected for models. The results show that utilization of the optimization algorithm enhances the precision in estimating the tensile fracture toughness of rocks. The R^2 improved to 0.93 when the whale optimization algorithm was used. On the other hand, the correlation factor reached 0.81 when the whale optimization algorithm was not implemented.

Keywords:
fracture toughness; artificial neural networks; whale optimization algorithm; size effect; tensile strength

1. Introduction

The study of fracture mechanics is a captivating area of research that delves into how objects withstand the growth of cracks and the numerous factors that impact the propagation of existing or emerging ones. This discipline has gained immense recognition in the study of metallic structures, especially in aerospace, marine, and nuclear engineering, and has even been applied to rocks and concrete structures. Despite the development of standard and straightforward fracture criteria over time, the field continues to face new challenges with advancements in science and industry. In rock mechanics, for instance, some older criteria cannot explain specific issues related to rock fractures due to the intrinsic complexity of quasi-brittle materials like rocks and concrete. These materials have unique characteristics, including various types of cracks that vary in size and shape, caused by diverse factors. Moreover, excessive loading on structures in real-world conditions is essential to their failure before reaching their theoretical strength (**Whittaker et al., 1992; Saxena, 1998; Bažant, 2000; Anderson, 2017**).

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It is obvious that cracks are present in all materials and can cause stress concentration and reduce the material's strength under different loading conditions. The stress intensity factor was introduced in 1952-54 to measure local stress around the crack tip and define fracture toughness (**Bažant and Yu, 2009**).

A rock or rock-like structure can experience various loading conditions. Understanding the structure's response to each loading condition is essential. Types of loading conditions based on crack propagation are (**Erarslan and Williams, 2013; Ebrahimi and Hosseini, 2022**):

- Tensile or opening mode (mode I),
- Shear or sliding mode (mode II),
- Tearing mode (mode III).

The loading type is generally a combination of different modes, which can be analyzed by considering loading modes from the first to the third mode (see **Figure 1**).

Based on their loading conditions, stress intensity coefficients KI, KII, and KIII can be calculated for brittle materials to determine stress intensity in modes I, II, and III. The critical value of these coefficients is commonly referred to as rock fracture toughness. Understanding fracture toughness and crack propagation in rock struc-

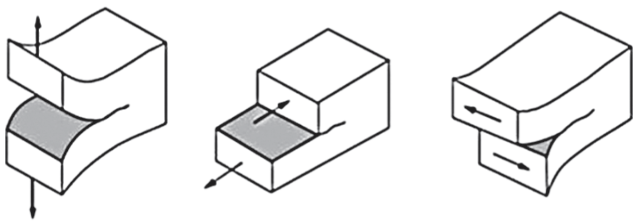

Figure 1: Schematic representation of various fracture modes (**Erarslan and Williams, 2013**)

tures is essential in various practical scenarios, including oil and gas extraction industries, tunnelling, mining, dams, and bridge designs. The investigation of fracture toughness in tension loading is an indispensable aspect of research, as rocks and rock-like materials tend to exhibit brittle behaviour in tension. Therefore, understanding the fracture toughness of such materials is critical for the development of effective engineering designs and the optimization of the performance of structural components (**Ghanbari et al., 2019**).

The ISRM (International Society for Rock Mechanics) has presented various experimental techniques to measure the tensile fracture toughness of rocks, categorized by three different methods: bending, direct tension, and compressive loading. Each approach has its distinct specimen preparation requirements for laboratory analysis (**Alkılıçgil, 2010; Dolatshahi and Molladavoodi, 2023**).

The most general experimental methods based on the type of loading for Mode I are as follows (**Franklin et al., 1988; Alkılıçgil, 2010; Kataoka et al., 2015**):

- Experimental methods such as short rod test (SR) as a method based on direct tensile loading conditions.
- Experimental methods such as the cracked straight-through Brazilian disc test (CSTBD), cracked chevron notched Brazilian disc test (CCNBD), and flattened Brazilian disc test (FBD) are methods based on compressive loading conditions.
- Experimental methods such as semi-circular bending test (SCB), straight edge cracked round bar bending test (SECRBB), and chevron bending test (CB) are based on bending loading conditions.

Various parameters affect the fracture toughness of rocks. These parameters include temperature, confining pressure, loading velocity, microscopic features, and the specimen's size and geometry. In addition, specimen preparation to determine this critical engineering parameter requires accuracy in the whole procedure with difficulty (**Miao et al., 2022**). Therefore, various experimental studies have been carried out by different researchers to provide a suitable approximation and estimate of tensile fracture toughness. For example, in an experimental study on eight sedimentary rocks, **Gunsallus et al. (1984)** proposed an experimental equation between compressive strength, Brazilian tensile strength, and point load index with rock fracture toughness Mode-I. **Whittaker et al. (1992)** presented that rock's tensile strength and fracture toughness have a direct relationship. In a laboratory study, **Zhang (2002)** proposed that the tensile fracture toughness of rock is related to the Brazilian tensile strength, and the result of his study was to provide an equation to estimate the tensile fracture toughness. Hosseini and Abdolghanizadeh presented an experimental relationship between rock tensile strength and fracture toughness Mode-I through the SCB method (**Hosseini and Abdolghanizadeh, 2017; Abdolghanizadeh et al., 2020**). In a similar study, **Hu et al. (2022)** investigated the effect of various temperatures on granite. They presented an experimental equation for estimating the tensile fracture toughness of granite by using longitudinal wave velocity, tensile strength, and the amount of heat applied to granite. **Shi et al. (2022)** drew out an empirical equation to predict fracture toughness Mode-I with the tensile strength of various rocks.

In addition to the experimental methods, different researchers have conducted many studies in which newer methods have been implemented. Researchers based their findings on datasets and the aid of techniques such as artificial neural network and deep machine learning (DML), fuzzy logic, or the use of statistical methods such as linear regression (LR) and non-linear regression (NLR) and the use of various metaheuristic algorithms to provide an accurate relationship for estimating the fracture toughness of rocks by including multiple factors (**Albaijan et al., 2023; Fakhri et al., 2023; Wiangkham et al., 2023**). For example, **Hamdia et al. (2015)** employed an artificial neural network (ANN) and adaptive neuro-fuzzy inference system (ANFIS) to predict the fracture energy of polymer nanocomposite. Their ANN and ANFIS models based on 115 experimental datasets and five input parameters were considered: the volume fraction of the nanofiller, the fracture energy of the matrix, the diameter of the nanoparticle, the elastic modulus of the nanocomposite's matrix and its yield strength. The results showed that the ANN and ANFIS models produced considerably superior outputs with higher coefficients of determination. The values of R^2 for the ANN and the ANFIS models were 0.925 and 0.937.

Wang et al. (2021) employed an artificial neural network (ANN) to predict the fracture toughness Mode-I of rocks based on crack properties, rock tensile strength, and radius of CCNBD specimens of 88 datasets. The results of this study showed that the capacity of ANN is more satisfactory than experimental relationships and more comprehensive to generalize other rocks.

Mahmoodzadeh et al. (2022) combined the support vector regression (SVR) method with six different metaheuristic optimization algorithms, including particle swarm optimization (PSO), grey wolf optimization (GWO), multiverse optimization (MO), moth flame optimization (MFO), sine cosine algorithm (SCA), and social spider optimization (SSO) to predict Mode-I rock fracture toughness. They used 250 datasets of rock frac-

Table 1: Some reviewed articles about the size effect on fracture toughness Mode-I

[Rf.]	Method	Material	Fracture toughness with an increase in in the dimensions
(Khoramishad et al., 2014)	CB	Limestone	Increase
(Ayatollahi et al., 2014)	CSTBD	Nayriz Marble	
(Aliha et al., 2015)	SCB	Asphalt concrete	
(Akbardoost, 2016)	MR	Marble	
(Jeong et al., 2017)	SCB	Granite	
(Zhang et al., 2021)	CSTBD	Sandstone	
(Muñoz-Ibáñez et al., 2021)	SCB	Sandstone, and granite	
(Davis et al., 2022)	CB	Quasi-brittle sandstone	
(Li et al., 2023)	CB	Anisotropic Shale	
(Pirmohammad et al., 2024)	SCB	Asphalt concrete	

ture toughness with the CCNBD method. The results showed that the hybrid model of SVR-PSO produced the most accurate results, and it was recommended to predict the Mode-I rock fracture toughness.

Fakhri et al. (2022) employed various machine learning methods to predict the fracture toughness of mixed modes of several types of concrete. Their models included the extreme standard gradient boosting process (XGboost), hybrid models of XGboost-PSO, XGboost-GWO, XGboost-imperialist competitive algorithm (ICA), XGboost-shuffled frog leaping algorithm (SFLA), and XGboost-genetic algorithm (GA) with utilizing 560 datasets obtained from the CSTBD test. Inputs of their models included concrete type, specimen diameter, thickness, crack length, failure loading, and crack angle. The results showed that the hybrid model of XGboost-PSO predicted the mixed-mode fracture toughness of concrete specimens best.

Afrasiabian and Eftekhari (2022) employed machine learning methods such as gene expression programming (GEP) to predict the fracture toughness Mode-I of rocks based on the mechanical properties of rocks, including Young's modulus, tensile strength, and uniaxial compressive strength. Their results, compared with multiple linear regression (MLR), showed that the GEP model produced exceptionally illustrative outputs with higher coefficients of determination. The values of R^2 for the GEP and the MLR models were 0.87 and 0.74, respectively.

Emami Meybodi et al. (2022) employed MLR, n-MLR, and SVR methods to predict the fracture toughness Mode-I and Mode-II of rocks based on the mechanical properties of rocks, including Young' modulus, tensile strength, and uniaxial compressive strength. Their results showed that the SVR method predicted fracture toughness better than the other methods.

The fracture toughness is dependent on the specimen size. Different researchers have conducted multiple studies that indicate the specimen size effect on the Mode-I toughness of a rock in the mostly experimental methods, some of which are reviewed in **Table 1.**

Based on the above literature, this study aims to evaluate the fracture toughness of Mode-I rocks based on their geometrical and the mean tensile strength with the ISRM suggestion size as a mechanical parameter to emphasize the potential of models considering the phenomenon of size effect on tensile fracture toughness. To achieve this, an artificial neural network and the WOA (Whale Optimization Algorithm) were utilized. The WOA has not been previously applied in the tensile fracture toughness prediction as a hybrid model approach. This research seeks to determine the effectiveness and accuracy of the WOA in estimating fracture toughness.

The general framework of this article is to collect ¸the geometrical and tensile strength data of different rocks to achieve the performance of both the artificial neural network model and the hybrid artificial neural network model with a whale optimization algorithm for weighting of the neurons in the hidden layer of the model considering the specimens' size effect on the tensile fracture toughness of rocks.

2. Methodology

Computational models known as Artificial Neural Networks (ANNs) function similarly to the human brain. ANNs consist of numerous processors connected by weighted connections, much like neurons. The output of each processor relies on the information at the node, which may be stored internally or received through links. Every processor receives input from various nodes and transmits its output to other nodes. Although a single processor is not particularly potent, they create a robust system when combined. The output of a single processor is a scalar output with a numerical value, which results from a simple nonlinear function of its inputs. The artificial neural network (ANN) is not a solution that relies solely on mathematical equations. Instead, it showcases information processing characteristics that enable it to approximate a given problem. ANNs have been widely used in complex nonlinear function mapping, image processing, pattern recognition classification, etc. Feed-forward networks are a common type of neural network (**Hopfield, 1988; Dongare et al., 2012; Pradeep and Samui, 2022**).

In this article, an artificial neural network consisting of four inputs and a hidden layer with eight neurons and one output is modelled. A whale optimization as a meta-heuristic algorithm has been implemented to determine the weight of each neuron and the impact of each parameter. The layer transfer function was employed with both hidden and output layers, and the number of neurons was determined through a rigorous trial and error process. To prevent the issue of overfitting, the early learning strategy was implemented. The combined model of the artificial neural network with the whale optimization algorithm has been coded in MATLAB software, and three critical points are necessary when checking to find a relationship to estimate the tensile fracture toughness with the ANN- metaheuristic algorithm, which are:

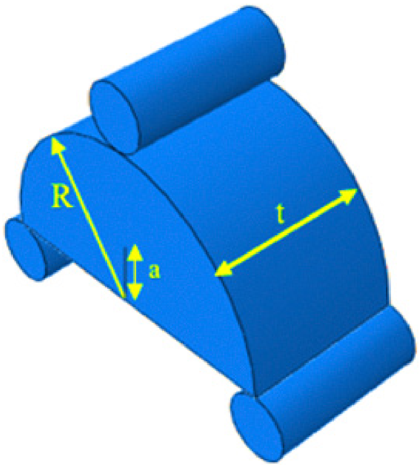

Figure 2: The semi-circular specimen under three-point bending (SCB)

a) Data collection: As mentioned earlier, the determination of tensile fracture toughness can be determined by various tests, so while collecting specimens, the data should be based on the same laboratory method. In this study, the data are collected based on the SCB laboratory method.
b) It is required to be most accurate in determining the inputs of an artificial neural network so that the relationship defined to estimate the output parameter is the most consistent with reality.
c) It is necessary to be knowledgeable about the general process of the optimization algorithm and inspect the results with the standard and default state of the artificial neural network without using the optimization algorithm. Optimization algorithms should always be used to improve the accuracy of a relationship (**Abdel-Basset et al., 2018**).

2.1 The SCB Method and Inputs of Models

One of the newest experimental methods to determine the fracture toughness of rocks was suggested by **Kuruppu et al.** in 2014 with the approval of ISRM. In this test, the studied rock specimen has a semi-circular geometry with a straight edge or chevron crack. The semi-circular model is subjected to three-point bending (see **Figure 2**). In addition to the tensile mode, the rock's combined modes of shear and tensile and pure shear mode fracture toughness can also be determined with this method (**Dolatshahi and Molladavoodi, 2023**).

For the ANN-WOA model, the inputs are based on the geometric characteristics of the specimen, including the thickness (t) and radius (R) of the sample, the length of the crack (a), and the mechanical properties of the rock is the mean tensile strength (σ_t) of the rock based on the Brazilian test according to the ISRM standard.

Table 2: Details of datasets of ANN and ANN-WOA models

Ref.	**Sample**	**N**	$\overline{\sigma_t}$ **(MPa)**	**R (mm)**	**a/R**	**t/2R**	K_{Ic} **(MPa.m$^{0.5}$)**
(Alkılıçgil, 2010)	Andesite	10	7	49.52-51.03	0.10-0.22	0.49-0.51	0.83-1.10
	Marble	7	5.13	36.63-51.50	0.15-0.27	0.49-0.51	0.47-0.64
(Kataoka et al., 2015; Jeong et al., 2017)	Granite (I)	15	10.30	35.70-36.80	0.48-0.50	0.25-0.28	1.97-2.48
	Granite (II)	17	11.40	35.20-37.10	0.49-0.50	0.25-0.28	1.62-2.03
(Xiao et al., 2021)	Granite	3	12.50	23.37-23.64	0.25	0.33-0.54	1.58-1.62
(Zhang et al., 2020)	Limestone	36	9.65	24.01-74.89	0.30-0.50	0.29-0.31	0.76-1.24
(Ghouli et al., 2021)	Limestone	6	5.42	25-300	0.50	0.05-0.40	0.67-1.02
	Marble	6	11.41	25-300	0.50	0.05-0.40	1.01-1.35
	Granite	6	9.50	25-300	0.50	0.05-0.40	0.88-1.18
(Aliha et al., 2012; Nejati et al., 2019)	Concrete	7	2.10-12.50	65	0.30	0.30	0.44-2.10

Where:
N is a number of data, $\overline{\sigma_t}$ is the average of tensile strength, R is the radius of specimens, a/R is the ratio of length to the radius of the specimens, t/2R is the ratio of thickness to diameters of SCB specimens, and K_{Ic} is tensile fracture toughness. All geometrical parameters are shown in **Figure 2**.

2.2. Data set

In this work, 113 datasets have been collected from various articles (see **Table 2**). 80% of all data is utilized for training the ANN-WOA, and the leftovers are used to test the model. The data selection has been made in such a way as to show the effect of altering the inherent tensile strength of the rock, as well as the specimen size and crack length effects.

According to the correlation coefficient method between the inputs and outputs of the collected data, it has been determined that the highest correlation is between the radius of the samples with the fracture toughness and the tensile strength with the fracture toughness for the above data.

2.3. Whale Optimization Algorithm (WOA)

The whale optimization algorithm is a meta-heuristic algorithm encouraged by the unique method of humpback whales' hunting called bubble network, presented by **Mirjalili et al. (2016)**. Humpback whales hunt at the water's surface, initiating the bubble-net feeding behaviour of humpback whales so the prey is close to the water's surface (see **Figure 3**). This metaheuristic algorithm is a nature-inspired, and population-based algorithm that has been utilized in various case studies. It consists of three phases: Siege prey, Operation, and Exploration phases.

- Siege prey: Whales can detect the location of the prey and know how to surround it. Since the site of the best prey in the target space of the whale has yet to be discovered, the whale optimization algorithm first selects the first prey for the whale as the most suitable target. Then other factors update their position targets (**Mafarja et al., 2017; Mirjalili et al., 2016**). The behaviour of whales can be modelled by **Equation 1**.

$$\begin{cases} \vec{D} = \left|\vec{C}.\vec{X}^*(t) - X(t)\right| \\ \vec{X}(t+1) = \vec{X}^*(t) - \vec{A}.\vec{D} \\ \vec{C} = 2.\vec{r} \\ \vec{A} = 2\vec{a}.\vec{r} - \vec{a} \end{cases} \tag{1}$$

where:

t: number of iterations,

$\vec{C}$, and $\vec{A}$: vector of coefficients,

$\vec{X}(n)$: vector of position,

$\vec{a}$: a vector whose value is between 0 to 2,

$\vec{r}$: a random pick between [0,1].

- Operation phase as bubble net attacking: the whale hunting operation is highly dependent on vector A, which is also dependent on the value of a. If the value of vector A is less than 1, based on what was explained in the previous step, the position of the whales is updated, and hunting takes place. If the value of vector A is one or greater than one, the method of updating the position of whales is changed according to **Equations 4** and **5**, and the hunting process takes place in the form of a spiral rotation (**Mirjalili et al., 2016; Mafarja et al., 2017; Mehranfar et al., 2019**).

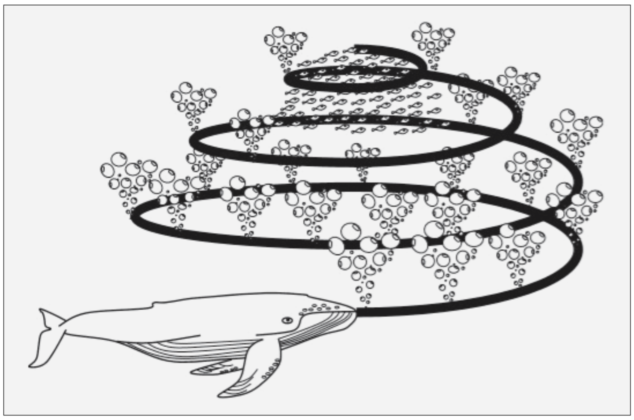

Figure 3: Bubble-net feeding behaviour of humpback whales (**Mirjalili et al., 2016**)

$$\vec{D'} = \left|\vec{X}^*(t) - X(t)\right| \tag{2}$$

$$\vec{X}(t+1) = \vec{D'}\, e^{bl}.\cos(2\pi l) + \vec{X}^*(t) \tag{3}$$

where:

I: a random number between [-1,1],

b: a constant value to determine the shape of the helix.

- Random exploration phase: In this method, the distance between the prey and the whale is first obtained from **Equations 4** and **5**, and then with the aid of **Equation 3**, the spiral movement of the whales towards the prey begins (**Mirjalili et al., 2016; Mafarja et al., 2017**).

$$\vec{D''} = \left|\vec{C}.\vec{X}^*_{random}(t) - X(t)\right| \tag{4}$$

$$\vec{X}(t+1) = \vec{X}^*_{random}(t) - \vec{A}.\vec{D} \tag{5}$$

To better understand how the whale optimization algorithm works, the flowchart of the whale algorithm is shown in **Figure 4**.

The Whale Optimization Algorithm (WOA) offers a set of simple yet powerful search mechanisms that enable the efficient identification of optimal solutions. Despite these benefits, however, the WOA, similar to other swarm intelligence algorithms, is susceptible to several challenges. These include the risk of falling into local optima, premature convergence, and low population diversity, all of which may hinder the algorithm's performance and limit its effectiveness. As such, it is important to be aware of these potential challenges and to take steps to address them as necessary to maximize the benefits of the WOA (**Nadimi-Shahraki et al., 2023**).

The critical point is checking the effectiveness and improvement of an ANN by using the WOA. Numerous

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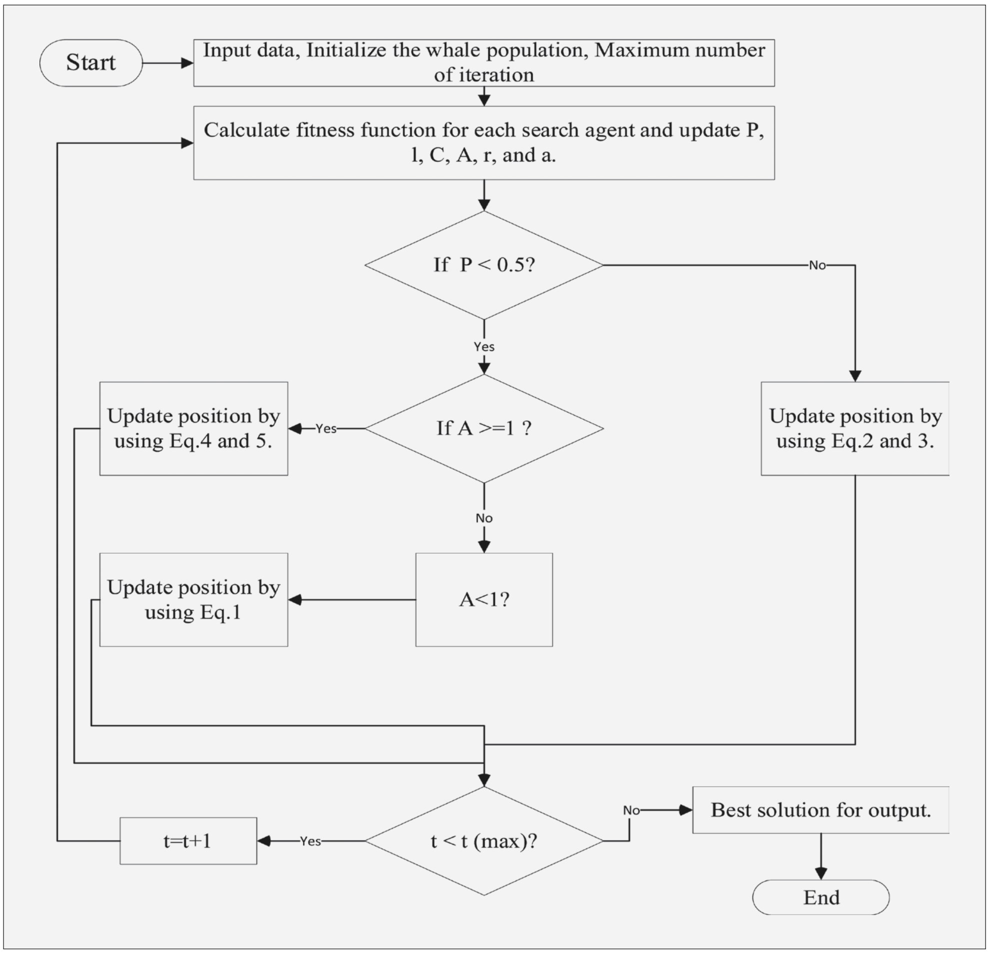


Figure 4: Flowchart of the WOA

optimization algorithms can cause more suitable improvement in the ANN. Furthermore, one of the debatable issues is that the features of the ANN can also affect the results. For instance, it can be used by increasing the number of hidden layers, and the change in the number of neurons in each hidden layer is even practical (**Goswami et al., 2020**).

As mentioned earlier, both models in this study have an identical configuration with one hidden layer of eight neurons This ensures that the models can be compared on the same physics. Various studies suggest that the number of neurons and hidden layers in the ANN may vary, affecting the model›s accuracy. In this study, the different neuron numbers were used for the hidden layer and the R^2 of the training data for each model of investigation and the optimal value of the number of neurons for the ANN and hybrid models have been selected to increase the speed and accuracy of the models (see **Figure 5**).

Several factors can impact the model's accuracy in some cases of using a meta-heuristic algorithm, like the WOA. These factors include the iteration rate, the number of whale populations, and their search range (**Mirjalili et al., 2016; Moayedi et al., 2019; Vaheddoost et al., 2020**). The study found that the number of whales and iteration values were optimal based on R^2 of training data. Higher values of whale population and iteration rate did not improve model accuracy (see **Figure 6** and **Figure 7**).

In this study, a hybrid ANN-WOA model consists of four inputs, one output, and one hidden layer with eight neurons. Also, 113 datasets, including tensile strength and geometrical parameters of the SCB specimens, were used in this study. In addition to the hybrid model with the same conditions, an ANN was also modelled using the whale optimization algorithm to clarify the difference. 80% of the data were used for training and 20% for testing of the model. The number of iterations of the whale algorithm is 500. Also, comparing two models in less than 100 iterations was checked to illustrate which models have been fitted well. A maximum number of 300 is set as a searcher factor in the hybrid model. The upper and lower limits of the algorithm are equal to 1 and zero, respectively. To evaluate and compare the performance and fitness of two of the ANN models and the hybrid ANN-WOA model, the main criterion employed in this study is to check the R^2 value of the relation between the actual tensile fracture toughness and the predicted values from each model. R^2 is obtained from **Equation 6**. For this purpose, the models were run in MATLAB software, and each run's results are given in **Table 3**. The coefficient of determination (R^2) for ANN

Figure 5: Effect of neuron number on the R^2 of training data

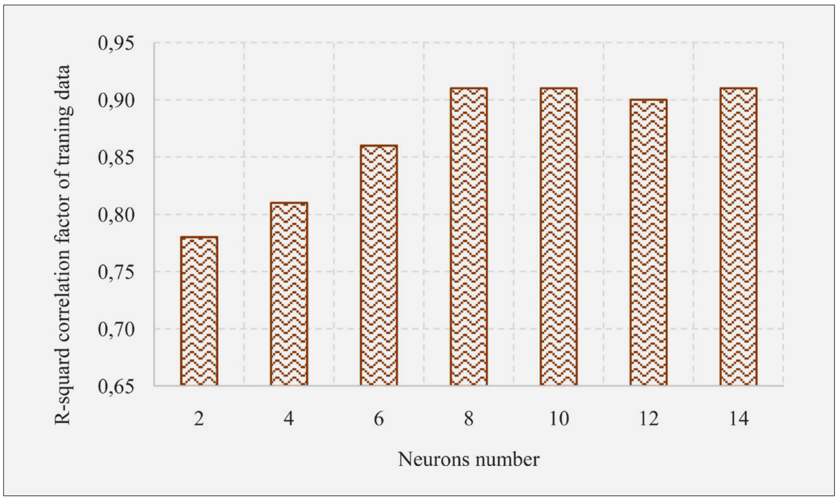


Figure 6: Effect of whale population on the R^2 of training data

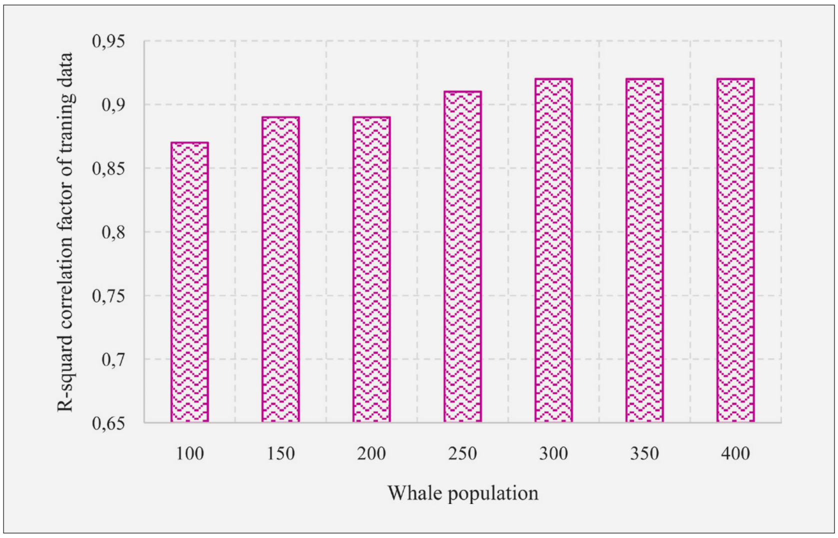
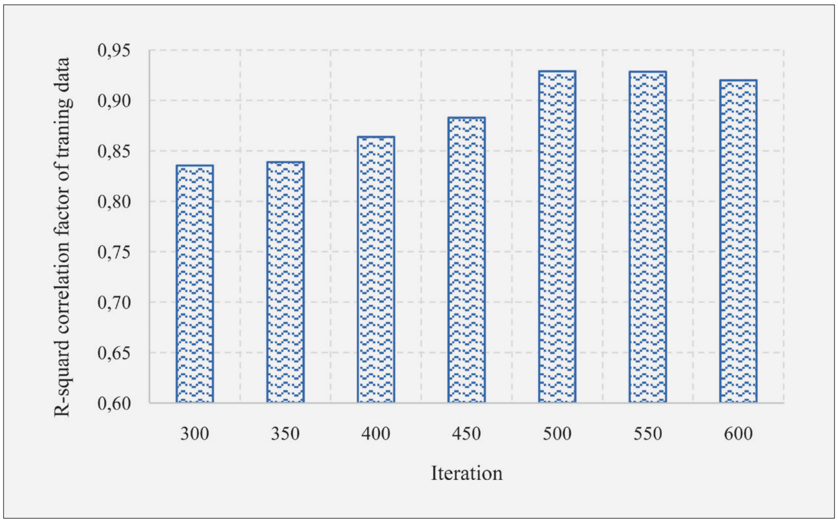


Figure 7: Effect of iteration on the R^2 of training data



and ANN-WOA models is calculated based on **Equation 6** as follows:

$$R^2 = 1 - \frac{\sum \left(y_{pr} - y_{ac} \right)^2}{\sum y_{ac}^2} \tag{6}$$

where:

y_{pr}: predicted values of models,
y_{ac}: actual values.

3. Results and discussion

The accurate determination of rock fracture toughness parameters under varying loading conditions is crucial for understanding the behaviour of rock structures during crack propagation. Given the poor performance of rocks and concrete under tensile loading conditions, it is imperative to pay close attention to the fracture toughness of these materials. However, the preparation of samples and the conduction of time-consuming tests present significant challenges, necessitating the use of estimation methods to determine this parameter.

This study utilized two factors specific to the geometry of the samples, as well as a parameter specific to the length of the crack and the average tensile stress as the rock strength parameter. The models presented are based on 113 datasets, and although the limited amount of data represents a constraint, the accuracy of the models in estimating fracture toughness was evaluated under low data conditions.

It is noteworthy that the estimation of fracture toughness parameters is critical in the design of rock structures and the determination of their safety margins. As such, the models presented herein represent a significant contribu-

Table 3: Results for the ANN and ANN-WOA models

Model	Train (R^2)	Test (R^2)
ANN	0.81	0.77
ANN-WOA	0.93	0.86

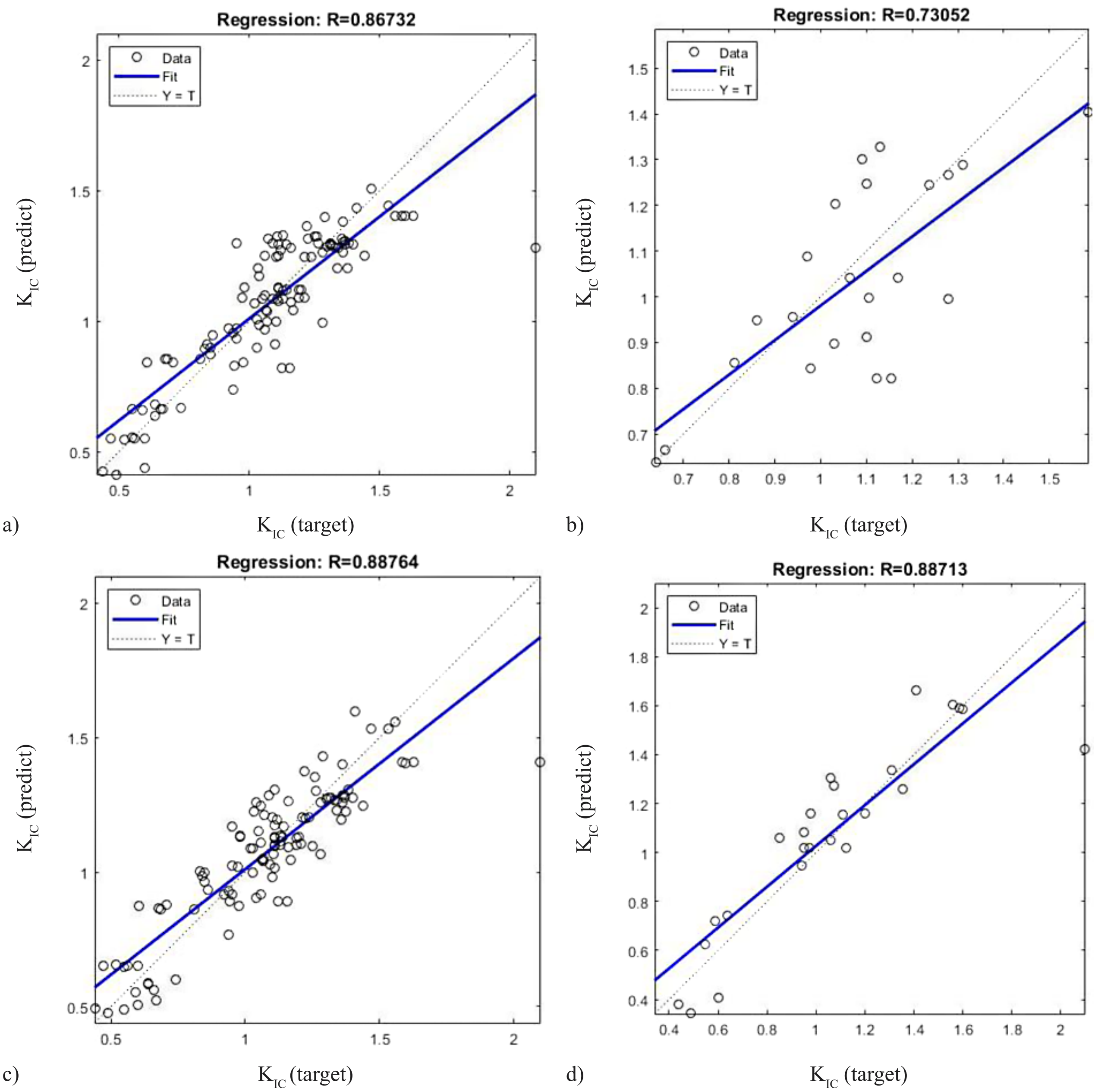


Figure 8: a) Regression of the ANN train, b) ANN test, c) the ANN-WOA train, and d) the ANN-WOA test

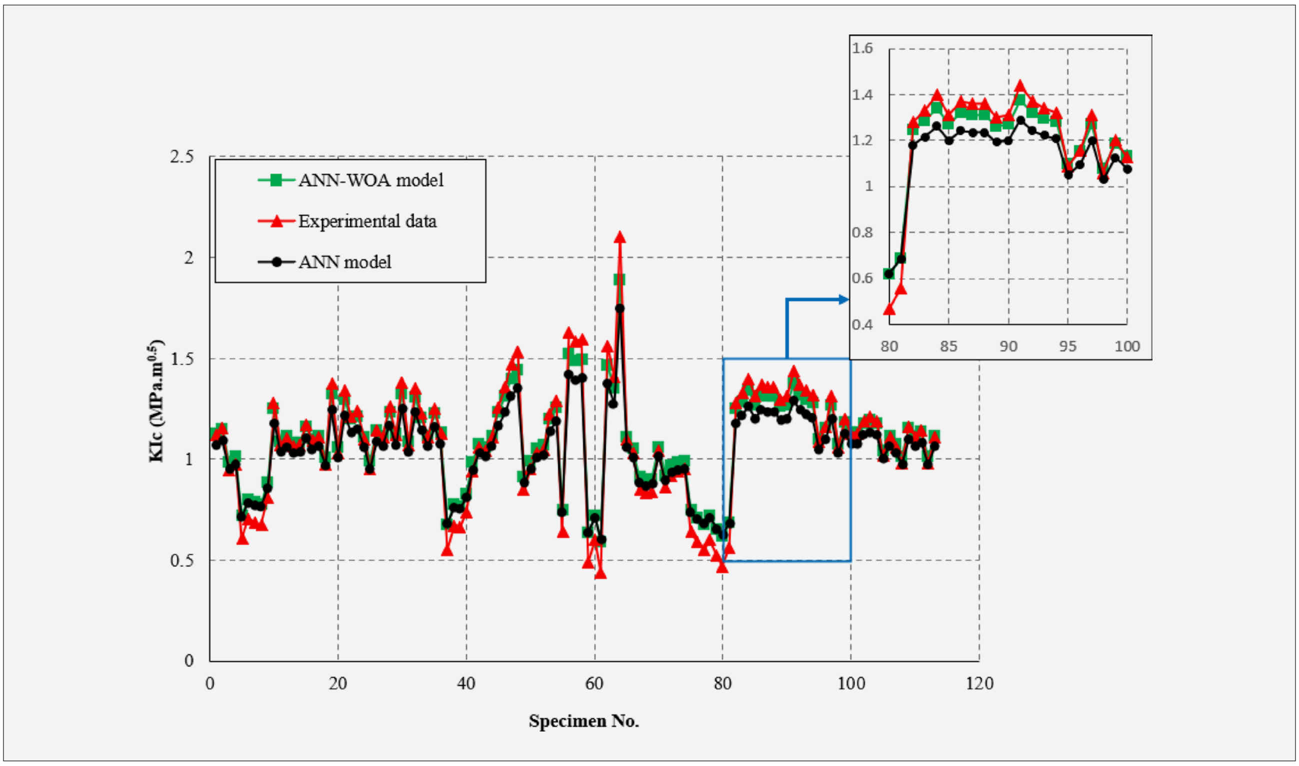


Figure 9: Comparing the results of the hybrid ANN-WOA and the ANN models with the experimental data

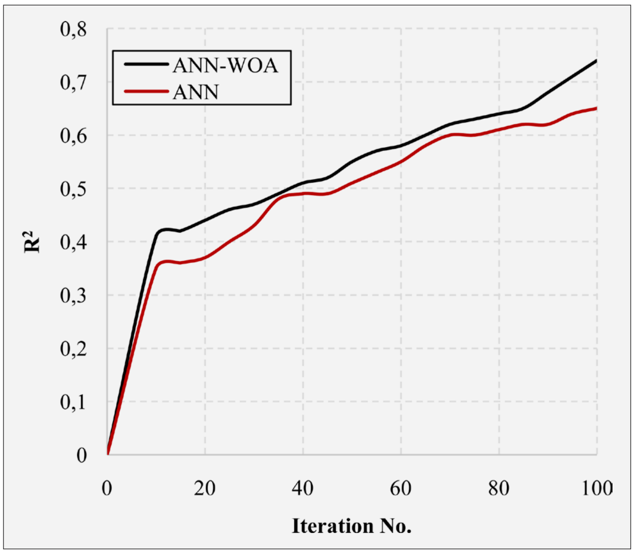


Figure 10: The R^2 of ANN, and hybrid ANN-WOA models in iteration less than 100

tion to the field, particularly in situations where the availability of data is limited. It is crucial to emphasize that ANNs require significant data for accurate training and testing results. With the scope of enhancement in certain subjects, some categories may have limited data availability. Notably, the data paucity may pose a challenge to the analysis and interpretation of results. However, carefully considering the available data and suitable statistical methods can assist in mitigating the impact of limited data. One of the most adequate manners to enhance the accuracy of the ANN is to utilize meta-heuristic algorithms. The hybrid model of the ANN combined with optimization algorithms adjusts the weighting of neurons of hidden layers to improve the model accuracy. The whale optimization algorithm stands out for its ability to evaluate search factors beyond just itself as the only perfect factor. This algorithm also uses a randomized approach to select data for model testing and training, resulting in a more comprehensive evaluation of the model's reliability and accuracy. **Table 3** presents the coefficient of determination (R^2) for the ANN and hybrid model of the ANN-WOA models for training and testing data.

Table 3 illustrates the findings of this study, which highlight that using the WOA to estimate the weight of neurons in the ANN has led to a significant improvement of 14.07% for the train and 10.71% for the test data's R^2 value. For a better understanding of the advancement in rock tensile fracture toughness estimation with helping of specimens' geometric parameters, crack length, and mean tensile strength of rocks in two models of the ANN and a hybrid model of the ANN-WOA, estimation of regression diagram and curve fitting of the ANN and ANN-WOA in train and test is obtained in **Figure 6** and **Figure 7** comparing the results of both models and the actual data.

It is evident in **Figure 7** that both models provide an acceptable estimate of the tensile fracture toughness of rocks in comparison to their experimental values. However, the hybrid model is more accurate than the ANN model.

Both models underwent 500 iterations. In addition, their respective fitness R^2 scores were evaluated for less than 100 iterations to assess their performance. This comparison can be used as a criterion to determine which model is more suitable for the given dataset with low iterations (see **Figure 10**).

Based on **Figure 10**, it is clear that the hybrid ANN-WOA model has a maximum accuracy of 18.9% higher than the ANN model in the number of iterations less than 100.

4. Conclusion

Tensile fracture toughness is one of the essential parameters in the design of a rock structure. This parameter can be determined through various laboratory methods. One of the concerns in determining this parameter is that it takes time and effort to prepare specimens. Therefore, researchers always try to provide suitable relations to estimate this parameter. One of these methods is establishing a relationship between the fracture toughness and other parameters of the rocks. In this study, 113 SCB specimens' geometrical parameters, crack length, and tensile strength data from various articles are collected and employed by the ANN and the hybrid model of the ANN-WOA. The results of this study show that the fracture toughness of rocks is dependent on the size of the specimen. Many studies by different researchers show that with an increase in the size of the SCB specimen, the fracture toughness of the rock increases, and there is also a relationship between the two parameters of tensile strength and the tensile fracture toughness of the rock. This study tries to estimate the fracture toughness Mode-I based on having the geometrical parameters of the specimen, crack length, and tensile strength of rocks with R^2 close to 1. The use of meta-heuristic algorithms in determining the weight of each neuron in the ANN permits to increase in the accuracy of the ANN models. The WOA has two essential features: the use of this algorithm for less data and the ability to search for factors randomly instead of only one factor. Employing the WOA in an ANN with one hidden layer and eight neurons increases the R^2-value of all data by 11.9%.

The present study demonstrated the practical implementation of two models to estimate the tensile fracture toughness of rock. In light of future research, it is recommended that these models can be further evaluated for their applicability in estimating the fracture toughness of modes II, III, and mixed modes, or through comparison with other hybrid models and diverse inputs. Such investigations would provide valuable insight to improve our understanding of the fracture mechanics of rock and enhance our capability to predict its strength and durability.

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SAŽETAK

Procjena lomne žilavosti stijene hibridnim ANN-WOA modelom

U inženjerstvu koje je vezano uz stijene koriste se različite tehnike za procjenu lomne žilavosti, koja je kritični parametar u ocjeni i projektiranju stabilnosti stijenske mase. Te metode obično uključuju laboratorijska ispitivanja i statističku analizu. Umjetne neuronske mreže također mogu uspostaviti korelacije između različitih skupova podataka. Primjena umjetne inteligencije postaje sve bitnija u svim područjima inženjerstva, uključujući i ona koja proučavaju mehaniku loma stijena. U ovome radu korištena je umjetna neuronska mreža sa skrivenim slojem i osam neurona te hibridna umjetna neuronska mreža s *whale* optimizacijskim algoritmom za određivanje lomne žilavosti stijena. Kako bi se razvili točni modeli, ova studija pažljivo je odabrala četiri temeljna parametra koji će poslužiti kao ulazni podatci. Ovi parametri uključuju polumjer, debljinu, duljinu pukotine i srednju vlačnu čvrstoću uzoraka. Također, za modele je prikupljeno 113 podataka o stijenama. Rezultati pokazuju da primjena optimizacijskoga algoritma povećava preciznost u procjeni lomne žilavosti stijena. Faktor međuovisnosti prije korištenja algoritma optimizacije iznosio je 0,81, a kada je primijenjen algoritam optimizacije, poboljšao se i iznosio je 0,93.

Ključne riječi:
lomna žilavost, umjetne neuronske mreže, *whale* optimizacijski algoritam, učinak veličine, vlačna čvrstoća

Author's contribution

Alireza Dolatshahi (PhD Candidate of Rock Mechanics): initialised the idea, provided the initial manuscript, completed the ANN and the hybrid Model, checked the results and tried to find the best cost of the models. **Hamed Molladavoodi** (Associate Professor) managed the whole process, supervised from the beginning to the end, and participated in all work stages.