Devčić, Kristina¹ Beljo, Ivana² Gaćina, Nikolina³

MINIMIZATION OF THE DIET COSTS ON THE THREE-DAY MENU EXAMPLE

Abstract:

Linear programming is a special field of applied mathematics that deals with solving optimization problems while certain conditions or constraints must be fulfilled. Aim of linear programming is to find a solution how to use or allocate a predetermined amount of resources with a predefined objective such as minimum cost, maximum revenue, maximum profit, etc. This paper presents the application of linear programming on the example of minimizing the diet costs. One of the basic human needs is the adequate daily intake of food in the body i.e. diet. In sufficient quantity food ensures the normal functioning of the human body, it gives energy for mental and physical work and raises the immune system. In this paper, the problem of diet is set to minimize the total cost of daily diet while the criteria for daily intake of a sufficient amount of all nutrients required for the normal functioning of the human body must be fulfilled.

Carbohydrates, fats and proteins give energy to the human body. Carbohydrates are first consumed, then fat, and only in the cases of starvation body translates carbohydrates into energy. Daily energy needs depend on gender, age, body mass, body height and total daily physical activity. Furthermore, a three-day menus for a group of young and middle-aged people, between 19 and 50, separately for a group of men and women is provided.

Keywords:

linear programming, costs, diet, minimization, nutrients

Author's data:

- ¹ Kristina, Devčić, prof., univ.spec.oec., viši predavač, Polytechnic Gospić, Bana Ivana Karlovića 16, 53000 Gospić, kdevcic@velegs-nikolatesla.hr;
- ² Ivana, Beljo, dipl.ing.mat., Polytechnic Šibenik, Trg Andrije Hebranga 11, 22000 Šibenik, ibeljo@vus.hr;
- ³ Nikolina, Gaćina, mag.ing. techn.aliment., Polytechnic Šibenik, Trg Andrije Hebranga 11, Šibenik, nikolina@vus.hr



Introduction

Nowadays, with the help of popular softwares such as WinQSB, LINDO, MS Office Excel and other, linear programming has found its application in a large number of scientific disciplines and areas of human activities such as production, transportation and distribution, marketing, telecommunications, financial investment and planning, employee scheduling and other. This paper presents the application of linear programming on the example of minimization of the costs of diet with the aim of meeting healthy and variety diet requirements.

We have witnessed the consequences of the global finance and economy crisis that began in 2007. Ten years after the start of the crisis, some national economies are still in the process of recovery as well as the economy of the Republic of Croatia. The recovery of the national economy is closely related both to the recovery of the real sector and to the growth of household consumption. House budgets will affect decisions about purchasing goods and services, including food. On the other hand, postulates of healthy and varied diet should not be neglected. The aim of this work is to minimize the costs of diet with the criteria for daily intake of a sufficient quantity of all nutrients required for the normal functioning of the human body are fulfilled.

Linear programming

The problem of linear programming is a special case of problem of mathematical programming in which the objective function is linear, and the constraints are expressed in the form of linear equations and/or inequalities [1]. The objective in the problem of linear programming is to maximize or minimize the objective function:

$$\max/\min z(x_1, x_2, ..., x_n) = c_1 x_1 + c_2 x_2 + ... + c_n x_n$$

with defined constraints

$$\begin{aligned} &a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1, \\ &a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2, \\ &\vdots \\ &a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m, \\ &x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0. \end{aligned}$$

The objective function (1) is linear, where c_j , j=1,2,...,n are coefficients of the objective function, and x_j , j=1,2,...,n are structural variables. The constraints are also linear and are expressed in the form of linear inequalities.

The problem of linear programming can be written as

$$\max z = \mathbf{c}^T \mathbf{x} \tag{3}$$

with defined constraints

$$Ax \le b$$
 (4) $x \ge 0$

where

$$\mathbf{c} = \begin{bmatrix} c_{1} \\ c_{2} \\ \vdots \\ c_{n} \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{bmatrix}. [5]$$



Linear programming addresses a large number of economic problems, they can refer to production, raw materials, labor force, market, supply, demand, imports, exports [6]. Also, there is a significant application of linear programming in the optimization of diet, transport and distribution, labor scheduling, choice of production assortment, ingredient mix problem and financial portfolio selection.

Nutrients

One of the basic human needs is the adequate daily intake of food in the body i.e. diet. In sufficient quantity food ensures the normal functioning of the human body, energy for mental and physical work and raises the immune system [5]. Nutrients include proteins, carbohydrates, fats, vitamins, minerals and water. Nutrients can be classified into two basic categories which are macronutrients and micronutrients. Macronutrients are carbohydrates. fats and proteins that are present in food in the largest share. Carbohydrates and fats supply body cells with energy, while proteins build cells and tissues [4]. Micronutrients are vitamins, minerals and trace elements that are present in food in very small quantities and are essential for the functioning of important body processes [2]. Balanced diet involves intake of 15-20% protein, 55-60% carbohydrate and 20-25% fat [3].

Proteins

Proteins are the main building elements of the cells and represent 16 to 19 percent of the body mass of the average adult man. They are essential for growth and regeneration of tissues and participate in all biochemical processes in the cells. An important function of the proteins is to replace damaged and dead cells, for example blood cells, kidneys, liver, muscles, hair, nails, teeth and bones.

The proteins are made of amino acids. The main sources of protein in food are meat, fish, eggs, milk, milk products, legumes, beans, lentils, wheat, rice, corn, barley, oats, rye.

Carbohydrates

Carbohydrates are present in food of plant origin, for example in fruits and vegetables, cereals and cereal products, milk, milk products, herbs, nuts, juices, jams, sweets, etc. According to the molecular composition, carbohydrates are divided into monosaccharide, disaccharides polysaccharides, and by degree of digestibility into digestive and indigestive. Starch, sugars, polyols, alcoholic sugars and etc are examples of digestive carbohydrates, while indigestive carbohydrates are found mainly in food of plant origin, for example cellulose, hemicelluloses, lignin, pectin, gum, and mucilage from plant juices. These substances are referred to as fibers.

Fats

The fats are rich, and with sugars, the cheapest source of energy. They enable the absorption of fat soluble substances, primarily vitamins A, D, E and K, which are fat soluble vitamins, and are also important for the synthesis of some hormones. The term fat refers to lipids, triglycerides, phospholipids, sterols, etc. The fat appears in two forms: liquid and solid aggregate state. Liquid fats are oils, and are derived from food of plant origin (sunflower, olives, oilseed rape, flax, wheat, corn and wheat germ). Solid fats are derived from animal products (pork fat, poultry fat and butter).

Vitamins and minerals

Vitamins and minerals in the organism are introduced in significantly smaller amounts



compared to the macronutrients, and are therefore called micronutrients. Micronutrients are essential for normal growth and development of the human body. Considering the solubility of the vitamins, we divide them into two groups: water-soluble vitamins (B-complex vitamin and vitamin C) and fat-soluble vitamins (A, D, E and K).

B-group vitamins are essential for converting carbohydrates, fats and proteins into energy as well as for their use for building and restoring the body tissue. They are found in dark green vegetables, beans, peas, cereals, meat, fish and eggs. Vitamin C is primarily an antioxidant, essential for the absorption of iron from food and the production of collagen that connects body cells. Citrus and kiwis are a rich source of vitamin C, and in vegetables in peppers in the largest amount. Vitamin D is the only vitamin that the human body can completely independently synthesize. One of the most important functions of vitamin D in the human body is absorption of calcium and phosphorus. It can be found in fish oil, eggs and milk. Vitamin A helps in growth and differentiation of epithelial tissue of the respiratory, urogenital and digestive system and in the effective functioning of the immune system, and is found in the liver, eggs and milk products.

From all minerals we can emphasize iodine and iron. Iodine is essential for normal growth and development of the thyroid gland, i.e. its main tyrosine hormone. Rich iodine sources are sea food such as sea salt, algae, fish and shellfish. The lack of iron in human diet is manifested as anemia. Animal origin food (primarily red meat) contains more useful iron than food of plant origin (legumes, integral cereals)

Proper diet

In order for the diet to be proper, it must fulfill the needs of daily energy intake and all nutrients necessary for the normal functioning of the body as well as its physical and mental health. There are numerous recommendations in the literature on what is considered a healthy diet. The most important recommendations include the following:

- take into account your body weight,
- have daily physical activity,
- eat various food following the food pyramid (Figure 1).
- eat cereal products, especially from integral grains,
 - eat fruits and vegetables daily,
- choose less grease food,
- choose less sweet food and drinks,
- put less salt in food,
- moderately consume alcoholic drinks,
- make sure that food is healthy [4].



Figure 1. Healthy diet pyramid.

Source: https://www.cdm.nhg.com.sg/Diabetes/ Pages/Eating-a-healthy-diet.aspx# (April 22, 2017)

Planning a healthy diet involves a lot of knowledge about the energy values of the food and the properties of each individual food item. Energy comes from the food by metabolizing fat, proteins and carbohydrates, and it is determined as a heat needed that one gram of water at a pressure of one atmosphere is warmed by 1 °C. Energy is measured in kilojoules (kJ) or kilocalories (kcal) where 1 kcal = 4.184 kJ or 1 kJ = 0.2388 kcal. Table 1 shows the energy value of one gram of each nutrient.



Type of nutrient	Energy value
Proteins	4 kcal/g (17kJ/g)
Fats	9 kcal/g (37kJ/g)
Carbohydrates	4 kcal/g (17kJ/g)
Vitamins and minerals	0

Table 1. Energy value of nutrients.

Source: Nutrition Value Guide, Ministry of Agriculture http://www.mps.hr/UserDocsImages/HRANA/Vodi%C4%8D%200%200%20navo%C4%91en ju%20hranjivih%20vrijednosti%20hrane%20-2.%20izdanje.pdf (April 15, 2017)

Table 1 shows that proteins, fats and carbohydrates have an energy value, while vitamins and minerals are not a source of energy. As we continue to observe the energy value of nutrients, we did not take vitamins and minerals into account, only proteins, fats and carbohydrates are included.

Analysis area

Carbohydrates, fats and proteins are the main sources of energy in the human body. Carbohydrates are first consumed, then fat, and only in the cases of starvation body translates proteins into energy. Daily energy needs depend on gender, age, body mass, body height and total daily physical activity.

In this paper younger and older adults (from 19 to 50 years) are observed, depending on gender, men and women. Table 2 shows the average daily energy intake of nutrients depending on the age and gender of the person.

Type of nutrient	Label of nutritient	Recommended average daily intake of nutrients			
		19 - 50 Men	19 - 50 Women		
Energy (kcal)	H ₁	2300	1900		
Carbohydr ates (g)	H ₂	316.25- 345	261.25-285		
Proteins (g)	Н3	57.5-86	47-71		
Fats (g)	H ₄	63-76	52-63		

Table 2: Recommended average daily intake of nutrients.

Source: author's calculation.

For healthy diet it is very important to be in accordance with the healthy diet pyramid and to be diverse. Table 3 covers 55 food items that are commonly part of daily diet. The values of energy (expressed in kilocalories), the amounts of protein, fat, carbohydrate (expressed in gram) per 100 gram of every food item are given. In addition to the above in table 3 the corresponding average prices of certain food items are also shown.

Food item (100g)	Label	Energy (kcal)	Carbohydrates (g)	Proteins (g)	. Eg	Price (Kuna)
Milk (0,9% milk fat)	A 1	40	4,7	3,3	0,9	0,65
Milk (3,2% milk fat)	A_2	66	4,7	3,3	3,2	0,62
Yogurt (2.8% milk fat)	A 3	40	5	4	4	1,20
Sour cream (12% milk fat)	A 4	192	3	3	18	2,20
Cream cheese (23% mm)	A_5	115	6	13	5	4,99
Fresh cow cheese	A_6	72	4	15	3	3,47
Gauda cheese	A 7	367	0	27	30	1,63
Mozzarella cheese	A_8	227	0	20	17	8,26
Liver pate	A 9	440	1	12	40	6,50
Ham	A ₁₀	147	0	23	6	6,68
Chicken breasts	A 11	110	0	23	1	7,50
Turkey breasts	A ₁₂	111	0	25	1	7,50
Pork steak	A 13	120	0	21	4	6,00
Mixed meat, ground	A 14	253	0	20	19	4,00
Egg	A 15	167	1	13	11	1,52
Hake	A ₁₆	88	0	17,2	0,85	5,05
Cod	A 17	76	0	17	1	5,20
Trout	A ₁₈	112	0	18	2	4,00
Sardine in oil	A 19	240	1	24	14	9,51
Tuna in oil	A ₂₀	303	0	24	21	13,82
Rice	A ₂₁	368	79	7	1	2,03
Oat flakes	A ₂₂	402	66	14	7	1,20
White bread	A ₂₃	237	47	7	2	1,48
Corn bread	A ₂₄	220	31	5	9	1,90
Rye bread	A ₂₅	222	47	7	1	1,82
Pasta	A ₂₆	390	72	13	3	1,85
Apple	A ₂₇	52	12	0	0	1,00
Pear	A ₂₈	55	12	0	0	1,70
Pineapple	A ₂₉	56	13	0	0	2,00
Orange	A ₃₀	54	9	1	0	0,90
Banana	A ₃₁	99	23	1	0	1,20
Dry fruits	A ₃₂	252	56	3	1	4,16
Jam	A33	261	66	0	0	2,38
Tomato	A 34	19	3	1	0	2,00



Food item (100g)	Label	Energy (kcal)	Carbohydrates (g)	Proteins (g)	Fat (g)	Price (Kuna)
Red onion	A ₃₅	42	9	1	0	0,50
Leek	A ₃₆	38	6	2	0	0,30
Pepper	A ₃₇	28	5	1	0	3,00
Mushrooms	A38	24	3	3	0	0,32
Salad	A39	14	2	1	0	0,30
Cucumber	A 40	10	2	1	0	1,30
Carrot	A ₄₁	35	7	1	0	0,70
Potato	A 42	85	19	2	0	0,85
Pork fat	A ₄₃	900	0	0	100	2,00
Butter	A 44	755	0	1	83	6,64
Margarine	A ₄₅	720	0	0	81	2,30
Sunflower oil	A 46	928	0	0	100	1,10
Pumpkin oil	A 47	900	0	0	100	12,28
Olive oil	A48	900	0	0	100	9,93
Chocolate for cooking	A 49	564	63	14	28	4,39
Orange juice	A ₅₀	47	11	1	0	0,95
Apple juice	A ₅₁	47	12	1	0	0,95
Bright beer	A ₅₂	45	4	1	0	1,07
Red wine	A ₅₃	66	0	0	0	3,19
Wight wine	A ₅₄	70	0	0	0	4,91
Boiled water	A ₅₅	0	0	0	0	0,58

Table 3: Energy value of nutrients, amount of carbohydrates, proteins, fats ad associated unit prices.

Izvor: http://www.vjezbaj.com/kalorijska-tablica/ (April 20, 2017)

http://www.fitness.com.hr/prehrana/nutricionizam/Tablica-kalorija.aspx (April 20, 2017)

https://www.konzum.hr (April 20, 2017)

In order to solve the diet problem by applying linear programming, it is necessary to determine the objective function as well as all the conditions and constraints that must be fulfilled. Let the choice of food items is made from Table 3 and let $x_1, x_2, ..., x_n$ are quantities of each food item $A_1, A_2, ..., A_n$, where n = 55. Furthermore, let $c_1, c_2, ..., c_n$ be unit prices (prices per 100 g) of each individual food item $A_1, A_2, ..., A_n$. The objective function is

$$\min z(x_1, x_2, ..., x_n) = c_1 x_1 + ... + c_n x_n.$$
 [6]



This function provides the minimum value of total food price required for one day, and the constraints that must be fulfilled are obtained from the recommended daily amounts of nutrients. Let b_j be recommended daily amounts of nutrients H_j and let a_{ij} be amounts of protein, fat, carbohydrates shown in Table 3, where $i=1,\dots,55$, and j=1,2,3,4. In the label a_{ij} the index i is a label for food item A_i , and the index j is a label for a type of nutrient H_j .

In order to recommended daily intake of all nutrients be fulfilled, the following conditions must be met

where j = 1,2,3,4 indicate the type of nutrient.

Additional conditions are also introduced

$$x_{i} \geq 0, \ \forall i = 1,...,55;$$

$$x_{i} \leq 0.3 \ \text{ za} \quad i = 1,2,50,51,55;$$

$$x_{i} \leq 0.2 \ \text{ za} \quad i = 3,4,33,52,53,54;$$

$$x_{i} \leq 0.5 \ \text{ za} \quad i = 5,9,32;$$

$$x_{i} \leq 1 \ \text{ za} \quad i = 6,7,8,19,20,22,...,25,35,36,41;$$

$$x_{i} \leq 0.1 \ \text{ za} \quad i = 10,46,...,49;$$

$$x_{i} \leq 2.5 \ \text{ za} \quad i = 11,...,14,38;$$

$$x_i \le 3.5$$
 za $i = 16,17,18$;
 $x_i \le 2$ za $i = 21,26,...,31,37,40,42$;
 $x_{34} \le 1.5$;
 $x_{39} \le 3$;
 $x_{15} \le 1.8$;
 $x_i \le 0.15$ za $i = 43,44,45$.

There is a condition of non negativity of structure variables x_i , $\forall i = 1,...,55$ to avoid meaningless solutions, since the amounts of food

cannot have negative values, and constraints to the maximum amounts of each food item in order to make the menus as diverse as possible have been introduced. In addition, in order to make the menus as varied as possible, with the daily constraints of the daily intake of certain food items, the restriction of the repeating of main food items in the three-day menu was introduced such as: types of milk and milk products, types of meat, type of fish and its products and types of bread. The diet problem is solved using Solver in MS Excel.

Three-days menus

Conducted analysis provided three-day menus for men and women between 19 and 50 years. When calculating, and respecting all the above constraints, it was taken into account that in three-days menus for both groups, at least one meal (lunch or dinner) is a fish, one vegetable meal (a meal that does not include meat and fish as well as their produces). Also, when calculation it was taken into account the minimum price of the daily menus since the lowest possible price violates the postulates of healthy diet.

Three-day diet menu for men between 19 and 50 years

For men aged 19 to 50 years with meeting of all the above constraints, the following three-day menu is obtained:

1st day - 22,21 kuna

BREAKFAST:

2 dcl orange juice, 13 g gauda cheese, 100 g corn bread, 1 apple, 10 g chocolate for cooking

LUNCH:

1 dcl sour cream, 50 g turkey breasts, 100 g pasta, 250 g mushrooms, 1 dcl olive oil, 3 dcl apple juice DINNER:



3 dcl milk, 100 g oat flakes, 50 g dry fruits

2nd day - 33,29 kuna

BREAKFAST:

2 dcl milk 3,2% mm, 10g margarine, 20g jam, 100 g white bread, 15 g dry fruits

LUNCH:

50 g pork steak, 2 potatoes, 1 onion, 1 tomato, 1 pepper, 1 cucumber, 100 g salad, 0,3 dcl sunflower oil, 2 dcl boiled water

DINNER:

0,5 dcl sour cream, 100 g pasta, 1 leek, 1 pepper, 3 dcl boiled water

3rd day - 42,07 kuna

BREAKFAST:

2 dcl yogurt, 50 g ham, 50 g rye bread, 10 g margarine, 1 banana

LUNCH:

150 g trout, 100 g white breaad, 1 tomato, 1 cucumber, 0,3 dcl sunflower oil, 2 dcl red wine

DINNER:

1 egg, 50 g tuna in oil, 100 g rice, 1 tomato, 2 dcl boiled water

Three-day menu for men is sorted by price, ranging from 22.21 kuna to 42.07 kuna.

Three-day diet menu for women between 19 and 50 years

For women aged 19 to 50 years with meeting of all the above constraints, the following weekly threeday diet menu is obtained:

1st day - 19,82 kuna

BREAKFAST:

2 dcl milk 3,2% mm, 50 g oat flakes, 10 g chocolate for cooking, 1 orange

LUNCH:

80 g mixed meat, 100 g pasta, 1 tomato, 1 onion, 0,1 dcl olive oil, 2 dcl apple juice DINNER:

0,8 dcl sour cream, 120 g mushroom, 1 leek, 1 potato, 2 dcl boiled water

2nd day - 31,10 kuna

BREAKFAST:

3 dcl orange juice, 30 g mozzarella, 80 g corn bread, 15 g tomato

LUNCH:

50 g pork steak, 2 potato, 1 onion, 200 g salad, 1 carrot, 0,3 dcl sunflower oil, 2 dcl orange juice

DINNER:

3 dcl yogurt, 30 g dry fruit, 1 banana

3rd day - 31,22 kuna

BREAKFAST:

2 dcl yogurt, 15 g gauda cheese, 15 g ham, 50 g rye bread, 1 apple

LUNCH:

100 g trout, 150 g rice, 1 leek, 0,2 dcl sunflower oil, 3 dcl boiled water

DINNER:

2 eggs, 1 tomato, 1 pepper, 0,1 dcl olive oil, 30 g rye bread, 2dcl boiled water

Three-day menu for men is sorted by price, ranging from 19.82 kuna to 31.22 kuna.

Conclusion

This paper deals with linear programming as a special case of mathematical programming. The objective function and constraints in the linear programming problem are given in the form of linear equations or inequalities. In this paper, special emphasis is placed on the application of linear programming with the aim of minimizing the diet costs on the example of a three-day menu, with the aim of meeting the conditions of healthy and varied diet. Objective function is defined as a total value of the daily diet, expressed in kunas. Constraints are



defined in accordance with healthy diet postulates on recommended daily intake of nutrients. The paper proposes three-day menus for men from 19 to 50 years and for women from 19 to 50 years. The results showed that the price of a healthy and varied daily menu for men ranges from 22.21 kuna to 42.07 kuna and for women from 19.82 kuna to 31.22 kuna. The reason for lower daily menu prices for women is related to a lower amount of recommended average daily intake of nutrients (carbohydrates, proteins and fat) than is the case with men.

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